

EXERCISE 6.4

Solve the following system of linear equations by Cramer's rule:

$$1. \quad x - 2y = 4$$

$$-3x + 5y = -7$$

Solution:

$$\text{Given } x - 2y = 4$$

$$-3x + 5y = -7$$

Let there be a system of n simultaneous linear equations and with n unknown given by

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

...

$$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$$

Let $D = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$

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Your Class. Your Pace.Let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \dots, x_n = \frac{D_n}{D} \text{ Provided that } D \neq 0$$

Now, here we have

$$x - 2y = 4$$

$$-3x + 5y = -7$$

So by comparing with the theorem, let's find D , D_1 and D_2

$$\Rightarrow D = \begin{vmatrix} 1 & -2 \\ -3 & 5 \end{vmatrix}$$

Solving determinant, expanding along 1st row

$$\Rightarrow D = 5(1) - (-3)(-2)$$

$$\Rightarrow D = 5 - 6$$

$$\Rightarrow D = -1$$

Again,

$$\Rightarrow D_1 = \begin{vmatrix} 4 & -2 \\ -7 & 5 \end{vmatrix}$$

Solving determinant, expanding along 1st row

$$\Rightarrow D_1 = 5(4) - (-7)(-2)$$

$$\Rightarrow D_1 = 20 - 14$$

$$\Rightarrow D_1 = 6$$

And

$$\Rightarrow D_2 = \begin{vmatrix} 1 & 4 \\ -3 & -7 \end{vmatrix}$$

Solving determinant, expanding along 1st row

$$\Rightarrow D_2 = 1(-7) - (-3)(4)$$

$$\Rightarrow D_2 = -7 + 12$$

$$\Rightarrow D_2 = 5$$

Thus by Cramer's Rule, we have

$$\Rightarrow x = \frac{D_1}{D}$$

$$\Rightarrow x = \frac{6}{-1}$$

$$\Rightarrow x = -6$$

And

$$\Rightarrow y = \frac{D_2}{D}$$

$$\Rightarrow y = \frac{5}{-1}$$

$$\Rightarrow y = -5$$

$$2. \quad 2x - y = 1$$

$$7x - 2y = -7$$

Solution:

Given $2x - y = 1$ and

$$7x - 2y = -7$$

Let there be a system of n simultaneous linear equations and with n unknown given by

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

...

$$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \dots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$

Let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \dots, x_n = \frac{D_n}{D} \text{ Provided that } D \neq 0$$

Now, here we have

$$2x - y = 1$$

$$7x - 2y = -7$$

So by comparing with the theorem, let's find D , D_1 and D_2

$$\Rightarrow D = \begin{vmatrix} 2 & -1 \\ 7 & -2 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} row

$$\Rightarrow D_1 = 1(-2) - (-7)(-1)$$

$$\Rightarrow D_1 = -2 - 7$$

$$\Rightarrow D_1 = -9$$

And

$$\Rightarrow D_2 = \begin{vmatrix} 2 & 1 \\ 7 & -7 \end{vmatrix}$$

Solving determinant, expanding along 1st row

$$\Rightarrow D_2 = 2(-7) - (7)(1)$$

$$\Rightarrow D_2 = -14 - 7$$

$$\Rightarrow D_2 = -21$$

Thus by Cramer's Rule, we have

$$\Rightarrow x = \frac{D_x}{D}$$

$$\Rightarrow x = \frac{-9}{3}$$

$$\Rightarrow x = -3$$

$$\text{And } \Rightarrow y = \frac{D_y}{D}$$

$$\Rightarrow y = \frac{-21}{3}$$

$$\Rightarrow y = -7$$

$$3. 2x - y = 17$$

$$3x + 5y = 6$$

Solution:

Given $2x - y = 17$ and

$$3x + 5y = 6$$

Let there be a system of n simultaneous linear equations and with n unknown given by

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

...

$$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \dots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$

Let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \dots, x_n = \frac{D_n}{D} \text{ Provided that } D \neq 0$$

