NCERT Solutions for Class-XII Maths

Chapter-10.4

NCERT Chemistry Class 12

1. Find $|\vec{a} \times \vec{b}|$, if $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$.

1. We have,

$\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{vmatrix}$$

$= \hat{i}(-14 + 14) - \hat{j}(2 - 21) + \hat{k}(-2 + 21) = 19\hat{j} + 19\hat{k}$

$\therefore |\vec{a} \times \vec{b}| = \sqrt{(19)^2 + (19)^2} = \sqrt{2 \times (19)^2} = 19\sqrt{2}$

2. Find a unit vector perpendicular to each of the vector $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$, $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$.

2. Given that

$\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and

$\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$

Let $\vec{p} = \vec{a} + \vec{b}$

$\Rightarrow \vec{p} = (3\hat{i} + 2\hat{j} + 2\hat{k}) + (\hat{i} + 2\hat{j} - 2\hat{k})$

$\Rightarrow \vec{p} = (4\hat{i} + 4\hat{j} + 0\hat{k})$

Let $\vec{q} = \vec{a} - \vec{b}$

$\Rightarrow \vec{q} = (3\hat{i} + 2\hat{j} + 2\hat{k}) - (\hat{i} + 2\hat{j} - 2\hat{k})$

$\Rightarrow \vec{q} = (2\hat{i} + 0\hat{j} + 4\hat{k})$

Now, we want to find a vector which is perpendicular to both $\vec{p}$ and $\vec{q}$. It is given by $\vec{p} \times \vec{q}$.

Let $\vec{r} = \vec{p} \times \vec{q}$

$$\vec{p}\times\vec{q}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & 4 & 0\\ 2 & 0 & 4\end{vmatrix}$$

Expanding along first row,

$\vec{p}\times\vec{q}=\hat{i}\left[(4\times4)-(0\times0)\right]-\hat{j}\left[(4\times4)-(2\times0)\right]+\hat{k}\left[(4\times0)-(4\times2)\right]$

$\Rightarrow \vec{p}\times\vec{q}=(16)\hat{i}-(16)\hat{j}+(-8)\hat{k}$

$\Rightarrow \vec{p}\times\vec{q}=16\hat{i}-16\hat{j}-4\hat{k}$

Therefore, $\vec{r}=16\hat{i}-16\hat{j}-8\hat{k}$

Now we want to find the unit vector. We know that unit vector means that the magnitude of the vector is 1(unit).

It is defined as $\hat{r}=\dfrac{\vec{r}}{|\vec{r}|}$

So, we find the magnitude of $\vec{a}$ first.

$\Rightarrow |\vec{r}|=\sqrt{x^2+y^2+z^2}=\sqrt{16^2+(-16)^2+(-8)^2}=\sqrt{576}=24$

$\hat{r}=\dfrac{\vec{r}}{|\vec{r}|}=\pm\dfrac{\left(16\hat{i}-16\hat{j}-8\hat{k}\right)}{24}=\pm\left(\dfrac{2}{3}\hat{i}-\dfrac{2}{3}\hat{j}-\dfrac{1}{3}\hat{k}\right)$

3. If a unit vector $\vec{a}$ makes angles $\dfrac{\pi}{3}$ with $\hat{i}$, $\dfrac{\pi}{4}$ with $\hat{j}$ and an acute angle θ with $\hat{k}$, then find θ and hence, the components of $\vec{a}$.

3. Let unit vector $\vec{a}$ have (a_1, a_2, a_3) components.

$\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$

Since $\vec{a}$ is a unit vector, $|\vec{a}|=1$.

Also, it is given that $\vec{a}$ makes angles $\dfrac{\pi}{3}$ with $\hat{i}$, $\dfrac{\pi}{4}$ with $\hat{j}$, and an acute angle θ with $\hat{k}$.

Then, we have:

$\cos\dfrac{\pi}{3}=\dfrac{a_1}{|\vec{a}|}\Rightarrow\dfrac{1}{2}=a_1\ \left[|\vec{a}|=1\right]$ $\cos\dfrac{\pi}{4}=\dfrac{a_2}{|\vec{a}|}\Rightarrow\dfrac{1}{\sqrt{2}}=a_2\ \left[|\vec{a}|=1\right]$ Also, $\cos\theta=\dfrac{a_3}{|\vec{a}|}$. 2

