# Exercise 20E

**Q. 1. Find the equation of the line which cuts off intercepts -3 and 5 on the x-axis and y-axis respectively.**

**Answer :** To Find: The equation of a line with intercepts -3 and 5 on the x-axis and y-axis respectively.

Given :Let a and b be the intercepts on x-axis and y-axis respectively.

Then, the x-intercept is a = -3

y-intercept is b = 5

**Formula used:**

we know that intercept form of a line is given by:

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{-3} + \frac{y}{5} = 1$$



5x-3y = -15

5x - 3y + 15 = 0

Hence 5x - 3y + 15 = 0 is the required equation of the given line.

**Q. 2. Find the equation of the line which cuts off intercepts 4 and -6 on the x-axis and y-axis respectively.**

**Answer :** To Find:The equation of the line with intercepts 4 and -6 on the x-axis and y-axis respectively.

Given : Let a and b be the intercepts on x-axis and y-axis respectively.

Then,x-intercept be a = 4

y-intercept be b = -6

**Formula used:**

we know that intercept form of a line is given by:

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{4} + \frac{y}{-6} = 1$$

-3x + 2y = -12

3x – 2y -12 = 0

Hence 3x – 2y -12 = 0 is the required equation of the given line.

**Q. 3. Find the equation of the line and cuts off equal intercepts on the coordinate axes and passes through the point (4,7).**

**Answer :** To Find: The equation of the line with equal intercepts on the coordinate axes and that passes through the point (4,7).

Given : Let a and b be two intercepts of x-axis and y-axis respectively.

Also,given that two intercepts are equal, i.e., a=b

And (4, 7) passes through the point (x, y)

**Formula used:**

Now since intercept form of a line is given:

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{4}{a} + \frac{7}{b} = 1$$

$$\frac{4+7}{a} = 1$$

a= 11 = b

Therefore, The required Equation of the line is $\frac{x}{11} + \frac{y}{11} = 1$

$\Rightarrow$ x + y = 11

**Q. 4. Find the equation of the line which passes through the point (3, -5) and cuts off intercepts on the axes which are equal in magnitude but opposite in sign.**

**Answer :** To Find: The equation of the line passing through (3, -5) and cuts off intercepts on the axes which are equal in magnitude but opposite in sign.

Given : Let a and b be two intercepts of x-axis and y-axis respectively.

According to the question a = -b or b = -a

And (3 , -5) passes through the point (x, y), thus satisfies the equation

**Formula used:**

Now since intercept form of the line is given by ,

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{3}{a} + \frac{-5}{-a} = 1$$



$$\frac{3+5}{a} = 1$$

a = 8 and b= -8

Equation of the line is $\frac{x}{8} + \frac{y}{-8} = 1$

$\Rightarrow$Hence ,the required equation of the line is $\frac{x}{8} - \frac{y}{8} = 1 \Rightarrow x - y = 8$

**Q. 5. Find the equation of the line passing through the point (2, 2) and cutting off intercepts on the axes, whose sum is 9.**

**Answer :** To Find: The equation of the line passing through the point (2, 2) and cutting off intercepts on the axes, whose sum is 9.

**Given :** Let a and b be two intercepts of x-axis and y-axis respectively.

sum of the intercepts is 9,i.e,a+b = 9

$\Rightarrow$ a = 9 – b or b = 9 – a

**Formula used:**

The equation of a line is given by:

$$\frac{x}{a}+\frac{y}{b}=1$$

The given point (2, 2) passing through the line and satisfies the equation of the line.

$$\frac{2}{a}+\frac{2}{9-a}=1$$



$2(9 - a) + 2a = 9a - a^2$

$18 - 2a + 2a = 9a - a^2$

$a^2 - 9a + 18 = 0$

$a^2 - 6a - 3a + 18 = 0$

$a(a - 6) - 3(a - 6) = 0$

$(a - 3)(a - 6) = 0$

a = 3, a = 6

when a = 3, b=6 and a=6, b=3

**case 1 :** when a=3 and b=6

Equation of the line : $\frac{x}{a}+\frac{y}{b}=1$

$$\frac{x}{3}+\frac{y}{6}$$

Hence, 2x + y = 6 is the required equation of the line.

**case 2 :** when a=6 and b=3

Equation of the line : $\frac{x}{a}+\frac{y}{b}=1$

$$\frac{x}{6}+\frac{y}{3}=1$$

Hence , x + 2y = 6 is the required equation of the line.