Now, here we have

$$2x - y = 17$$

$$3x + 5y = 6$$

So by comparing with the theorem, let's find D , D_1 and D_2

$$\Rightarrow D = \begin{vmatrix} 2 & -1 \\ 3 & 5 \end{vmatrix}$$

Solving determinant, expanding along 1st row

$$\Rightarrow D_1 = 17(5) - (6)(-1)$$

$$\Rightarrow D_1 = 85 + 6$$

$$\Rightarrow D_1 = 91$$

$$\Rightarrow D_2 = \begin{vmatrix} 2 & 17 \\ 3 & 6 \end{vmatrix}$$

Solving determinant, expanding along 1st row

$$\Rightarrow D_2 = 2(6) - (17)(3)$$

$$\Rightarrow D_2 = 12 - 51$$

$$\Rightarrow D_2 = -39$$

Thus by Cramer's Rule, we have

$$\Rightarrow x = \frac{D_1}{D}$$

$$\Rightarrow x = \frac{91}{13}$$

$$\Rightarrow x = 7$$

$$\text{And } \Rightarrow y = \frac{D_2}{D}$$

$$\Rightarrow y = \frac{-39}{13}$$

$$\Rightarrow y = -3$$

$$4. \quad 3x + y = 19$$

$$3x - y = 23$$

Solution:

Let there be a system of n simultaneous linear equations and with n unknown given by

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

!!!

$$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \dots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$

Let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

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Then,

$$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \dots, x_n = \frac{D_n}{D} \text{ Provided that } D \neq 0$$

Now, here we have

$$3x + y = 19$$

$$3x - y = 23$$

So by comparing with the theorem, let's find D , D_1 and D_2

$$\Rightarrow D = \begin{vmatrix} 3 & 1 \\ 3 & -1 \end{vmatrix}$$

Solving determinant, expanding along 1st row

$$\Rightarrow D = 3(-1) - (3) \quad (1)$$

$$\Rightarrow D = -3 - 3$$

$$\Rightarrow D = -6$$

Again,

$$\Rightarrow D_1 = \begin{vmatrix} 19 & 1 \\ 23 & -1 \end{vmatrix}$$

Solving determinant, expanding along 1st row

$$\Rightarrow D_1 = 19(-1) - (23)(1)$$

$$\Rightarrow D_1 = -19 - 23$$

$$\Rightarrow D_1 = -42$$

$$\Rightarrow D_2 = \begin{vmatrix} 3 & 19 \\ 3 & 23 \end{vmatrix}$$

Solving determinant, expanding along 1st row

$$\Rightarrow D_2 = 3(23) - (19)(3)$$

$$\Rightarrow D_2 = 69 - 57$$

$$\Rightarrow D_2 = 12$$

Thus by Cramer's Rule, we have

$$\Rightarrow x = \frac{D_1}{D}$$

$$\Rightarrow x = \frac{-42}{-6}$$

$$\Rightarrow x = 7$$

$$\text{And } \Rightarrow y = \frac{D_2}{D}$$

$$\Rightarrow y = \frac{12}{-6}$$

$$\Rightarrow y = -2$$

$$5. 2x - y = -2$$

$$3x + 4y = 3$$

Solution:

Given $2x - y = -2$ and

$$3x + 4y = 3$$

Let there be a system of n simultaneous linear equations and with n unknown given by

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

...

$$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$

Let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \dots, x_n = \frac{D_n}{D} \text{ Provided that } D \neq 0$$

Now, here we have

$$2x - y = -2$$

$$3x + 4y = 3$$

So by comparing with the theorem, let's find D , D_1 and D_2

$$\Rightarrow D = \begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} row

$$\Rightarrow D = 2(4) - (3)(-1)$$

$$\Rightarrow D = 8 + 3$$

$$\Rightarrow D = 11$$

Again,

$$\Rightarrow D_1 = \begin{vmatrix} -2 & -1 \\ 3 & 4 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} row

$$\Rightarrow D_1 = -2(4) - (3)(-1)$$

$$\Rightarrow D_1 = -8 + 3$$

$$\Rightarrow D_1 = -5$$

$$\Rightarrow D_2 = \begin{vmatrix} 2 & -2 \\ 3 & 3 \end{vmatrix}$$

Solving determinant, expanding along 1st row

$$\Rightarrow D_2 = 3(2) - (-2)(3)$$

$$\Rightarrow D_2 = 6 + 6$$

$$\Rightarrow D_2 = 12$$

Thus by Cramer's Rule, we have

$$\Rightarrow x = \frac{D_1}{D}$$

$$\Rightarrow x = \frac{-5}{11}$$

$$\text{And } \Rightarrow y = \frac{D_2}{D}$$

$$\Rightarrow y = \frac{12}{11}$$

$$6. 3x + ay = 4$$

$$2x + ay = 2, a \neq 0$$

Solution:

Given $3x + ay = 4$ and

$2x + ay = 2, a \neq 0$

Let there be a system of n simultaneous linear equations and with n unknown given by