$\Rightarrow a_3=\cos\theta$

Now, $|a|=1$

$\Rightarrow \sqrt{a_1^2+a_2^2+a_3^2}=1$

$$\Rightarrow \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \cos^2\theta = 1$$

$$\Rightarrow \frac{1}{4} + \frac{1}{2} + \cos^2\theta = 1$$

$$\Rightarrow \frac{3}{4} + \cos^2\theta = 1$$

$$\Rightarrow \cos^2\theta = 1 - \frac{3}{4} = \frac{1}{4}$$

$$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

$$\therefore a_3 = \cos\frac{\pi}{3} = \frac{1}{2}$$

Hence, $\theta = \frac{\pi}{3}$ and the components of $\vec{a}$ are $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}\right)$

4. Show that $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2(\vec{a} \times \vec{b})$

4. We solve for the left-hand side,

L.H.S.=

$$\left(\vec{a} - \vec{b}\right) \times \left(\vec{a} + \vec{b}\right) = \left(\vec{a} \times \vec{a}\right) + \left(\vec{a} \times \vec{b}\right) - \left(\vec{b} \times \vec{a}\right) - \left(\vec{b} \times \vec{b}\right)$$

We know $\vec{a} \times \vec{a} = 0$ and $\vec{b} \times \vec{b} = 0$

Also, $\vec{b} \times \vec{a} = -\left(\vec{a} \times \vec{b}\right)$

Putting these in our equation, we get

$$\Rightarrow \left(\vec{a} - \vec{b}\right) \times \left(\vec{a} + \vec{b}\right) = 0 + \left(\vec{a} \times \vec{b}\right) + \left(\vec{a} \times \vec{b}\right) - 0$$

$$\Rightarrow \left(\vec{a} - \vec{b}\right) \times \left(\vec{a} + \vec{b}\right) = 2\left(\vec{a} \times \vec{b}\right)$$

which is equal to R.H.S.

Hence proved.

5. Find λ and μ if $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + \lambda\hat{j} + \mu\hat{k}) = \vec{0}$.

5. $\left(2\hat{i} + 6\hat{j} + 27\hat{k}\right) \times \left(\hat{i} + \lambda\hat{j} + \mu\hat{k}\right) = \vec{0}$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & 27 \\ 1 & \lambda & \mu \end{vmatrix} = 0\hat{i} + 0\hat{j} + 0\hat{k}$$

$\Rightarrow \hat{i}(6\mu - 27\lambda) - \hat{j}(2\mu - 27) + \hat{k}(2\lambda - 6) = 0\hat{i} + 0\hat{j} + 0\hat{k}$

On comparing the corresponding components, we have:

$6\mu - 27\lambda = 0$

$2\mu - 27 = 0$

$2\lambda - 6 = 0$

Now,

$2\lambda - 6 = 0 \Rightarrow \lambda = 3$

$2\mu - 27 = 0 \Rightarrow \mu = \frac{27}{2}$

Hence, $\lambda = 3$ and $\mu = \frac{27}{2}$.

6. Given that $\vec{a}.\vec{b} = 0$ and $\vec{a} \times \vec{b} = \vec{0}$. What can you conclude about the vectors $\vec{a}$ and $\vec{b}$?

6. Given that

$\vec{a}.\vec{b} = 0$

$\Rightarrow |\vec{a}|.|\vec{b}|.\cos\theta = 0$ (where θ is the angle between the vectors)

$\Rightarrow |\vec{a}| = 0$ or $|\vec{b}| = 0$ or $\cos\theta = 0$

Also given that, $\vec{a} \times \vec{b} = 0$

$\Rightarrow |\vec{a}|.|\vec{b}|.\sin\theta = 0$ (where θ is the angle between the vectors)

$\Rightarrow |\vec{a}| = 0$ or $|\vec{b}| = 0$ or $\sin\theta = 0$

As there is no value of θ for which both sin θ and cos θ are zero.

Therefore, the condition for which $\vec{a}.\vec{b} = 0$ and $\vec{a} \times \vec{b} = 0$ is:

$|\vec{a}| = 0$ or $|\vec{b}| = 0$.

7. Let the vectors $\vec{a}, \vec{b}, \vec{c}$ be given as $a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, b_1\hat{i} + b_2\hat{j} + b_3\hat{k}, c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$. Then show that $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$.

7. We have,

$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}, \vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

$(\vec{b} + \vec{c}) = (b_1 + c_1)\hat{i} + (b_2 + c_2)\hat{j} + (b_3 + c_3)\hat{k}$

Now, $\vec{a}\times\left(\vec{b}+\vec{c}\right)\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1+c_1 & b_2+c_2 & b_3+c_3\end{vmatrix}$

$=\hat{i}\left[a_2\left(b_3+c_3\right)-a_3\left(b_2+c_2\right)\right]-\hat{j}\left[a_1\left(b_3+c_3\right)-a_3\left(b_1+c_1\right)\right]+\hat{k}\left[a_1\left(b_2+c_2\right)-a_2\left(b_1+c_1\right)\right]$

