Exercise 7(A)

Solution 1:

(i)

$$3^3 \times (243)^{-\frac{2}{3}} \times 9^{-\frac{1}{3}} = 3^3 \times (3\times3\times3\times3\times3)^{-\frac{2}{3}} \times (3\times3)^{-\frac{1}{3}}$$

$$= 3^3 \times (3^5)^{-\frac{2}{3}} \times (3^2)^{-\frac{1}{3}}$$

$$= 3^3 \times 3^{\left(-\frac{10}{3}\right)} \times 3^{-\frac{2}{3}} \qquad [(a^m)^n = a^{mn}]$$

$$= 3^{3-\frac{10}{3}-\frac{2}{3}} \qquad [a^m \times a^n \times a^o = a^{m+n+o}]$$

$$= 3^{\frac{9-10-2}{3}}$$

$$= 3^{\frac{9-12}{3}}$$

$$= 3^{-\frac{3}{3}}$$

$$= 3^{-1}$$

$$= \frac{1}{3}$$

(ii)

$$5^{-4} \times (125)^{\frac{5}{3}} \div (25)^{-\frac{1}{2}} = 5^{-4} \times (5\times5\times5)^{\frac{5}{3}} \div (5\times5)^{-\frac{1}{2}}$$

$$= 5^{-4} \times (5^3)^{\frac{5}{3}} \div (5^2)^{-\frac{1}{2}}$$

$$= 5^{-4} \times \left(5^{3\times\frac{5}{3}}\right) \div \left(5^{2\times\left(-\frac{1}{2}\right)}\right)$$

$$= \frac{5^{-4} \times 5^5}{5^{-1}}$$

$$= \frac{5^{5-4}}{5^{-1}}$$

$$= \frac{5^1}{5^{-1}}$$

$$= 5^{1-(-1)}$$

$$= 5^2$$

$$= 5\times5$$

$$= 25$$

(iii)

$$\left(\frac{27}{125}\right)^{\frac{2}{3}} \times \left(\frac{9}{25}\right)^{-\frac{3}{2}} = \left(\frac{3\times3\times3}{5\times5\times5}\right)^{\frac{2}{3}} \times \left(\frac{3\times3}{5\times5}\right)^{-\frac{3}{2}}$$

$$= \left[\left(\frac{3}{5}\right)^{3}\right]^{\frac{2}{3}} \times \left[\left(\frac{3}{5}\right)^{2}\right]^{-\frac{3}{2}}$$

$$= \left(\frac{3}{5}\right)^{3\times\frac{2}{3}} \times \left(\frac{3}{5}\right)^{2\times\left(-\frac{3}{2}\right)}$$

$$= \left(\frac{3}{5}\right)^{2} \times \left(\frac{3}{5}\right)^{-3}$$

$$= \left(\frac{3}{5}\right)^{2-3}$$

$$= \left(\frac{3}{5}\right)^{-1}$$

$$= \frac{1}{\frac{3}{5}}$$

$$= \frac{5}{3}$$

(iv)

$$7^0 \times (25)^{-\frac{3}{2}} - 5^{-3} = 7^0 \times (5\times5)^{-\frac{3}{2}} - 5^{-3}$$

$$= 7^0 \times \left(5^2\right)^{-\frac{3}{2}} - \frac{1}{5^3}$$

$$= 7^0 \times 5^{2\times\left(-\frac{3}{2}\right)} - \frac{1}{5^3}$$

$$= 7^0 \times 5^{-3} - \frac{1}{5^3}$$

$$= 1\times5^{-3} - \frac{1}{5^3}$$

$$= \frac{1}{5^3} - \frac{1}{5^3}$$

$$= \frac{1-1}{5\times5\times5}$$

$$= \frac{0}{125}$$

$$= 0$$

(v)

$$\left(\frac{16}{81}\right)^{-\frac{3}{4}} \times \left(\frac{49}{9}\right)^{\frac{3}{2}} \div \left(\frac{343}{216}\right)^{\frac{2}{3}}$$

$$= \left(\frac{2\times2\times2\times2}{3\times3\times3\times3}\right)^{-\frac{3}{4}} \times \left(\frac{7\times7}{3\times3}\right)^{\frac{3}{2}} \div \left(\frac{7\times7\times7}{6\times6\times6}\right)^{\frac{2}{3}}$$