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

...

$$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \dots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$

Let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \dots, x_n = \frac{D_n}{D} \text{ Provided that } D \neq 0$$

$$3x + ay = 4$$

$$2x + ay = 2, a \neq 0$$

So by comparing with the theorem, let's find D , D_1 and D_2

$$\Rightarrow D = \begin{vmatrix} 3 & a \\ 2 & a \end{vmatrix}$$

Solving determinant, expanding along 1st row

$$\Rightarrow D = 3(a) - (2)(a)$$

$$\Rightarrow D = 3a - 2a$$

$$\Rightarrow D = a$$

Again,

$$\Rightarrow D_1 = \begin{vmatrix} 4 & a \\ 2 & a \end{vmatrix}$$

Solving determinant, expanding along 1st row

$$\Rightarrow D_1 = 4(a) - (2)(a)$$

$$\Rightarrow D = 4a - 2a$$

$$\Rightarrow D = 2a$$

$$\Rightarrow D_2 = \begin{vmatrix} 3 & 4 \\ 2 & 2 \end{vmatrix}$$

Solving determinant, expanding along 1st row

$$\Rightarrow D_2 = 3(2) - (2)(4)$$

$$\Rightarrow D = 6 - 8$$

$$\Rightarrow D = -2$$

Thus by Cramer's Rule, we have

$$\Rightarrow x = \frac{D_1}{D}$$

$$\Rightarrow x = \frac{2a}{a}$$

$$\Rightarrow x = 2$$

$$\Rightarrow y = \frac{D_2}{D}$$

$$\Rightarrow y = \frac{-2}{a}$$

$$7. 2x + 3y = 10$$

$$x + 6y = 4$$

Solution:

Let there be a system of n simultaneous linear equations and with n unknown given by

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

!!!

$$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$

Let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \dots, x_n = \frac{D_n}{D} \text{ Provided that } D \neq 0$$

Now, here we have

$$2x + 3y = 10$$

$$x + 6y = 4$$

So by comparing with the theorem, let's find D , D_1 and D_2

$$\Rightarrow D = \begin{vmatrix} 2 & 3 \\ 1 & 6 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} row

$$\Rightarrow D = 2(6) - (3)(1)$$

$$\Rightarrow D = 12 - 3$$

$$\Rightarrow D = 9$$

Again,

$$\Rightarrow D_1 = \begin{vmatrix} 10 & 3 \\ 4 & 6 \end{vmatrix}$$

Solving determinant, expanding along 1st row

$$\Rightarrow D_1 = 10(6) - (3)(4)$$

$$\Rightarrow D = 60 - 12$$

$$\Rightarrow D = 48$$

$$\Rightarrow D_2 = \begin{vmatrix} 2 & 10 \\ 1 & 4 \end{vmatrix}$$

Solving determinant, expanding along 1st row

$$\Rightarrow D_2 = 2(4) - (10)(1)$$

$$\Rightarrow D_2 = 8 - 10$$

$$\Rightarrow D_2 = -2$$

Thus by Cramer's Rule, we have

$$\Rightarrow x = \frac{D_1}{D}$$

$$\Rightarrow x = \frac{48}{9}$$

$$\Rightarrow x = \frac{16}{3}$$

$$\Rightarrow y = \frac{D_2}{D}$$

$$\Rightarrow y = \frac{-2}{9}$$

$$\Rightarrow y = \frac{-2}{9}$$

$$8. 5x + 7y = -2$$

$$4x + 6y = -3$$

Solution:

Let there be a system of n simultaneous linear equations and with n unknown given by

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

...

$$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \dots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$

Let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \dots, x_n = \frac{D_n}{D} \text{ Provided that } D \neq 0$$

Now, here we have

$$5x + 7y = -2$$

$$4x + 6y = -3$$

So by comparing with the theorem, let's find D , D_1 and D_2

$$\Rightarrow D = \begin{vmatrix} 5 & 7 \\ 4 & 6 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} row

$$\Rightarrow D = 5(6) - (7)(4)$$

$$\Rightarrow D = 30 - 28$$

$$\Rightarrow D = 2$$

Again,

$$\Rightarrow D_1 = \begin{vmatrix} -2 & 7 \\ -3 & 6 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} row

$$\Rightarrow D_1 = -2(6) - (7)(-3)$$

$$\Rightarrow D_1 = -12 + 21$$

$$\Rightarrow D_1 = 9$$

$$\Rightarrow D_2 = \begin{vmatrix} 5 & -2 \\ 4 & -3 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} row

