NCERT Solutions for Class-XI Maths

Chapter-1 Exercise-Miscellaneous

NCERT Math Class 11

1. Decide, among the following sets, which sets are subsets of one and another:

$A=\{x:x\in R \text{ and } x \text{ satisfy } x^2-8x+12=0\}, B=\{2,4,6\},$

$C=\{2,4,6,8\ldots\}, D=\{6\}$

1. $A=\{x:x\in R \text{ and } x \text{ satisfies } x^2-8x+12=0\}$

2 and 6 are the only solutions of $x^2-8x+12=0$.

$\therefore A=\{2,6\}$

$B=\{2,4,6\}, C=\{2,4,6,8\ldots\}, D=\{6\}$

$\therefore D\subset A\subset B\subset C$

Hence, $A\subset B, A\subset C, B\subset C, D\subset A, D\subset B, D\subset C$

2. In each of the following, determine whether the statement is true or false. If it is true, prove it. If it is false, give an example:

(i) If x ∈ A and A ∈ B, then x ∈ B

(ii) If A ⊂ B and B ∈ C, then A ∈ C

(iii) If A ⊂ B and B ⊂ C, then A ⊂ C

(iv) If A ⊄ B and B ⊄ C, then A ⊄ C

(v) If x ∈ A and A ⊄ B, then x ∈ B

(vi) If A ⊂ B and x ∉ B, then x ∉ A

2. (i) If x ∈ A and A ∈ B, then x ∈ B

Solution: Let us assume A = {1, 2}

And, B = {1, {1, 2}, {3}}

So, $2\in\{1,2\}$

And, $\{1,2\}\in\{\{3\},1,\{1,2\}\}$

$\therefore A\in B$

But, $2\notin\{\{3\},1,\{1,2\}\}$

Hence, the given statement is false

(ii) If A ⊂ B and B ∈ C, then A ∈ C

Solution: Let us assume, A {2}

B = {0, 2}
And, C = {1, {0, 2}, 3}
It is given in the question that,
$A \subset B$
$\therefore B \in C$
But, $A \notin C$
Hence, the givgen statement is false

(iii) If A $\subset$ B and B $\subset$C, then A $\subset$ C
Solution: It is given in the question that,
$A \subset B \text{and} B \subset C$
Let us assume, $x \in A$
So, $x \in B$
And, $x \in C$
$\therefore A \subset C$
Hence, the given statement is correct

(iv) If A $\not\subset$ B and B $\not\subset$C, then A $\not\subset$ C
Solution: It is given in the question that,
A $\not\subset$ B
And, B $\not\subset$ C
Let us now assume, A = {1, 2}
B = {0, 6, 8}
And, C = {0, 1, 2, 6, 9}
$\therefore A \subset C$
Hence, the given statement is false

(v) If x $\in$ A and A $\not\subset$B, then x $\in$ B
Solution: It is given in the question that,
$x \in A$
And, A $\not\subset$ B
Let us now assume, A = {3, 5, 7}
And, B = {3, 4, 6}
As, $5 \in A$
And, $A \not\subset B$
$\therefore 5 \notin B$
Hence, the given statement is false

(vi) If A $\subset$ B and x $\notin$ B, then x $\notin$ A

Solution: It is given in the question that,
$A \subset B$
And, $x \notin B$
Let us suppose, $x \in A$ then we have:
$x \in B$
But it is given in the question that, $x \notin B$
$\therefore x \notin A$
Hence, the given statement is true

3. Let A, B and C be the sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$. show that $B = C$
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3. Let, A, B and C be the sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$.
To show: $B = C$
Let $x \in B$
$\Rightarrow x \in A \cup B [B \subset A \cup B]$
$\Rightarrow x \in A \cup C [A \cup B = A \cup C]$
$\Rightarrow x \in A$ or $x \in C$
Case I $x \in$
A
Also, $x \in B$
$x \in A \cap B$
$\Rightarrow x \in A \cap C [Q\ A \cap B = A \cap C]$
$\therefore x \in A$ and $x \in C$
$\therefore x \in C$
$\therefore B \subset C$
Similarly, we can show that $C \subset B$.
$\therefore B = C$

4. Show that the following four conditions are equivalent:

(i) $A \subset B$(ii) $A - B = \phi$ (iii) $A \cup B = B$
(iv) $A \cap B = A$

4. Here, first we will prove (i) ⇔ (ii)
Where, (i) = $A \subset B$ and (ii) = $A - B \neq \phi$
Let us assume that $A \subset B$
Now, we need to prove $A - B \neq \phi$
If possible, let $A - B \neq \phi$
Thus, there exists $X \in A$, $X \neq B$, but this is impossible as $A \subset B$