Therefore, 2x + y = 6 is the required equation of the line when a=3 and b=6.And , x + 2y = 6 is the required equation of the line when a=6 and b=3.

**Q. 6. Find the equation of the line which passes through the point (22, -6) and whose intercept on the x-axis exceeds the intercept on the y-axis by 5.**



**Answer : To** Find:The equation of the line that passes through the point (22, -6) and intercepts on the x-axis exceeds the intercept on the y-axis by 5.

**Given :** let x-intercept be a and y-intercept be b.

According to the question : a = b + 5

**Formula used:** And the given point satisfies the equation of the line, so

$$\frac{x}{a}+\frac{y}{b}=1$$

$$\frac{22}{b+5}+\frac{-6}{b}=1$$

$22b - 6b - 30 = b^2 + 5b$

$11b - 30 = b^2$

$b^2 - 11b + 30 = 0$

$b^2$ -6b -5b +30 =0

b(b-6) -5(b-6) =0

(b-5) (b-6) =0

The values are b=5 ,b=6

When b=5 then a=10

and b=6 then a=11

**case 1 :** when b=5 and a=10

Equation of the line : $\frac{x}{a}+\frac{y}{b}=1$

$\frac{x}{10}+\frac{y}{5}=1$

$\frac{x+2y}{10}=1$

Hence, x + 2y = 10 is the required equation of the line.

**case 2 :** when b=6 and a=11

Equation of the line : $\frac{x}{a}+\frac{y}{b}=1$

$\frac{x}{11}+\frac{y}{6}=1$

$\frac{6x+11y}{66}=1$

Hence, 6x + 11y = 66 is the required equation of the line.

Therefore, x + 2y = 10 is the required equation of the line when b=5 and a=10 .And 6x + 11y = 66 is the required equation of the line when b=6 and a=11.

**Q. 7. Find the equation of the line whose portion intercepted between the axes is bisected at the point (3, -2).**

**Answer :** To Find: The equation of the line whose portion intercepted between the axes is bisected at the point (3, -2).

**Formula used:**

Let the equation of the line be

$$\frac{x}{a}+\frac{y}{b}=1$$

Since it is given that this equation , whose portion is intercepted between the axes is bisected i.e.; is divided into ratio 1:1 .

Let A(a,0) and B(0,b) be the points foring the coordinate axis.

⟹ a and b are intercepts of x and y-axis respectively.



By using mid-point formula (m:n = 1:1)

$$(x, y) = (\frac{y_1+x_1}{2}, \frac{y_2+x_2}{2}) = \left(\frac{a}{2}, \frac{b}{2}\right)$$

Since given point (3 , -2) divides coordinate axis in 1:1 ratio

(x , y) = (3 , -2)

$$\Rightarrow \frac{a}{2}=3 \text{ and } \frac{b}{2}=-2$$

a=6 b=-4

equation of the line : $\frac{x}{a} + \frac{y}{b} = 1$

$$\frac{x}{6} + \frac{y}{-4} = 1$$

-4x +6y = -24

-2x +3y = -12

Hence the required equation of the line is 2x -3y = 12.

**Q. 8. Find the equation of the line whose portion intercepted between the coordinate axes is divided at the point (5, 6) in the ratio 3 : 1.**

**Answer :** To Find: The equation of the line whose portion intercepted between the coordinate axes is divided at the point (5, 6) in the ratio 3 : 1.

Given : The coordinate axes is divided in the ratio 3 : 1

$(x_1 , y_1)$ = A(a,0)

$(x_2 , y_2)$ = B(0, b)

Where a and b are intercepts of the line.

**Formula used:**

The equation of the line is :

The equation of the line is : $\frac{x}{a} + \frac{y}{b} = 1$

And the co-ordinate axis is divided at (5,6) , thus by using Section formula

$$(x , y) = \left( \frac{my_1 + nx_1}{m+n} , \frac{my_2 + nx_2}{m+n} \right)$$

$$= \left(\frac{3*0+a}{4}, \frac{3b}{4}\right) = \left(\frac{a}{4}, \frac{3b}{4}\right)$$

(5,6) divides the co-ordinate axis, thus (x,y)= (5,6).

$$\frac{a}{4} = 5 \Rightarrow a = 20,\ \frac{3b}{4} = 6 \Rightarrow b = 8$$

Equation of the line becomes $\frac{x}{20} + \frac{y}{8} = 1$

8x +20y = 160

2x +5y = 40

Hence the required equation of the line is 2x +5y = 40.