$$\Rightarrow D_2 = -3(5) - (-2)(4)$$

$$\Rightarrow D_2 = -15 + 8$$

$$\Rightarrow D_2 = -7$$

Thus by Cramer's Rule, we have

$$\Rightarrow x = \frac{D_1}{D}$$

$$\Rightarrow x = \frac{9}{2}$$

$$\Rightarrow x = \frac{9}{2}$$

$$\Rightarrow y = \frac{D_2}{D}$$

$$\Rightarrow y = \frac{-7}{2}$$

$$\Rightarrow y = \frac{-7}{2}$$

$$9. 9x + 5y = 10$$

$$3y - 2x = 8$$

Solution:

Let there be a system of n simultaneous linear equations and with n unknown given by

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

...

$$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \dots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$

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Let D_1 be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \dots, x_n = \frac{D_n}{D} \text{ Provided that } D \neq 0$$

Now, here we have

$$9x + 5y = 10$$

$$3y - 2x = 8$$

So by comparing with the theorem, let's find D , D_1 and D_2

$$\Rightarrow D = \begin{vmatrix} 9 & 5 \\ -2 & 3 \end{vmatrix}$$

Solving determinant, expanding along 1st row

$$\Rightarrow D = 3(9) - (5)(-2)$$

$$\Rightarrow D = 27 + 10$$

$$\Rightarrow D = 37$$

Again,

$$\Rightarrow D_1 = \begin{vmatrix} 10 & 5 \\ 8 & 3 \end{vmatrix}$$

Solving determinant, expanding along 1st row

$$\Rightarrow D_1 = 10(3) - (8)(5)$$

$$\Rightarrow D_1 = 30 - 40$$

$$\Rightarrow D_1 = -10$$

$$\Rightarrow D_2 = \begin{vmatrix} 9 & 10 \\ -2 & 8 \end{vmatrix}$$

Solving determinant, expanding along 1st row

$$\Rightarrow D_2 = 9(8) - (10)(-2)$$

$$\Rightarrow D_2 = 72 + 20$$

$$\Rightarrow D_2 = 92$$

Thus by Cramer's Rule, we have

$$\Rightarrow x = \frac{D_1}{D}$$

$$\Rightarrow x = \frac{-10}{37}$$

$$\Rightarrow x = \frac{-10}{37}$$

$$\Rightarrow y = \frac{D_2}{D}$$

$$\Rightarrow y = \frac{92}{37}$$

$$\Rightarrow y = \frac{92}{37}$$

10. $x + 2y = 1$

$3x + y = 4$

Solution:

Let there be a system of n simultaneous linear equations and with n unknown given by

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

...

$$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$

Let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \dots, x_n = \frac{D_n}{D} \text{ Provided that } D \neq 0$$

Now, here we have

$$x + 2y = 1$$

$$3x + y = 4$$

So by comparing with theorem, now we have to find D , D_1 and D_2

$$\Rightarrow D = \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} row

$$\Rightarrow D = 1(1) - (3)(2)$$

$$\Rightarrow D = 1 - 6$$

$$\Rightarrow D = -5$$

Again,

$$\Rightarrow D_1 = \begin{vmatrix} 1 & 2 \\ 4 & 1 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} row

$$\Rightarrow D_1 = 1(1) - (2)(4)$$

$$\Rightarrow D_1 = 1 - 8$$

$$\Rightarrow D_1 = -7$$

$$\Rightarrow D_2 = \begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix}$$

Solving determinant, expanding along 1st row

$$\Rightarrow D_2 = 1(4) - (1)(3)$$

$$\Rightarrow D_2 = 4 - 3$$

$$\Rightarrow D_2 = 1$$

Thus by Cramer's Rule, we have

$$\Rightarrow x = \frac{D_1}{D}$$

$$\Rightarrow x = \frac{-7}{-5}$$

$$\Rightarrow x = \frac{7}{5}$$

$$\Rightarrow y = \frac{D_2}{D}$$

$$\Rightarrow y = \frac{1}{-5}$$

$$\Rightarrow y = -\frac{1}{5}$$

Solve the following system of linear equations by Cramer's rule:

$$11. \quad 3x + y + z = 2$$

$$2x - 4y + 3z = -1$$

$$4x + y - 3z = -11$$

Solution:

Let there be a system of n simultaneous linear equations and with n unknown given by

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

...

