

EXERCISE 7.5

Factorize each of the following expressions:

1. $16x^2 - 25y^2$

Solution:

We have,

$$16x^2 - 25y^2$$

$$(4x)^2 - (5y)^2$$

By using the formula $(a^2 - b^2) = (a + b)(a - b)$ we get,

$$(4x + 5y)(4x - 5y)$$

2. $27x^2 - 12y^2$

Solution:

We have,

$$27x^2 - 12y^2$$

By taking 3 as common we get,

$$3 [(3x)^2 - (2y)^2]$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$3 (3x + 2y)(3x - 2y)$$

3. $144a^2 - 289b^2$

Solution:

We have,

$$144a^2 - 289b^2$$

$$(12a)^2 - (17b)^2$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$(12a + 17b)(12a - 17b)$$

4. $12m^2 - 27$

Solution:

We have,

$$12m^2 - 27$$

By taking 3 as common we get,

$$3 (4m^2 - 9)$$

$$3 [(2m)^2 - 3^2]$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$3 (2m + 3)(2m - 3)$$

5. $125x^2 - 45y^2$

Solution:

We have,

$$125x^2 - 45y^2$$

By taking 5 as common we get,

$$5 (25x^2 - 9y^2)$$

$$5 [(5x)^2 - (3y)^2]$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$5 (5x + 3y) (5x - 3y)$$

6. $144a^2 - 169b^2$

Solution:

We have,

$$144a^2 - 169b^2$$

$$(12a)^2 - (13b)^2$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$(12a + 13b) (12a - 13b)$$

7. $(2a - b)^2 - 16c^2$

Solution:

We have,

$$(2a - b)^2 - 16c^2$$

$$(2a - b)^2 - (4c)^2$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$(2a - b + 4c) (2a - b - 4c)$$

8. $(x + 2y)^2 - 4(2x - y)^2$

Solution:

We have,

$$(x + 2y)^2 - 4(2x - y)^2$$

$$(x + 2y)^2 - [2(2x - y)]^2$$

By using the formula $(a^2 - b^2) = (a + b)(a - b)$ we get,

$$[(x + 2y) + 2(2x - y)] [(x + 2y) - 2(2x - y)]$$

$$(x + 4x + 2y - 2y) (x - 4x + 2y + 2y)$$

$$(5x) (4y - 3x)$$

9. $3a^5 - 48a^3$

Solution:

We have,

$$3a^5 - 48a^3$$

By taking 3 as common we get,

$$3a^3 (a^2 - 16)$$

$$3a^3 (a^2 - 4^2)$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$3a^3 (a + 4)(a - 4)$$

10. $a^4 - 16b^4$

Solution:

We have,

$$a^4 - 16b^4$$

$$(a^2)^2 - (4b^2)^2$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$(a^2 + 4b^2)(a^2 - 4b^2)$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$(a^2 + 4b^2)(a + 2b)(a - 2b)$$

11. $x^8 - 1$

Solution:

We have,

$$x^8 - 1$$

$$(x^4)^2 - (1)^2$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$(x^4 + 1)(x^4 - 1)$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$(x^4 + 1)(x^2 + 1)(x - 1)(x + 1)$$

12. $64 - (a + 1)^2$

Solution:

We have,

$$64 - (a + 1)^2$$

$$8^2 - (a + 1)^2$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$[8 + (a + 1)][8 - (a + 1)]$$

$$(a + 9)(7 - a)$$

13. $36l^2 - (m + n)^2$

Solution:

We have,

$$36l^2 - (m + n)^2$$

$$(6l)^2 - (m + n)^2$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$(6l + m + n)(6l - m - n)$$

$$14. 25x^4y^4 - 1$$

Solution:

We have,

$$25x^4y^4 - 1$$

$$(5x^2y^2)^2 - (1)^2$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$(5x^2y^2 - 1)(5x^2y^2 + 1)$$

$$15. a^4 - 1/b^4$$

Solution:

We have,

$$a^4 - 1/b^4$$

$$(a^2)^2 - (1/b^2)^2$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$(a^2 + 1/b^2)(a^2 - 1/b^2)$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$(a^2 + 1/b^2)(a - 1/b)(a + 1/b)$$

$$16. x^3 - 144x$$

Solution:

We have,

$$x^3 - 144x$$

$$x [x^2 - (12)^2]$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$x(x + 12)(x - 12)$$

$$17. (x - 4y)^2 - 625$$

Solution:

We have,

$$(x - 4y)^2 - 625$$

$$(x - 4y)^2 - (25)^2$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$(x - 4y + 25)(x - 4y - 25)$$

$$18. 9(a - b)^2 - 100(x - y)^2$$

Solution:

We have,

$$9(a - b)^2 - 100(x - y)^2$$

$$[3(a - b)]^2 - [10(x - y)]^2$$

By using the formula $(a^2 - b^2) = (a - b)(a + b)$

$$[3(a - b) + 10(x + y)][3(a - b) - 10(x - y)]$$

$$[3a - 3b + 10x - 10y][3a - 3b - 10x + 10y]$$

19. $(3 + 2a)^2 - 25a^2$

Solution:

We have,

$$(3 + 2a)^2 - 25a^2$$

$$(3 + 2a)^2 - (5a)^2$$

By using the formula $(a^2 - b^2) = (a - b)(a + b)$

$$(3 + 2a + 5a)(3 + 2a - 5a)$$

$$(3 + 7a)(3 - 3a)$$

$$(3 + 7a)3(1 - a)$$

20. $(x + y)^2 - (a - b)^2$

Solution:

We have,

$$(x + y)^2 - (a - b)^2$$

By using the formula $(a^2 - b^2) = (a - b)(a + b)$

$$[(x + y) + (a - b)][(x + y) - (a - b)]$$

$$(x + y + a - b)(x + y - a + b)$$

21. $1/16x^2y^2 - 4/49y^2z^2$

Solution:

We have,

$$1/16x^2y^2 - 4/49y^2z^2$$

$$(1/4xy)^2 - (2/7yz)^2$$

By using the formula $(a^2 - b^2) = (a - b)(a + b)$

$$(xy/4 + 2yz/7)(xy/4 - 2yz/7)$$

$$y^2(x/4 + 2/7z)(x/4 - 2/7z)$$

22. $75a^3b^2 - 108ab^4$

Solution:

We have,

$$75a^3b^2 - 108ab^4$$

$$3ab^2 (25a^2 - 36b^2)$$

$$3ab^2 [(5a)^2 - (6b)^2]$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$3ab^2 (5a + 6b)(5a - 6b)$$

23. $x^5 - 16x^3$

Solution:

We have,

$$x^5 - 16x^3$$

$$x^3 (x^2 - 16)$$

$$x^3 (x^2 - 4^2)$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$x^3 (x + 4)(x - 4)$$

24. $50/x^2 - 2x^2/81$

Solution:

We have,

$$50/x^2 - 2x^2/81$$

$$2 (25/x^2 - x^2/81)$$

$$2 [(5/x)^2 - (x/9)^2]$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$2 (5/x + x/9)(5/x - x/9)$$

25. $256x^3 - 81x$

Solution:

We have,

$$256x^3 - 81x$$

$$x (256x^2 - 81)$$

$$x [(16x^2)^2 - 9^2]$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$x (4x + 3)(4x - 3)(16x^2 + 9)$$

26. $a^4 - (2b + c)^4$

Solution:

We have,

$$a^4 - (2b + c)^4$$

$$(a^2)^2 - [(2b + c)^2]^2$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$[a^2 + (2b + c)^2][a^2 - (2b + c)^2]$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$
 $[a^2 + (2b + c)^2] [a + 2b + c] [a - 2b - c]$

27. $(3x + 4y)^4 - x^4$

Solution:

We have,

$$(3x + 4y)^4 - x^4$$

$$[(3x + 4y)^2]^2 - (x^2)^2$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$[(3x + 4y)^2 + x^2] [(3x + 4y)^2 - x^2]$$

$$[(3x + 4y)^2 + x^2] [3x + 4y + x] [3x + 4y - x]$$

$$[(3x + 4y)^2 + x^2] [4x + 4y] [2x + 4y]$$

$$[(3x + 4y)^2 + x^2] 8[x + 2y] [x + y]$$

28. $p^2q^2 - p^4q^4$

Solution:

We have,

$$p^2q^2 - p^4q^4$$

$$(pq)^2 - (p^2q^2)^2$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$(pq + p^2q^2)(pq - p^2q^2)$$

$$p^2q^2(1 + pq)(1 - pq)$$

29. $3x^3y - 243xy^3$

Solution:

We have,

$$3x^3y - 243xy^3$$

$$3xy(x^2 - 81y^2)$$

$$3xy[x^2 - (9y)^2]$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$(3xy)(x + 9y)(x - 9y)$$

30. $a^4b^4 - 16c^4$

Solution:

We have,

$$a^4b^4 - 16c^4$$

$$(a^2b^2)^2 - (4c^2)^2$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$(a^2b^2 + 4c^2)(a^2b^2 - 4c^2)$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$
 $(a^2b^2 + 4c^2)(ab + 2c)(ab - 2c)$

31. $x^4 - 625$

Solution:

We have,

$$x^4 - 625$$

$$(x^2)^2 - (25)^2$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$(x^2 + 25)(x^2 - 25)$$

$$(x^2 + 25)(x^2 - 5^2)$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$(x^2 + 25)(x + 5)(x - 5)$$