**Q. 9. A straight line passes through the point (5, -2) and the portion of the line intercepted between the axes is divided at this point in the ratio 2 : 3. Find the equation of the line.**



**Answer :** Given : The ratio of the line intercepted between the axes is 2 :3

Let $(x_1, y_1)$ = A(a,0)

And $(x_2, y_2)$ = B(0, b)

Where a and b are intercepts of the line.

**Formula used:**

The equation of the line is : $\frac{x}{a} + \frac{y}{b} = 1$

And the co-ordinate axis is divided at (5,-2) , thus by using Section formula

$$(x, y) = \left(\frac{my_1 + nx_1}{m+n}, \frac{my_2 + nx_2}{m+n}\right)$$

$$= \left(\frac{2*0+3a}{5}, \frac{2b+3*0}{5}\right) = \left(\frac{3a}{5}, \frac{2b}{5}\right)$$

(5,-2) divides the co-ordinate axis, thus (x,y)= (5,-2).

$\frac{3a}{5} = 5 \Rightarrow a = 25/3$ , $\frac{2b}{4} = -2 \Rightarrow b = -5$

Equation of the line becomes $\frac{x}{a} + \frac{y}{b} = 1$

$$\frac{x}{25/3} + \frac{y}{-5} = 1$$

$$\frac{3x}{25} - \frac{y}{5} = 1$$

$$\frac{3x - 5y}{25} = 1$$



Hence ,3x – 5y = 25 is the required equation of the line.

**Q. 10. If the straight line $\frac{x}{a} + \frac{y}{b} = 1$ passes through the points (8, -9) and (12, -15), find the values of a and b.**

**Answer :** To Find: The values of a and b when the line $\frac{x}{a} + \frac{y}{b} = 1$ passes through the points (8, -9) and (12, -15).

Given : the equation of the line : $\frac{x}{a} + \frac{y}{b} = 1$ equation 1

Also (8, -9) passes through equation 1

$$\frac{8}{a} - \frac{9}{b} = 1$$

8b - 9a = ab equation 2

And (12, -15) passes through equation 1

$$\frac{12}{a} - \frac{15}{b} = 1$$

12b - 15a = ab equation 3

Solving equation 2 and 3

a= 2.

Put a=2 in equation 2

8b- 9a = ab

8b -18 = 2b

6b =18 ⟹ b=3

Hence the values of a and b are 2 and 3 respectively.

## Exercise 20F

**Q. 1 A. Find the equation of the line for which**

**p = 3 and ∝ = 450**



**Answer :** To Find:The equation of the line.

Given: p = 3 and ∝ = 450

Here p is the perpendicular that makes an angle ∝ with positive direction of x-axis , hence the equation of the straight line is given by:

**Formula used:**

X cos ∝ + y sin ∝ =p

X cos 450 + y sin 450 =3

i.e; cos 450 = cos (360 +90) = cos 90 [ ∵ cos(360+ x) = cos x]

similarly, sin 450 = sin (360 +90) = sin 90 [ ∵ sin(360+ x) = sin x]

hence, x cos 90 + y sin 90 =3

x × (0)+ y ×1=3

Hence the required equation of the line is y=3.

**Q. 1 B. Find the equation of the line for which**

**p = 5 and ∝ = 1350**

**Answer :** Given: p = 5 and ∝ = 1350

Here p is the perpendicular that makes an angle ∝ with positive direction of x-axis , hence the equation of the straight line is given by:

**Formula used:**

$x \cos \propto + y \sin \propto = p$

$x \cos 1350 + y \sin 1350 = 5$

i.e; $\cos 1350 = \cos ((4 \times 360) - 90) = \cos((4 \times 2\pi) - 90) = \cos 90$

similarly, $\sin 1350 = \sin ((4 \times 360) - 90) = \sin((4 \times 2\pi) - 90) = -\sin 90$

hence, $x \cos 90 + y (-\sin 90) = 5$

$x \times (0) - y \times 1 = 5$



Hence The required equation of the line is y=-5.