$$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \dots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$

Let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

Then,

$$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \dots, x_n = \frac{D_n}{D} \text{ Provided that } D \neq 0$$

Now, here we have

$$3x + y + z = 2$$

$$2x - 4y + 3z = -1$$

$$4x + y - 3z = -11$$

So by comparing with the theorem, let's find D , D_1 , D_2 and D_3

$$\Rightarrow D = \begin{vmatrix} 3 & 1 & 1 \\ 2 & -4 & 3 \\ 4 & 1 & -3 \end{vmatrix}$$

Solving determinant, expanding along 1st row

$$\Rightarrow D = 3[(-4)(-3) - (3)(1)] - 1[(2)(-3) - 12] + 1[2(-4) - 4]$$

$$\Rightarrow D = 3[12 - 3] - [-6 - 12] + [2 + 16]$$

$$\Rightarrow D = 27 + 18 + 18$$

$$\Rightarrow D = 63$$

Again,

$$\Rightarrow D_1 = \begin{vmatrix} 2 & 1 & 1 \\ -1 & -4 & 3 \\ -11 & 1 & -3 \end{vmatrix}$$

Solving determinant, expanding along 1st row

$$\Rightarrow D_1 = 2[(-4)(-3) - (3)(1)] - 1[(-1)(-3) - (-11)(3)] + 1[(-1) - (-4)(-11)]$$

$$\Rightarrow D_1 = 2[12 - 3] - 1[3 + 33] + 1[-1 - 44]$$

$$\Rightarrow D_1 = 2[9] - 36 - 45$$

$$\Rightarrow D_1 = 18 - 36 - 45$$

$$\Rightarrow D_1 = -63$$

Again

$$\Rightarrow D_2 = \begin{vmatrix} 3 & 2 & 1 \\ 2 & -1 & 3 \\ 4 & -11 & -3 \end{vmatrix}$$

Solving determinant, expanding along 1st row

$$\Rightarrow D_2 = 3[3 + 33] - 2[-6 - 12] + 1[-22 + 4]$$

$$\Rightarrow D_2 = 3[36] - 2[-18] - 18$$

$$\Rightarrow D_2 = 126$$

$$\Rightarrow D_3 = \begin{vmatrix} 3 & 1 & 2 \\ 2 & -4 & -1 \\ 4 & 1 & -11 \end{vmatrix}$$

Solving determinant, expanding along 1st row

$$\Rightarrow D_3 = 3[44 + 1] - 1[-22 + 4] + 2[2 + 16]$$

$$\Rightarrow D_3 = 3[45] - 1(-18) + 2(18)$$

$$\Rightarrow D_3 = 135 + 18 + 36$$

$$\Rightarrow D_3 = 189$$

Thus by Cramer's Rule, we have

$$\Rightarrow x = \frac{D_1}{D}$$

$$\Rightarrow x = \frac{-63}{63}$$

$$\Rightarrow x = -1$$

$$\Rightarrow y = \frac{D_2}{D}$$

$$\Rightarrow y = \frac{126}{63}$$

$$\Rightarrow y = 2$$

$$\Rightarrow z = \frac{D_3}{D}$$

$$\Rightarrow z = \frac{107}{63}$$

$$\Rightarrow z = 3$$

12. $x - 4y - z = 11$

$2x - 5y + 2z = 39$

$-3x + 2y + z = 1$

Solution:

Given,

$$x - 4y - z = 11$$

$$2x - 5y + 2z = 39$$

$$-3x + 2y + z = 1$$

Let there be a system of n simultaneous linear equations and with n unknown given by

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

...

$$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$

Let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \dots, x_n = \frac{D_n}{D} \text{ Provided that } D \neq 0$$

Now, here we have

$$x - 4y - z = 11$$

$$2x - 5y + 2z = 39$$

$$-3x + 2y + z = 1$$

So by comparing with theorem, now we have to find D , D_1 and D_2

$$\Rightarrow D = \begin{vmatrix} 1 & -4 & -1 \\ 2 & -5 & 2 \\ -3 & 2 & 1 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} row

$$\Rightarrow D = 1[(-5)(1) - (2)(2)] + 4[(2)(1) + 6] - 1[4 + 5(-3)]$$

$$\Rightarrow D = 1[-5 - 4] + 4[8] - [-11]$$

$$\Rightarrow D = -9 + 32 + 11$$

$$\Rightarrow D = 34$$

Again,

$$\Rightarrow D_1 = \begin{vmatrix} 11 & -4 & -1 \\ 39 & -5 & 2 \\ 1 & 2 & 1 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} row