32. $x^4 - 1$

Solution:

We have,

$$x^4 - 1$$

$$(x^2)^2 - (1)^2$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$(x^2 + 1)(x^2 - 1)$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$(x^2 + 1)(x + 1)(x - 1)$$

33. $49(a - b)^2 - 25(a + b)^2$

Solution:

We have,

$$49(a - b)^2 - 25(a + b)^2$$

$$[7(a - b)]^2 - [5(a + b)]^2$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$[7(a - b) + 5(a + b)][7(a - b) - 5(a + b)]$$

$$(7a - 7b + 5a + 5b)(7a - 7b - 5a - 5b)$$

$$(12a - 2b)(2a - 12b)$$

$$2(6a - b)2(a - 6b)$$

$$4(6a - b)(a - 6b)$$

34. $x - y - x^2 + y^2$

Solution:

We have,

$$x - y - x^2 + y^2$$

$$x - y - (x^2 - y^2)$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$x - y - (x + y)(x - y)$$

$$(x - y)(1 - x - y)$$

35. $16(2x - 1)^2 - 25y^2$

Solution:

We have,

$$16(2x - 1)^2 - 25y^2$$

$$[4(2x - 1)]^2 - (5y)^2$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$(8x + 5y - 4)(8x - 5y - 4)$$

36. $4(xy + 1)^2 - 9(x - 1)^2$

Solution:

We have,

$$4(xy + 1)^2 - 9(x - 1)^2$$

$$[2(xy + 1)]^2 - [3(x - 1)]^2$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$(2xy + 2 + 3x - 3)(2xy + 2 - 3x + 3)$$

$$(2xy + 3x - 1)(2xy - 3x + 5)$$

37. $(2x + 1)^2 - 9x^4$

Solution:

We have,

$$(2x + 1)^2 - 9x^4$$

$$(2x + 1)^2 - (3x^2)^2$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$(2x + 1 + 3x^2)(2x + 1 - 3x^2)$$

$$(3x^2 + 2x + 1)(-3x^2 + 2x + 1)$$

38. $x^4 - (2y - 3z)^2$

Solution:

We have,

$$x^4 - (2y - 3z)^2$$

$$(x^2)^2 - (2y - 3z)^2$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$(x^2 + 2y - 3z)(x^2 - 2y + 3z)$$

39. $a^2 - b^2 + a - b$

Solution:

We have,

$$a^2 - b^2 + a - b$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$(a + b)(a - b) + (a - b)$$

$$(a - b)(a + b + 1)$$

40. $16a^4 - b^4$

Solution:

We have,

$$16a^4 - b^4$$

$$(4a^2)^2 - (b^2)^2$$

$$(4a^2 + b^2)(4a^2 - b^2)$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$(4a^2 + b^2)(2a + b)(2a - b)$$

41. $a^4 - 16(b - c)^4$

Solution:

We have,

$$a^4 - 16(b - c)^4$$

$$(a^2)^2 - [4(b - c)^2]^2$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$[a^2 + 4(b - c)^2][a^2 - 4(b - c)^2]$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$[a^2 + 4(b - c)^2][(a + 2b - 2c)(a - 2b + 2c)]$$

42. $2a^5 - 32a$

Solution:

We have,

$$2a^5 - 32a$$

$$2a(a^4 - 16)$$

$$2a[(a^2)^2 - (4)^2]$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$2a(a^2 + 4)(a^2 - 4)$$

$$2a(a^2 + 4)(a^2 - 2^2)$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$2a(a^2 + 4)(a + 2)(a - 2)$$

43. $a^4b^4 - 81c^4$

Solution:

We have,

$$a^4b^4 - 81c^4$$

$$(a^2b^2)^2 - (9c^2)^2$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$(a^2b^2 + 9c^2)(a^2b^2 - 9c^2)$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$(a^2b^2 + 9c^2)(ab + 3c)(ab - 3c)$$

44. $xy^9 - yx^9$

Solution:

We have,

$$xy^9 - yx^9$$

$$-xy(x^8 - y^8)$$

$$-xy[(x^4)^2 - (y^4)^2]$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$-xy(x^4 + y^4)(x^4 - y^4)$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$-xy(x^4 + y^4)(x^2 + y^2)(x^2 - y^2)$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$-xy(x^4 + y^4)(x^2 + y^2)(x + y)(x - y)$$

45. $x^3 - x$

Solution:

We have,

$$x^3 - x$$

$$x(x^2 - 1)$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$x(x + 1)(x - 1)$$

46. $18a^2x^2 - 32$

Solution:

We have,

$$18a^2x^2 - 32$$

$$2[(3ax)^2 - (4)^2]$$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$$2(3ax + 4)(3ax - 4)$$