$$= \left[\left(\frac{2}{3}\right)^{4}\right]^{-\frac{3}{4}} \times \left[\left(\frac{7}{3}\right)^{2}\right]^{\frac{3}{2}} \div \left[\left(\frac{7}{6}\right)^{3}\right]^{\frac{2}{3}}$$

$$= \left(\frac{2}{3}\right)^{4\times\left(-\frac{3}{4}\right)} \times \left(\frac{7}{3}\right)^{2\times\frac{3}{2}} \div \left(\frac{7}{6}\right)^{3\times\frac{2}{3}}$$

$$= \left(\frac{2}{3}\right)^{-3} \times \left(\frac{7}{3}\right)^{3} \div \left(\frac{7}{6}\right)^{2}$$

$$= \frac{1}{\left(\frac{2}{3}\right)^{3}} \times \left(\frac{7}{3}\right)^{3} \times \frac{1}{\left(\frac{7}{6}\right)^{2}}$$

$$= \frac{1}{\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}} \times \frac{7}{3}\times\frac{7}{3}\times\frac{7}{3}\times\frac{1}{\frac{7}{6}\times\frac{7}{6}}$$

$$= \frac{1\times3\times3\times3}{2\times2\times2} \times \frac{7}{3}\times\frac{7}{3}\times\frac{7}{3}\times\frac{1\times6\times6}{7\times7}$$

$$= \frac{7\times3\times3}{2}$$

$$= \frac{63}{2}$$

$$= 31.5$$

Solution 2:

(i)

$$\left(8x^3 \div 125y^3\right)^{\frac{2}{3}} = \left(\frac{8x^3}{125y^3}\right)^{\frac{2}{3}}$$

$$= \left(\frac{2x\times2x\times2x}{5y\times5y\times5y}\right)^{\frac{2}{3}}$$

$$= \left[\left(\frac{2x}{5y}\right)^{3}\right]^{\frac{2}{3}}$$

$$= \left(\frac{2x}{5y}\right)^{3\times\frac{2}{3}}$$

$$= \left(\frac{2x}{5y}\right)^{2}$$

$$= \frac{2x}{5y}\times\frac{2x}{5y}$$

$$= \frac{4x^2}{25y^2}$$

(ii)

$$(a+b)^{-1}.\left(a^{-1}+b^{-1}\right)=\frac{1}{(a+b)}\times\left(\frac{1}{a}+\frac{1}{b}\right)$$

$$=\frac{1}{(a+b)}\times\left(\frac{b+a}{ab}\right)$$

$$=\frac{1}{(a+b)}\times\frac{(a+b)}{ab}$$

$$=\frac{1}{ab}$$

(iii)

$$\frac{5^{n+3}-6\times5^{n+1}}{9\times5^{n}-5^{n}\times2^{2}}=\frac{5^{n+1}\times5^{2}-6\times5^{n+1}}{9\times5^{n}-5^{n}\times2^{2}}$$

$$=\frac{5^{n+1}\times\left(5^{2}-6\right)}{5^{n}\times(9-4)}$$

$$=\frac{5^{n}\times5^{1}\times(25-6)}{5^{n}\times(9-4)}$$

$$=\frac{5^{1}\times19}{5}$$

$$=19$$

(iv)

$$\left(3x^{2}\right)^{-3}\times\left(x^{9}\right)^{\frac{2}{3}}=\frac{1}{\left(3x^{2}\right)^{3}}\times x^{9\times\frac{2}{3}}$$

$$=\frac{1}{3^{3}x^{2\times3}}\times x^{6}$$

$$=\frac{1}{27x^{6}}\times x^{6}$$

$$=\frac{1}{27}$$

Solution 3:

(i)

$$\sqrt{\frac{1}{4}}+(0.01)^{-\frac{1}{2}}-(27)^{\frac{2}{3}}=\sqrt{\frac{1}{2}\times\frac{1}{2}}+(0.1\times0.1)^{-\frac{1}{2}}-(3\times3\times3)^{\frac{2}{3}}$$

$$=\frac{1}{2}+\left[(0.1)^2\right]^{-\frac{1}{2}}-\left(3^2\right)^{\frac{2}{3}}$$

$$=\frac{1}{2}+(0.1)^{2\times\left(-\frac{1}{2}\right)}-3^{3\times\frac{2}{3}}$$

$$=\frac{1}{2}+(0.1)^{-1}-3^2$$

$$=\frac{1}{2}+\frac{1}{0.1}-9$$

$$=\frac{1}{2}+\frac{10}{1}-9$$

$$=\frac{1+20-18}{2}$$

$$=\frac{3}{2}$$

(ii)

$$\left(\frac{27}{8}\right)^{\frac{2}{3}}-\left(\frac{1}{4}\right)^{-2}+5^0=\left(\frac{3\times3\times3}{2\times2\times2}\right)^{\frac{2}{3}}-\left(\frac{1\times1}{2\times2}\right)^{-2}+5^0$$

$$=\left[\left(\frac{3}{2}\right)^3\right]^{\frac{2}{3}}-\left[\left(\frac{1}{2}\right)^2\right]^{-2}+1$$

$$=\left(\frac{3}{2}\right)^{3\times\frac{2}{3}}-\left(\frac{1}{2}\right)^{2\times(-2)}+1$$

$$=\left(\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^{-4}+1$$

$$=\frac{3}{2}\times\frac{3}{2}-\frac{1}{\left(\frac{1}{2}\right)^4}+1$$

$$=\frac{9}{4}-\frac{1}{\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}}+1$$

$$=\frac{9}{4}-\frac{1}{\frac{1}{16}}+1$$

$$=\frac{9}{4}-16+1$$

$$=\frac{9-64+4}{4}$$

$$=\frac{-51}{4}$$

Solution 4:

(i)

$$\left(\frac{3^{-4}}{2^{-8}}\right)^{\frac{1}{4}} = \left(\frac{2^{8}}{3^{4}}\right)^{\frac{1}{4}}$$

$$= \frac{\left(2^{8}\right)^{\frac{1}{4}}}{\left(3^{4}\right)^{\frac{1}{4}}}$$

$$= \frac{2^{8\times\frac{1}{4}}}{3^{4\times\frac{1}{4}}}$$

$$= \frac{2^{2}}{3}$$

$$= \frac{4}{3}$$

(ii)

$$\left(\frac{27^{-3}}{9^{-3}}\right)^{\frac{1}{5}} = \left(\frac{9^{3}}{27^{3}}\right)^{\frac{1}{5}}$$

$$= \left(\frac{\left(3^{2}\right)^{3}}{\left(3^{3}\right)^{3}}\right)^{\frac{1}{5}}$$

$$= \left[\left(\frac{3^{2}}{3^{3}}\right)^{3}\right]^{\frac{1}{5}}$$

$$= \left[\left(\frac{1}{3}\right)^{3}\right]^{\frac{1}{5}}$$

$$= \left(\frac{1}{3}\right)^{3\times\frac{1}{5}}$$

$$= \frac{1}{3^{\frac{3}{5}}}$$

(iii)

$$(32)^{-\frac{2}{5}} \div (125)^{-\frac{2}{3}} = \frac{(32)^{-\frac{2}{5}}}{(125)^{-\frac{2}{3}}}$$

$$= \frac{(125)^{\frac{2}{3}}}{(32)^{\frac{2}{5}}}$$

$$= \frac{(5 \times 5 \times 5)^{\frac{2}{3}}}{(2 \times 2 \times 2 \times 2 \times 2)^{\frac{2}{5}}}$$

$$= \frac{\left(5^3\right)^{\frac{2}{3}}}{\left(2^5\right)^{\frac{2}{5}}}$$

$$= \frac{5^2}{2^2}$$

$$= \frac{25}{4}$$

$$= 6\frac{1}{4}$$

(iv)

$$\left[1-\left\{1-(1-n)^{-1}\right\}^{-1}\right]^{-1} = \frac{1}{\left[1-\left\{1-(1-n)^{-1}\right\}^{-1}\right]^{+1}}$$