**Q. 1 C. Find the equation of the line for which**

**p = 8 and ∝ = 1500**

**Answer :** Given: p = 8 and ∝ = 1500

Here p is the perpendicular that makes an angle ∝ with positive direction of x-axis , hence the equation of the straight line is given by:

**Formula used:**

$x \cos \propto + y \sin \propto = p$

$x \cos 1500 + y \sin 1500 = 8$

i.e; $\cos 1500 = \cos ((4 \times 360) + 60) = \cos((4 \times 2\pi) + 60) = \cos 60$

similarly, $\sin 1500 = \sin ((4 \times 360) + 60) = \sin((4 \times 2\pi) + 60) = \sin 60$

$x \times (1/2) + y \times (\sqrt{3}/2) = 8$

Hence The Required equation of the line is x + √3 y = 16.

**Q. 1 D. Find the equation of the line for which**

**p = 3 and ∝ = 2250**

**Answer : Given:** p = 3 and ∝ = 2250

Here p is the perpendicular that makes an angle ∝ with positive direction of x-axis , hence the equation of the straight line is given by:

**Formula used:**

x cos ∝ + y sin ∝ =p

x cos 2250 + y sin 2250 =3

i.e; cos 2250 = cos ((6×360) +90) = cos((6×2π )+ 90)= cos 90

similarly, sin 2250 = sin ((6×60) + 90) = sin((6×2π ) +90)= sin 90

hence, x cos 90 + y sin 90 =3

x × (0)+y ×1=3

Hence The required equation of the line is y=3.

**Q. 1 E. Find the equation of the line for which**

**p = 2 and ∝ = 3000**

**Answer :** Given: p = 2 and ∝ = 3000

Here p is the perpendicular that makes an angle ∝ with positive direction of x-axis , hence the equation of the straight line is given by:

**Formula used:**

X cos ∝ + y sin ∝ =p

X cos 3000 + y sin 3000 =2

i.e; cos 3000 = cos ((8×360) +120) = cos((8×2π )+120)= cos 120 =cos(180-60)=cos60

similarly, sin 3000 = sin ((8×360) +120) = sin((8×2π )+120)= sin 120

= sin(180-60) = -sin 60

hence, x cos 60 + y (–sin 60) =2

x × (1/2)- y × (√3/2)=2

Hence The required equation of the line is x - √3y = 4

**Q. 1 F. Find the equation of the line for which**

**p = 4 and ∝ = 1800**

**Answer :** Given: p = 4 and ∝ = 1800

Here p is the perpendicular that makes an angle ∝ with positive direction of x-axis , hence the equation of the straight line is given by:

**Formula used:**

x cos ∝ + y sin ∝ =p

x cos 1800 + y sin 1800 =4

i.e; cos 1800 = cos (5 × 360) = cos(5 × 2π )= cos 360 = 1

similarly, sin 1800 = sin (5 × 360) = sin(5 × 2π )= sin 360 =0

hence, x × 1+ y ×0=4

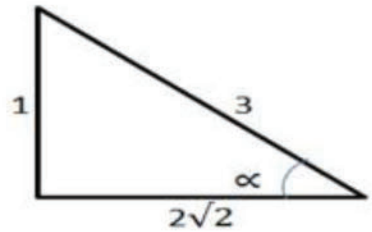
Hence The required equation of the line is x=4.

**Q. 2. The length of the perpendicular segment from the origin to a line is 2 units and the inclination of this perpendicular is ∝ such that $\sin \propto = \frac{1}{3}$ and ∝ is acute. Find the equation of the line.**

**Answer :** To Find: The equation of the line .

Given : p=2 units and $\sin \propto = \frac{1}{3}$.

Since $\sin \propto = \frac{\text{opp}}{\text{hyp}} = \frac{1}{3}$




**Using Pythagoras theorem:**

adj = $\sqrt{9-1}$ = $\sqrt{8}$ = 2$\sqrt{2}$ units.

i.e; cos ∝ = $\frac{\text{adj}}{\text{hyp}}$ = $\frac{2\sqrt{2}}{3}$

Formula used:

equation of the line: x cos ∝ +y sin ∝ =p

x × ($\frac{2\sqrt{2}}{3}$) +y × ( $\frac{1}{3}$) = 2

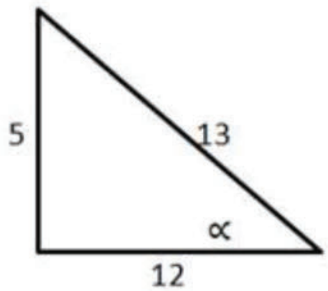
Hence ,2$\sqrt{2}$ x + y = 6 Or $\sqrt{8}$ x +y =6 is the required equation of the line.