∴ $A - B = \phi$

And $A \subset B \Rightarrow A - B \neq \phi$

Let us assume that $A - B \neq \phi$

Now, to prove: $A \subset B$

Let $X \in A$

It can be concluded that $X \in B$ (if $X \notin B$, then $A - B \neq \phi$)

Thus, $A - B = \phi \Rightarrow A \subset B$

∴(i) ⇔ (ii)

Let us assume that $A \subset B$

To prove: $A \cup B = B$

$\Rightarrow B \subset A \cup B$

Let us assume that, $x \in A \cup B$

$\Rightarrow X \in A$ or $X \in B$

Taking Case I: $X \in B$

$A \cup B = B$

Taking Case II: $X \in A$

$\Rightarrow X \in B$ $(A \subset B)$

$\Rightarrow A \cup B \subset B$

Let $A \cup B = B$

Let us assume that $X \in A$

$\Rightarrow X \in A \cup B$ $(A \subset A \cup B)$

$\Rightarrow X \in B$ $(A \cup B = B)$

∴$A \subset B$

Thus, (i) ⇔ (iii)

Now, to prove (i) ⇔ (iv)

Let us assume that $A \subset B$

It can be observed that $A \cap B \subset A$

Let $X \in A$

To show: $X \in A \cap B$

Since, $A \subset B$ and $X \in B$

Thus, $X \in A \cap B$

$\Rightarrow A \subset A \cap B$

$\Rightarrow A = A \cap B$

Similarly, let us assume that $A \cap B = A$

Let $X \in A$

$\Rightarrow X \in A \cap B$

$\Rightarrow X \in B$ and $X \in A$

$\Rightarrow$ $A \subset B$

$\therefore$ (i) $\Leftrightarrow$ (iv)

Hence, proved that (i)$\Leftrightarrow$ (ii)$\Leftrightarrow$ (iii)$\Leftrightarrow$ (iv)

5. Show that if $A \subset B$, then $C - B \subset C - A$.

5. Let $A \subset B$

To show: $C - B \subset C - A$

Let $x \in C - B$

$\Rightarrow x \in C$ and $x \notin B$

$\Rightarrow x \in C$ and $x \notin A[A \subset B]$

$\Rightarrow x \in C - A$

$\therefore C - B \subset C - A$

6. Assume that P (A) = P (B). Show that A = B

6. We have to show that: A = B

Let us now assume, P (A) = P (B)

Let, $x \in A$

$A \in P(A) = P(B)$

$\therefore$ For any $C \in P(B)$ we have, $x \in C$

Now, we have:

$C \subset B$

$\therefore$ $x \in B$

$\therefore$ $A \subset B$

Similarly, we have:

$B \subset A$

$\therefore$ A = B

7. Is it true that for any sets A and $B, P(A) \cup P(B) = P(A \cup B)$? Justify your answer.

7. False

Let $A = \{0,1\}$ and $B = \{1,2\}$

$\therefore A \cup B = \{0,1,2\}$

$P(A) = \{\Phi, \{0\}, \{1\}, \{0,1\}\}$

$P(B) = \{\Phi, \{1\}, \{2\}, \{1,2\}\}$

$P(A \cup B) = \{\Phi, \{0\}, \{1\}, \{2\}, \{0,1\}, \{1,2\}, \{0,2\}, \{0,1,2\}\}$