$$= \frac{1}{1 - \frac{1}{1 - (1-n)^{-1}}}$$

$$= \frac{1}{1 - \frac{1}{1 - \frac{1}{(1-n)}}}$$

$$= \frac{1}{1 - \frac{1}{\frac{1(1-n)-1}{(1-n)}}}$$

$$= \frac{1}{1 - \frac{1}{\frac{1-n-1}{(1-n)}}}$$

$$= \frac{1}{1 - \frac{1}{\frac{-n}{(1-n)}}}$$

$$= \frac{1}{1 - \frac{(1-n)}{-n}}$$

$$= \frac{1}{1 + \frac{(1-n)}{n}}$$

$$= \frac{1}{\frac{n + (1-n)}{n}}$$

$$= \frac{1}{\frac{n + 1 - n}{n}}$$

$$= \frac{n}{1}$$

$$= n$$

Solution 5:

$2160 = 2^a \times 3^b \times 5^c$

$\Rightarrow 2\times2\times2\times2\times3\times3\times3\times5 = 2^a \times 3^b \times 5^c$

$\Rightarrow 2^4 \times 3^3 \times 5^1 = 2^a \times 3^b \times 5^c$

$\Rightarrow 2^a \times 3^b \times 5^c = 2^4 \times 3^3 \times 5^1$

Comparing powers of 2,3 and 5 on the both sides of equation, we have

a=4;b=3 and c=1

Hence value of $3^a \times 2^{-b} \times 5^{-c} = 3^4 \times 2^{-3} \times 5^{-1}$

$$= 3\times3\times3\times3\times\frac{1}{2^3}\times\frac{1}{5}$$

$$= 81\times\frac{1}{2\times2\times2}\times\frac{1}{5}$$

$$= 81\times\frac{1}{8}\times\frac{1}{5}$$

$$= \frac{81}{40}$$

$$= 2\frac{1}{40}$$

Solution 6:

$1960 = 2^a \times 5^b \times 7^c$

$\Rightarrow 2\times2\times2\times5\times7\times7 = 2^a \times 5^b \times 7^c$

$\Rightarrow 2^3 \times 5^1 \times 7^2 = 2^a \times 5^b \times 7^c$

$\Rightarrow 2^a \times 5^b \times 7^c = 2^3 \times 5^1 \times 7^2$

Comparing powers of 2,5 and 7 on the both sides of equation, we have

a=3;b=1 and c=2

Hence value of $2^{-a} \times 7^b \times 5^{-c} = 2^{-3} \times 7^1 \times 5^{-2}$

$$= \frac{1}{2^3}\times7\times\frac{1}{5^2}$$

$$= \frac{1}{8}\times7\times\frac{1}{5\times5}$$

$$= \frac{7}{200}$$

Solution 7:

(i)

$$\frac{8^{3a}\times 2^5\times 2^{2a}}{4\times 2^{11a}\times 2^{-2a}} = \frac{\left(2^3\right)^{3a}\times 2^5\times 2^{2a}}{2^2\times 2^{11a}\times 2^{-2a}}$$

$$= \frac{2^{3\times 3a}\times 2^5\times 2^{2a}}{2^2\times 2^{11a}\times 2^{-2a}}$$

$$= \frac{2^{9a}\times 2^5\times 2^{2a}}{2^2\times 2^{11a}\times 2^{-2a}}$$

$$= 2^{9a+5+2a-2-11a+2a}$$

$$= 2^{2a+3}$$

(ii)

$$\frac{3\times 27^{n+1}+9\times 3^{3n-1}}{8\times 3^{3n}-5\times 27^n} = \frac{3\times(3\times 3\times 3)^{n+1}+3\times 3\times 3^{3n-1}}{2\times 2\times 2\times 3^{3n}-5\times(3\times 3\times 3)^n}$$

$$= \frac{3\times\left(3^3\right)^{n+1}+3^2\times 3^{3n-1}}{2^3\times 3^{3n}-5\times\left(3^3\right)^n}$$

$$= \frac{3\times 3^{3n+3}+3^{3n+1}}{2^3\times\left(3^3\right)^n-5\times\left(3^3\right)^n}$$