**Q. 3. Find the equation of the line which is at a distance of 3 units from the origin such that tan $\propto = \frac{5}{12}$,where ∝ is the acute angle which this perpendicular makes with the positive direction of the x-axis.**

**Answer :** To Find:The equation of the line.

Given : $\propto = \frac{5}{12}$ and p =3 units.

Since $\tan \propto = \frac{\text{opp}}{\text{adj}} = \frac{5}{12}$




**Using Pythagoras theorem :**

hyp = $\sqrt{25 + 144} = \sqrt{169}$ = 13 units.

From the figure: $\cos \propto = \frac{\text{adj}}{\text{hyp}} = \frac{12}{13}$ and $\sin \propto = \frac{\text{opp}}{\text{hyp}} = \frac{5}{13}$



Formula used:

equation of the line: $x \cos \propto + y \sin \propto = p$

$x \times \left(\frac{12}{13}\right) + y \times \left(\frac{5}{13}\right) = 5$

Hence ,12x + 5y = 65 is the required equation of the line.

## Exercise 20G

**Q. 1. Reduce the equation 2x – 3y – 5 = 0 to slope-intercept form, and find from it the slope and y-intercept.**

**Answer** : Given equation is 2x – 3y – 5 = 0

We can rewrite it as 2x – 5 = 3y

$$\Rightarrow 3y = 2x - 5$$

$$\Rightarrow y = \frac{2}{3}x - \frac{5}{3}$$

This equation is in the slope-intercept form i.e. it is the form of

$y = m \times x + c$, where m is the slope of the line and c is y-intercept of the line

Therefore, $m = \frac{2}{3}$ and $c = -\frac{5}{3}$

**Conclusion:**

Slope is $\frac{2}{3}$ and y – intercept is $-\frac{5}{3}$

**Q. 2. Reduce the equation 5x + 7y – 35 = 0 to slope-intercept form, and hence find the slope and the y-intercept of the line**



**Answer :** Given equation is 5x + 7y -35 = 0

We can rewrite it as 7y = 35 - 5x

⇒7y = - 5x + 35

$$\Rightarrow y = -\frac{5}{7}x + 5$$

This equation is in the slope-intercept form i.e. it is the form of

$y = m \times x + c$, where m is the slope of the line and c is y-intercept of the line

Therefore, $m = -\frac{5}{7}$ and c = 5

Conclusion: Slope is $-\frac{5}{7}$ and y-intercept is 5

**Q. 3. Reduce the equation y + 5 = 0 to slope-intercept form, and hence find the slope and the y-intercept of the line.**

**Answer :** Given equation is y + 5 = 0

We can rewrite it as y = -5

This equation is in the slope-intercept form, i.e. it is the form of

$y = m \times x + c$, where m is the slope of the line and c is y-intercept of the line

Therefore, m = 0 and c = - 5

**Conclusion:** Slope is 0 and y-intercept is -5

**Q. 4. Reduce the equation 3x – 4y + 12 = 0 to intercepts form. Hence, find the length of the portion of the line intercepted between the axes**

**Answer :** Given equation is 3x - 4y +12 = 0

We can rewrite it as 3x - 4y = -12

$$\Rightarrow \frac{3}{-12}x + \frac{4}{12}y = 1$$

$$\Rightarrow \frac{x}{-4} + \frac{y}{3} = 1$$

This equation is in the slope intercept form i.e. in the form

$\frac{x}{a} + \frac{y}{b} = 1$

Where, x-intercept = -4 and y-intercept = 3

Two points are: (-4, 0) on the x-axis and (0, 3) on y-axis

We know distance between two points $(x_1, y_1), (x_2, y_2)$ is

$$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Length of the line

$$= \sqrt{(-4-0)^2 + (0-3)^2}$$

$$= \sqrt{16+9}$$

$$= \sqrt{25}$$

= 5

**Q. 5. Reduce the equation 5x – 12y = 60 to intercepts form. Hence, find the length of the portion of the line intercepted between the axes**