$=\hat{i}\left[a_2b_3+a_2c_3-a_3b_2-a_3c_2\right]+\hat{j}\left[-a_1b_3-a_1c_3+a_3b_1+a_3c_1\right]+$

$\hat{k}\left[a_1b_2+a_1c_2-a_2b_1-a_2c_1\right]\ldots(1)$

$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$

$=\hat{i}\left[a_2b_3-a_3b_2\right]+\hat{j}\left[b_1a_3-a_1b_3\right]+\hat{k}\left[a_1b_2-a_2b_1\right]$

$\vec{a}\times\vec{c}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ c_1 & c_2 & c_3\end{vmatrix}$

$=\hat{i}\left[a_2c_3-a_3c_2\right]+\hat{j}\left[a_3c_1-a_1c_3\right]+\hat{k}\left[a_1c_2-a_2c_1\right]$

On adding (2) and (3), we get:

$\left(\vec{a}\times\vec{b}\right)+\left(\vec{a}\times\vec{c}\right)=\hat{i}\left[a_2b_3+a_2c_3-a_3b_2-a_3c_2\right]+\hat{j}\left[b_1a_3+a_3c_1-a_1b_3-a_1c_3\right]$

$+\hat{k}\left[a_1b_2+a_1c_2-a_2b_1-a_2c_1\right]$

Now, from (1) and (4), we have:

$\vec{a}\times\left(\vec{b}+\vec{c}\right)=\vec{a}\times\vec{b}+\vec{a}\times\vec{c}$ Hence, the given result is proved.

8. If either $\vec{a}=\vec{0}$ or $\vec{b}=\vec{0}$, then $\vec{a}\times\vec{b}=\vec{0}$. Is the converse true? Justify your answer with an example.

8. $\left|\vec{a}\times\vec{b}\right|=\left|\vec{a}\right|.\left|\vec{b}\right|.\sin\theta$ (where θ is the angle between the vectors)

If $\vec{a}=\vec{0}$,

$\Rightarrow\vec{a}\times\vec{b}=(0).\left|\vec{b}\right|.\sin\theta=0$

Similarly, If $\vec{b}=\vec{0}$,

$\Rightarrow\vec{a}\times\vec{b}=\left|\vec{a}\right|.(0).\sin\theta=0$

Now, If $\vec{a} \times \vec{b} = \vec{0}$

$\Rightarrow |\vec{a}|.|\vec{b}|.\sin\theta = 0$ (where θ is the angle between the vectors)

$\Rightarrow |\vec{a}| = 0$ or $|\vec{b}| = 0$ or $\sin\theta = 0$

$\Rightarrow \sin\theta = 0$ implies $\theta = 90°$

This implies that the converse is not always true. The vectors may not be zero but the angle between is 90°, i.e. the vectors are perpendicular.

9. Find the area of the triangle with vertices A(1, 1, 2), B(2, 3, 5) and C(1, 5, 5).

9. The vertices of triangle ABC are given as $A(1,1,2), B(2,3,5)$, and $C(1,5,5)$. The adjacent sides $\overrightarrow{AB}$ and $\overrightarrow{BC}$ of $\triangle ABC$ are given as:

$\overrightarrow{AB} = (2-1)\hat{i} + (3-1)\hat{j} + (5-2)\hat{k} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\overrightarrow{BC} = (1-2)\hat{i} + (5-3)\hat{j} + (5-5)\hat{k} = -\hat{i} + 2\hat{j}$

Area of $\triangle ABC = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{BC}\right|$

$$\overrightarrow{AB} \times \overrightarrow{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -1 & 2 & 0 \end{vmatrix} = \hat{i}(-6) - \hat{j}(3) + \hat{k}(2+2) = -6\hat{i} - 3\hat{j} + 4\hat{k}$$

$\therefore \left|\overrightarrow{AB} \times \overrightarrow{BC}\right| = \sqrt{(-6)^2 + (-3)^2 + 4^2} = \sqrt{36+9+16} = \sqrt{61}$

Hence, the area of $\triangle ABC$ is $\frac{\sqrt{61}}{2}$ square units.