$P(A) \cup P(B) = \{\Phi, \{0\}, \{1\}, \{0,1\}, \{2\}, \{1,2\}\}$

$\therefore P(A) \cup P(B) \neq P(A \cup B)$

8. Show that for any sets A and B,
A = (A ∩ B) È (A – B) and A ∪ (B – A) = (A ∪ B)

8. To Prove: $A = (A \cap B) \cup (A - B)$
Proof: Let $X \in A$
Now, we need to show that $X \in (A \cap B) \cup (A - B)$
In Case I,
$X \in (A \cap B)$
$\Rightarrow X \in (A \cap B) \subset (A \cup B) \cup (A - B)$
In Case II,
$X \notin A \cap B$
$\Rightarrow X \notin B$ or $X \notin A$
$\Rightarrow X \notin B\ (X \notin A)$
$\Rightarrow X \notin A - B \subset (A \cup B) \cup (A - B)$
$\therefore A \subset (A \cap B) \cup (A - B)$ (i)
It can be concluded that, $A \cap B \subset A$ and $(A - B) \subset A$
Thus, $(A \cap B) \cup (A - B) \subset A$ (ii)
Equating (i) and (ii),
$A = (A \cap B) \cup (A - B)$
Now, we need to show, $A \cup (B - A) \subset A \cup B$
Let us assume that,
$X \in A \cup (B - A)$
$X \in A$ or $X \in (B - A)$
$\Rightarrow X \in A$ or ($X \in B$ and $X \notin A$)
$\Rightarrow$ ($X \in A$ or $X \in B$) and ($X \in A$ and $X \notin A$)
$\Rightarrow X \in (B \cup A)$
$\therefore A \cup (B - A) \subset (A \cup B)$ (iii)
Now, to prove: $(A \cup B) \subset A \cup (B - A)$
Let $y \in A \cup B$
$y \in A$ or $y \in B$
($y \in A$ or $y \in B$) and ($X \in A$ and $X \notin A$)
$\Rightarrow y \in A$ or ($y \in B$ and $y \notin A$)
$\Rightarrow y \in A \cup (B - A)$
Thus, $A \cup B \subset A \cup (B - A)$ (iv)
$\therefore$Using (iii) and (iv), we get:
$A \cup (B - A) = A \cup B$

9. Using properties of sets show that
(i) $A\cup(A\cap B)=A$ (ii) $A\cap(A\cup B)=A$.

9. (i) To show: $A\cup(A\cap B)=A$
We know that
$A\subset A$
$A\cap B\subset A$
$\therefore A\cup(A\cap B)\subset A\ldots(1)$
Also, $A\subset A\cup(A\cap B)\ldots(2)$
$\therefore$ From (1) and $(2), A\cup(A\cap B)=A$
(ii) To show: $A\cap(A\cup B)=A$
$A\cap(A\cup B)=(A\cap A)\cup(A\cap B)$
$=A\cup(A\cap B)$
$=A$ { from (1) }

10. Show that A ∩ B = A ∩ C need not imply B = C.
10. Let us assume, A = {0, 1}
B = {0, 2, 3}
And, C = {0, 4, 5}
Now, accordingly we have:
$A\cap B=\{0\}$
And, $A\cap C=\{0\}$
$\therefore A\cap B=A\cap C=\{0\}$
Also, $B\neq C$ $(\text{As}, 2\in B \text{ and } 2\notin C)$

11. Let A and B be sets. If $A\cap X=B\cap X=\Phi$ and $A\cup X=B\cup X$ for some set X, show that $A=B$.
(Hints $A=A\cap(A\cup X), B=B\cap(B\cup X)$ and use distributive law)

11. Let A and B be two sets such that $A\cap X=B\cap X=f$ and $A\cup X=B\cup X$ for some set X.
To show: $A=B$
It can be seen that
$A=A\cap(A\cup X)=A\cap(B\cup X)[A\cup X=B\cup X]$
$=(A\cap B)\cup(A\cap X)$ [Distributive law]

$=(A\cap B)\cup\Phi[A\cap X=\Phi]$

$=A\cap B$

Now, $B=B\cap(B\cup X)$

$=B\cap(A\cup X)[A\cup X=B\cup X]$

$=(B\cap A)\cup(B\cap X)$ [Distributive law]

$=(B\cap A)\cup\Phi[B\cap X=\Phi]$

$=B\cap A$

$=A\cap B$

Hence, from (1) and (2), we obtain $A=B$.

12. Find sets A, B and C such that A ∩ B, B ∩ C and A ∩ C are non-empty sets and A ∩ B ∩ C = f.

12. Let us assume, A {0, 1}

B = {1, 2}

And, C = {2, 0}

Now, accordingly we have:

$A\cap B=\{1\}$,

$B\cap C=\{2\}$

And, $A\cap C=\{0\}$

$\therefore$ $A\cap B, B\cap C$ and $A\cap C$ are not empty sets

Hence, $A\cap B\cap C=\varnothing$

13. In a survey of 600 students in a school, 150 students were found to be taking tea and 225 taking coffee, 100 were taking both tea and coffee. Find how many students were taking neither tea nor coffee?

13. Let U be the set of all students who took part in the survey.

Let T be the set of students taking tea.

Let C be the set of students taking coffee.

Accordingly, $n(U)=600, n(T)=150, n(C)=225, n(T\cap C)=100$

To find: Number of student taking neither tea nor coffee i.e., we have to find $n(T'\cap C')$

.