**Answer :** Given equation is 5x - 12y = 60

We can rewrite it as

$$\frac{5}{60}x - \frac{12}{60}y = 1$$

$$\Rightarrow \frac{x}{12} - \frac{y}{5} = 1$$

$$\Rightarrow \frac{x}{12} + \frac{y}{-5} = 1$$

This equation is in the slope intercept form i.e. in the form

$$\frac{x}{a} + \frac{y}{b} = 1$$

Where, x-intercept = 12 and y-intercept = -5

Two points are: (12, 0) on the x-axis and (0,-5) on y-axis

We know the distance between two points $(x_1, y_1), (x_2, y_2)$ is

$$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Length of the line

$$=\sqrt{(12-0)^2+(0+5)^2}$$

$$=\sqrt{144+25}$$

$$=\sqrt{169}$$

= 13

**Q. 6. Find the inclination of the line:**

**(i) $x + \sqrt{3}y + 6 = 0$**
**(ii) 3x + 3y + 8 = 0**
**(iii) $\sqrt{3}x - y - 4 = 0$**

**Answer :**

(i) Given equation is $x + \sqrt{3}y + 6 = 0$



We can rewrite it as $\sqrt{3}y = -x - 6$

$$\Rightarrow y = \frac{-1}{\sqrt{3}}x + \frac{-6}{\sqrt{3}}$$

It is in the form of $y = x \times \tan\alpha + c$

Where $\tan\alpha = -\frac{1}{\sqrt{3}}$ and $c = -\frac{6}{\sqrt{3}}$

The inclination of the line is $\alpha$

Therefore $\alpha = \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

$= \frac{5\pi}{6}\ 3x + 3y = 8$

Conclusion: Inclination $x + \sqrt{3}y + 6 = 0$ of the line is $\frac{5\pi}{6}$

$3y = 8 - 3x$

(ii) Given equation is



We can rewrite it as

$\Rightarrow y = -x + \frac{-3}{8}$

It is in the form of $y = x \times \tan\alpha + c$

Where $\tan\alpha = -1$ and $c = -\frac{3}{8}$

Therefore $\alpha = \tan^{-1}(-1)$

$= \frac{3\pi}{4}$

Conclusion: Inclination of line 3x + 3y + 8 = 0 is $\frac{3\pi}{4}$

(iii) Given equation is $\sqrt{3}x - y - 4 = 0$

We can rewrite it as $y = \sqrt{3}x - 4$

It is in the form of $y = x \times \tan\alpha + c$

Where $\tan\alpha = \sqrt{3}$ and c = -4



$\Rightarrow \alpha = \tan^{-1}(\sqrt{3})$

$= \frac{\pi}{3}$

Conclusion: Inclination of the line is $\frac{\pi}{3}$

**Q. 7. Reduce the equation x + y - √2 = 0 to the normal form x cos ∝ + y sin ∝ = p, and hence find the values of ∝ and p.**

**Answer :**

Given equation is $x + y - \sqrt{2} = 0$

If the equation is in the form of ax + by = c, to get into the normal form, we should divide it by $\sqrt{a^2+b^2}$, so now

Divide by $\sqrt{1+1}=\sqrt{2}$

$$\Rightarrow \frac{x}{\sqrt{2}}+\frac{y}{\sqrt{2}}=1$$

This is in the form of $x\cos\alpha+y\sin\alpha=p$

$$\cos\alpha=\frac{1}{\sqrt{2}} \Rightarrow \alpha=\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$\Rightarrow \alpha=\frac{\pi}{4}$ And



$$\Rightarrow p=1$$

Conclusion: $\alpha=\frac{\pi}{4}$ and p =1

**Q. 8. Reduce the equation $x+\sqrt{3}y-4=0$ to the normal form x cos ∝ + y sin ∝ = p, and hence find the values of ∝ and p.**

**Answer :** Given equation is

$$x+\sqrt{3}y-4=0$$

If the equation is in the form of ax + by = c, to get into the normal form, we should divide it by $\sqrt{a^2+b^2}$, so now

Divide by $\sqrt{\sqrt{3}^2+1^2}=2$

Now we get $\Rightarrow \frac{x}{2}+\frac{\sqrt{3}y}{2}=1$

This is in the form of $x\cos\alpha+y\sin\alpha=p$

**Q. 8. Reduce the equation** $x+\sqrt{3}y-4=0$ **to the normal form x cos ∝ + y sin ∝ = p, and hence find the values of ∝ and p.**

**Answer :** Given equation is $x+\sqrt{3}y-4=0$

If the equation is in the form of ax + by = c, to get into the normal form, we should divide it by $\sqrt{a^2+b^2}$ , so now