10. Find the area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.

10. Given

$\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and

$\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$

We know that $\text{Area(parallelogram)} = |\vec{a} \times \vec{b}|$

So, we find $|\vec{a} \times \vec{b}|$,

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 3\\ 2 & -7 & 1\end{vmatrix}$$

Expanding along first row,

$\vec{a}\times\vec{b}=\hat{i}\left[(-1\times1)-(-7\times3)\right]-\hat{j}\left[(1\times1)-(2\times3)\right]+\hat{k}\left[(-7\times1)-(-1\times2)\right]$

$\Rightarrow \vec{a}\times\vec{b}=(20)\hat{i}-(-5)\hat{j}+(-5)\hat{k}$

$\Rightarrow \vec{a}\times\vec{b}=20\hat{i}+5\hat{j}-5\hat{k}$

$\therefore \left|\vec{a}\times\vec{b}\right|=\sqrt{x^2+y^2+z^2}=\sqrt{20^2+5^2+(-5)^2}=\sqrt{450}=15\sqrt{2}$

$\therefore$ Area (parallelogram) $=\left|\vec{a}\times\vec{b}\right|=15\sqrt{2}$ sq. units

11. Let the vectors $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}|=3$ and $|\vec{b}|=\frac{\sqrt{2}}{3}$, then $\vec{a}\times\vec{b}$ is a unit vector, if the angle between $\vec{a}$ and $\vec{b}$ is

(A) $\pi/6$ (B) $\pi/4$

(C) $\pi/3$ (D) $\pi/2$

11. It is given that $|\vec{a}|=3$ and $|\vec{b}|=\frac{\sqrt{2}}{3}$

We know that $\vec{a}\times\vec{b}=|\vec{a}||\vec{b}|\sin\theta\hat{n}$, where is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ and θ is the angle between $\vec{a}$ and $\vec{b}$.

Now, $\vec{a}\times\vec{b}$ is a unit vector if $|\vec{a}\times\vec{b}|=1$

$|\vec{a}\times\vec{b}|=1$

$\Rightarrow |\vec{a}||\vec{b}|\sin\theta\hat{n}|=1$

$\Rightarrow |\vec{a}||\vec{b}||\sin\theta|=1$

$\Rightarrow 3\times\frac{\sqrt{2}}{3}\times\sin\theta=1$

$\Rightarrow \sin\theta=\frac{1}{\sqrt{2}}$

$\Rightarrow \theta=\frac{\pi}{4}$

Hence, $\vec{a}\times\vec{b}$ is a unit vector if the angle between $\vec{a}$ and $\vec{b}\frac{\pi}{4}$. The correct answer is B.

12. Area of a rectangle having vertices A, B, C and D with position vectors $-\hat{i}+\frac{1}{2}\hat{j}+4\hat{k}, \hat{i}+\frac{1}{2}\hat{j}+4\hat{k}, \hat{i}-\frac{1}{2}\hat{j}+4\hat{k}$ and $-\hat{i}-\frac{1}{2}\hat{j}+4\hat{k}$, respectively is

(A) $\frac{1}{2}$ (B) 1

(C) 2 (D) 4

12. Let

$\vec{a} = -\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}$,

$\vec{b} = \hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}$

$\vec{c} = \hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}$

$\vec{d} = -\hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}$

Now we want to find the $\overrightarrow{AB}$

$\overrightarrow{AB} = \vec{B} - \vec{A}$

$\Rightarrow \overrightarrow{AB} = \left(\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}\right) - \left(-\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}\right)$

$\Rightarrow \overrightarrow{AB} = \left(2\hat{i} + 0\hat{j} + 0\hat{k}\right)$

Now we want to find the $\overrightarrow{AC}$

$\overrightarrow{AD} = \vec{D} - \vec{A}$

$\Rightarrow \overrightarrow{AD} = \left(-\hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}\right) - \left(-\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}\right)$

$\Rightarrow \overrightarrow{AD} = \left(0\hat{i} - 1\hat{j} + 0\hat{k}\right)$

We know that $\text{Area}(\text{rectangle}) = \left|\overrightarrow{AB} \times \overrightarrow{AD}\right|$

So, we find $\left|\overrightarrow{AB} \times \overrightarrow{AD}\right|$,

$$\overrightarrow{AB} \times \overrightarrow{AD} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix}$$

Expanding along first row,

$$\overrightarrow{AB} \times \overrightarrow{AD} = \hat{i}\left[(0\times 0) - (-1\times 0)\right] - \hat{j}\left[(2\times 0) - (0\times 0)\right] + \hat{k}\left[(-1\times 2) - (0\times 0)\right]$$

$\Rightarrow \overrightarrow{AB} \times \overrightarrow{AD} = (0)\hat{i} - (0)\hat{j} + (-2)\hat{k}$

$\Rightarrow \overrightarrow{AB} \times \overrightarrow{AD} = 0\hat{i} + 0\hat{j} - 2\hat{k}$

$\therefore \left|\overrightarrow{AB} \times \overrightarrow{AD}\right| = \sqrt{x^2 + y^2 + z^2} = \sqrt{0^2 + 0^2 + (-2)^2} = 2$

$\therefore \text{Area}(\text{rectangle}) = \left|\overrightarrow{AB} \times \overrightarrow{AD}\right| = 2\text{sq. units}$