$$= \frac{3^{3n+3+1}+3^{3n+1}}{2^3\times\left(3^3\right)^n-5\times\left(3^3\right)^n}$$

$$= \frac{3^{3n+4}+3^{3n+1}}{2^3\times\left(3^3\right)^n-5\times\left(3^3\right)^n}$$

$$= \frac{3^{3n}\times 3^4+3^{3n}\times 3^1}{2^3\times\left(3^3\right)^n-5\times\left(3^3\right)^n}$$

$$= \frac{3^{3n}\left(3^4+3^1\right)}{\left(3^3\right)^n(8-5)}$$

$$= \frac{3^{3n}\left(3^4+3^1\right)}{3^{3n}\times 3}$$

$$= \frac{3\times 3\times 3\times 3+3}{3}$$

$$= \frac{81+3}{3}$$

$$= \frac{84}{3}$$

$$= 28$$

Solution 8:

$$\left(\frac{a^m}{a^{-n}}\right)^{m-n} \times \left(\frac{a^n}{a^{-\ell}}\right)^{n-\ell} \times \left(\frac{a^\ell}{a^{-m}}\right)^{\ell-m}$$

$$= \left(a^m \times a^n\right)^{m-n} \times \left(a^n \times a^\ell\right)^{n-\ell} \times \left(a^\ell \times a^m\right)^{\ell-m}$$

$$= \left(a^{m+n}\right)^{m-n} \times \left(a^{n+\ell}\right)^{n-\ell} \times \left(a^{\ell+m}\right)^{\ell-m}$$

$$= a^{m^2-n^2} \times a^{n^2-\ell^2} \times a^{\ell^2-m^2}$$

$$= a^{m^2-n^2+n^2-\ell^2+\ell^2-m^2}$$

$$= a^0$$

$$= 1$$

Solution 9:

$$a = x^{m+n}.x^l$$

$$b = x^{n+l}.x^m$$

$$c = x^{l+m}.x^n$$

LHS

$$a^{m-n}.b^{n-l}.c^{l-m}$$

$$= \left(x^{m+n}.x^l\right)^{m-n}.\left(x^{n+l}.x^m\right)^{n-l}.\left(x^{l+m}.x^n\right)^{l-m}$$ [Substituting a,b,c in LHS]

$$= x^{(m+n)(m-n)}.x^{l(m-n)}.x^{(n+l)(n-l)}.x^{m(n-l)}.x^{(l+m)(l-m)}.x^{n(l-m)}$$

$$= x^{m^2-n^2+ml-nl+n^2-l^2+mn-nl+l^2-m^2+nl-mn}$$

$$= x^0$$

$$= 1 = \text{RHS}$$

Solution 10:

(i)

$$\left(\frac{x^a}{x^b}\right)^{a^2+ab+b^2} \times \left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2} \times \left(\frac{x^c}{x^a}\right)^{c^2+ca+a^2}$$

$$= \left(x^{a-b}\right)^{a^2+ab+b^2} \times \left(x^{b-c}\right)^{b^2+bc+c^2} \times \left(x^{c-a}\right)^{c^2+ca+a^2}$$

$$= x^{a^3-b^3} \times x^{b^3-c^3} \times x^{c^3-a^3}$$

$$= x^{a^3-b^3+b^3-c^3+c^3-a^3}$$

$$= x^0$$

$$= 1$$

(ii)

$$\left(\frac{x^a}{x^{-b}}\right)^{a^2-ab+b^2} \times \left(\frac{x^b}{x^{-c}}\right)^{b^2-bc+c^2} \times \left(\frac{x^c}{x^{-a}}\right)^{c^2-ca+a^2}$$

$$= \left(x^{a+b}\right)^{a^2-ab+b^2} \times \left(x^{b+c}\right)^{b^2-bc+c^2} \times \left(x^{c+a}\right)^{c^2-ca+a^2}$$

$$= x^{a^3+b^3} \times x^{b^3+c^3} \times x^{c^3+a^3}$$

$$= x^{a^3+b^3+b^3+c^3+c^3+a^3}$$

$$= x^{\left(a^3+b^3+b^3+c^3+c^3+a^3\right)}$$

$$= x^{2\left(a^3+b^3+c^3\right)}$$