Divide by

$$\sqrt{\sqrt{3}^2+1^2}=2$$

Now we get

$$\Rightarrow \frac{x}{2}+\frac{\sqrt{3}y}{2}=1$$

This is in the form of

$$x\cos\alpha+y\sin\alpha=p$$

Where

$$\cos\alpha=\frac{1}{2}\Rightarrow \alpha=\frac{\pi}{3}$$

And p = 1

Conclusion: $\alpha = \frac{\pi}{3}$ and p =1

**Q. 9. Reduce each of the following equations to normal form :**

**(i) x + y - 2 = 0**
**(ii)** $x + y + \sqrt{2} = 0$
**(iii) x + 5 = 0**
**(iv) 2y – 3 =0**
**(v) 4x + 3y - 9 = 0**

**Answer :**

$\Rightarrow x + y = 2$

If the equation is in the form of ax + by = c, to get into the normal form we should divide it by $\sqrt{a^2 + b^2}$ , so now

Divide by $\sqrt{1^2 + 1^2} = \sqrt{2}$



$$\Rightarrow \frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = \frac{2}{\sqrt{2}}$$

$$\Rightarrow \frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = \sqrt{2}$$

This is in the form of $x\cos\alpha + y\sin\alpha = p$, where $\alpha = \frac{\pi}{4}$ and $p = \sqrt{2}$

Conclusion: $\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = \sqrt{2}$ is the normal form of x + y - 2 = 0

(ii) $x + y + \sqrt{2} = 0$

$\Rightarrow x + y = -\sqrt{2}$

If the equation is in the form of ax + by = c, to get into the normal form, we should divide it by $\sqrt{a^2 + b^2}$, so now

Divide by $\sqrt{1^2 + 1^2} = \sqrt{2}$

Our new equation is $\frac{x}{-\sqrt{2}} + \frac{y}{-\sqrt{2}} = 1$



This is in the form of $x\cos\alpha + y\sin\alpha = p$, where $\alpha = \frac{5\pi}{4}$ and p =1

Conclusion: $\frac{x}{-\sqrt{2}} + \frac{y}{-\sqrt{2}} = 1$ is the normal form of $x + y + \sqrt{2} = 0$

(iii) $\Rightarrow -x = 5$

If the equation is in the form of ax + by = c, to get into the normal form, we should divide it by $\sqrt{a^2 + b^2}$, so now

Divide the equation by $\sqrt{1^2 + 0^2} = 1$

Our new equation is $-x = 5$

This is in the form of $x\cos\alpha + y\sin\alpha = p$ , where $\alpha = \pi$ and p = 5

Conclusion: $-x = 5$ is the normal form of $x + 5 = 0$

(iv) $\Rightarrow 2y = 3$

If the equation is in the form of ax + by = c, to get into the normal form, we should divide it by $\sqrt{a^2 + b^2}$ , so now

Divide by $\sqrt{2^2 + 0^2} = 2$

Our new equation is $y = \frac{3}{2}$



This is in the form of $x\cos\alpha + y\sin\alpha = p$ , where $\alpha = \frac{\pi}{2}$ and $p = \frac{3}{2}$

Conclusion: $y = \frac{3}{2}$ is the normal form of $2y = 3$

(v) $\Rightarrow 4x + 3y - 9 = 0$

If the equation is in the form of ax + by = c, to get into the normal form, we should divide it by $\sqrt{a^2 + b^2}$ , so now

Divide by $\sqrt{4^2 + 3^2} = 5$

Our new equation is $\frac{4}{5}x+\frac{3}{5}y=\frac{9}{5}$

This is in the form of $x\cos\alpha+y\sin\alpha=p$, where

$\alpha=\sin^{-1}\left(\frac{3}{5}\right)$ or $\alpha=\cos^{-1}\left(\frac{4}{5}\right)$ and $p=\frac{9}{5}$

Conclusion: $\frac{4}{5}x+\frac{3}{5}y=\frac{9}{5}$ is the normal form of $4x+3y-9=0$

## Exercise 20H

**Q. 1. Find the distance of the point (3, -5) from the line 3x – 4y = 27**

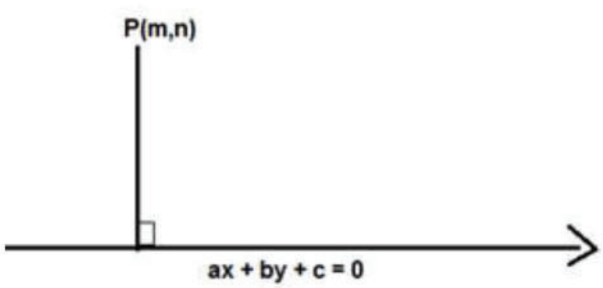
**Answer :** Given: Point (3,-5) and line 3x – 4y = 27