$n(T'\cap C')=n(T\cup C)'$

$=n(U)-n(T\cup C)$

$=n(U)-[n(T)+n(C)-n(T\cap C)]$

$=600-[150+225-100]$

$=600-275$

$=325$

Hence, 325 students were taking neither tea nor coffee.

14. In a group of students, 100 students know Hindi, 50 know English and 25 know both. Each of the students knows either Hindi or English. How many students are there in the group?

14. Let us assume U be the set of all students in the group

Let E be the set of students who know English

And, let H be the set of the students who know Hindi

$\therefore H\cup E=U$

Given that, number of students who know Hindi n (H) = 100

Number of students who knew English, n (E) = 50

Number of students who know both, $n(H\cap E)=25$

We have to find the total number of students in the group i.e. n (U)

$\therefore n(U)=n(H)+n(E)-n(H\cap E)$

= 100 + 50 – 25

= 125

$\therefore$ Total number of students in the group = 125 students

15. In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspaper T, 26 read newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, 3 read all three newspapers. Find:

(i) the number of people who read at least one of the newspapers.

(ii) the number of people who read exactly one newspaper.

15. Let A be the set of people who read newspaper H. Let B be the set of people who read newspaper T. Let C be the set of people who read newspaper I. Accordingly,

$n(A)=25, n(B)=26, n(C)=26, n(A\cap C)=9, n(A\cap B)=11,$

$n(B\cap C)=8, n(A\cap B\cap C)=3$

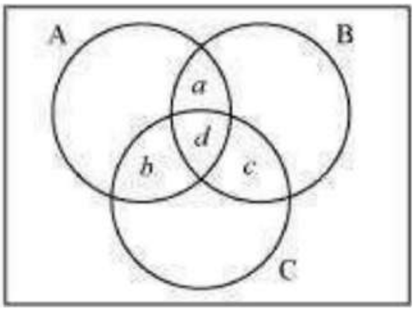
Let U be the set of people who took part in the survey.

(i) Accordingly,

$n(A\cup B\cup C)=n(A)+n(B)+n(C)-n(A\cap B)-n(B\cap C)-n(C\cap A)+n(A\cap B\cap C)=25+26+26-11-8-9+3=52$

Hence, 52 people read at least one of the newspapers.

(ii) Let a be the number of people who read newspapers H and T only.



Let b denote the number of people who read newspapers I and H only.
Let c denote the number of people who read newspapers T and I only.
Let d denote the number of people who read all three newspapers.

Accordingly, $d = n(A \cap B \cap C) = 3$

Now, $n(A \cap B) = a + d$

$n(B \cap C) = c + d$

$n(C \cap A) = b + d$

$\therefore a + d + c + d + b + d = 11 + 8 + 9 = 28$

$\Rightarrow a + b + c + d = 28 - 2d = 28 - 6 = 22$

Hence, $(52 - 22) = 30$ people read exactly one newspaper.

16. In a survey it was found that 21 people liked product A, 26 liked product B and 29 liked product C. If 14 people liked products A and B, 12 people liked products C and A, 14 people liked products B and C and 8 liked all the three products. Find how many liked product C only.

16. Let us first assume A, B and C be the set of people who like product A, product B and product C respectively

Now, it is given in the question that:

Number of students who like product A, n (A) = 21

Number of students who like product B, n (B) = 26

Number of students who like product A, n (C) = 29

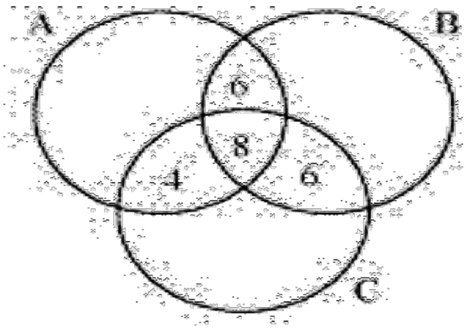
Number of students who like both product A and B, $n(A \cap B) = 14$

Number of students who like both product A and B, $n(C \cap A) = 12$

Number of students who like both product A and B, $n(B \cap C) = 14$

Number of students who like all three product, $n(A \cap B \cap C) = 8$

On the basis of given condition the venn diagram for the following question can be drawn as follows:



From the venn diagram, it can clearly be seen that:
Number of students who only like product C = {29 – (4 + 8 + 6)}
= {29 – 18}
= 11 students