**To find:** The distance of the point (3, -5) from the line 3x – 4y = 27

**Formula used:**

We know that the distance between a point P(m,n) and a line ax + by + c = 0 is given by,

$D=\frac{|am+bn+c|}{\sqrt{a^2+b^2}}$




The equation of the line is 3x – 4y - 27=0

Here m= 3 and n = -5 , a = 3 , b = -4 , c = -27

$$D=\frac{|3(3)-4(-5)-27|}{\sqrt{3^2+4^2}}$$

$$D=\frac{|9+20-27|}{\sqrt{9+16}}=\frac{|29-27|}{\sqrt{25}}=\frac{|2|}{5}$$

$$D=\frac{2}{5}$$

The distance of the point (3, -5) from the line 3x – 4y = 27 is $\frac{2}{5}$ units

**Q. 2. Find the distance of the point (-2, 3) from the line 12x = 5y + 13.**

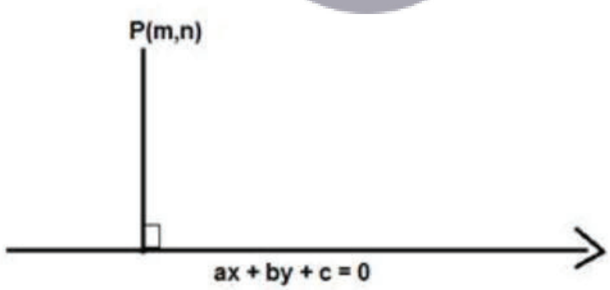
**Answer :** Given: Point (-2,3) and line 12x – 5y = 13

To find: The distance of the point (-2, 3) from the line 12x – 5y = 13

**Formula used:** We know that the distance between a point P (m,n) and a line ax + by + c = 0 is given by,



$$D=\frac{|am+bn+c|}{\sqrt{a^2+b^2}}$$




The given equation of the line is 12x – 5y - 13=0

Here m= -2 and n = 3 , a = 12 , b = -5 , c = -13

$$D=\frac{|12(-2)-5(3)-13|}{\sqrt{12^2+5^2}}$$

$$D=\frac{|-24-15-13|}{\sqrt{144+25}}=\frac{|-52|}{\sqrt{169}}=\frac{|-52|}{13}=\frac{52}{13}=4$$

D = 4

The distance of the point (-2, 3) from the line 12x = 5y + 13 is 4 units

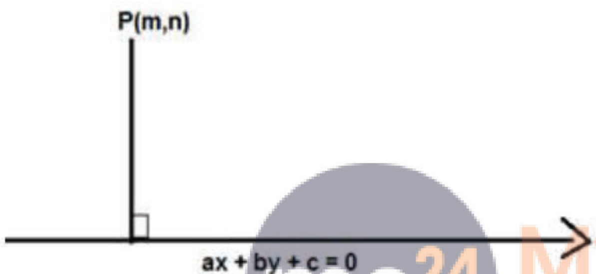
**Q. 3. Find the distance of the point (-4, 3) from the line 4(x + 5) = 3(y – 6).**

**Answer :** Given: Point (-4,3) and line 4(x + 5) = 3(y – 6)

To find: The distance of the point (-4, 3) from the line 4(x + 5) = 3(y – 6)

**Formula used:** We know that the distance between a point P (m,n) and a line ax + by + c = 0 is given by,

$$D=\frac{|am+bn+c|}{\sqrt{a^2+b^2}}$$






The equation of the line is 4x + 20 = 3y – 18

4x – 3y + 38 = 0

Here m= -4 and n = 3 , a = 4 , b = -3 , c = 38

$$D=\frac{|4(-4)-3(3)+38|}{\sqrt{4^2+3^2}}$$

$$D=\frac{|-16-9+38|}{\sqrt{16+9}}=\frac{|-25+38|}{\sqrt{25}}=\frac{|13|}{5}$$

$$D=\frac{13}{5}$$

The distance of the point (-4, 3) from the line 4(x + 5) = 3(y – 6) is $\frac{13}{5}$ units