$$\Rightarrow D_1 = 11[(-5)(1) - (2)(2)] + 4[(39)(1) - (2)(1)] - 1[2(39) - (-5)(1)]$$

$$\Rightarrow D_1 = 11[-5 - 4] + 4[39 - 2] - 1[78 + 5]$$

$$\Rightarrow D_1 = 11[-9] + 4(37) - 83$$

$$\Rightarrow D_1 = -99 - 148 - 45$$

$$\Rightarrow D_1 = -34$$

Again

$$\Rightarrow D_2 = \begin{vmatrix} 1 & 11 & -1 \\ 2 & 39 & 2 \\ -3 & 1 & 1 \end{vmatrix}$$

Solving determinant, expanding along 1st row

$$\Rightarrow D_2 = 1[39 - 2] - 11[2 + 6] - 1[2 + 117]$$

$$\Rightarrow D_2 = 1[37] - 11(8) - 119$$

$$\Rightarrow D_2 = -170$$

And,

$$\Rightarrow D_3 = \begin{vmatrix} 1 & -4 & 11 \\ 2 & -5 & 39 \\ -3 & 2 & 1 \end{vmatrix}$$

Solving determinant, expanding along 1st row

$$\Rightarrow D_3 = 1[-5 - (39)(2)] - (-4)[2 - (39)(-3)] + 11[4 - (-5)(-3)]$$

$$\Rightarrow D_3 = 1[-5 - 78] + 4(2 + 117) + 11(4 - 15)$$

$$\Rightarrow D_3 = -83 + 4(119) + 11(-11)$$

$$\Rightarrow D_3 = 272$$

Thus by Cramer's Rule, we have

$$\Rightarrow x = \frac{D_1}{D}$$

$$\Rightarrow x = \frac{-34}{34}$$

$$\Rightarrow x = -1$$

Again,

$$\Rightarrow y = \frac{D_2}{D}$$

$$\Rightarrow y = \frac{-170}{34}$$

$$\Rightarrow y = -5$$

$$\Rightarrow z = \frac{D_3}{D} = (272/34) = 8$$

$$13. 6x + y - 3z = 5$$

$$x + 3y - 2z = 5$$

$$2x + y + 4z = 8$$

Solution:

Given

$$6x + y - 3z = 5$$

$$x + 3y - 2z = 5$$

$$2x + y + 4z = 8$$

Let there be a system of n simultaneous linear equations and with n unknown given by

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

...

$$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$

Let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \dots, x_n = \frac{D_n}{D} \text{ Provided that } D \neq 0$$

Now, here we have

$$6x + y - 3z = 5$$

$$x + 3y - 2z = 5$$

$$2x + y + 4z = 8$$

So by comparing with theorem, now we have to find D , D_1 and D_2

$$\Rightarrow D = \begin{vmatrix} 6 & 1 & -3 \\ 1 & 3 & -2 \\ 2 & 1 & 4 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} Row

$$\Rightarrow D = 6[(4)(3) - (1)(-2)] - 1[(4)(1) + 4] - 3[1 - 3(2)]$$

$$\Rightarrow D = 6[12 + 2] - [8] - 3[-5]$$

$$\Rightarrow D = 84 - 8 + 15$$

$$\Rightarrow D = 91$$

Again, Solve D_1 formed by replacing 1st column by B matrices

Here

$$B = \begin{vmatrix} 5 \\ 5 \\ 8 \end{vmatrix}$$

$$\Rightarrow D_1 = \begin{vmatrix} 5 & 1 & -3 \\ 5 & 3 & -2 \\ 8 & 1 & 4 \end{vmatrix}$$

Solving determinant, expanding along 1st Row

$$\Rightarrow D_1 = 5[(4)(3) - (-2)(1)] - 1[(5)(4) - (-2)(8)] - 3[(5) - (3)(8)]$$

$$\Rightarrow D_1 = 5[12 + 2] - 1[20 + 16] - 3[5 - 24]$$

$$\Rightarrow D_1 = 5[14] - 36 - 3(-19)$$

$$\Rightarrow D_1 = 70 - 36 + 57$$

$$\Rightarrow D_1 = 91$$

Again, Solve D_2 formed by replacing 1st column by B matrices

Here

$$B = \begin{vmatrix} 5 \\ 5 \\ 8 \end{vmatrix}$$

$$\Rightarrow D_2 = \begin{vmatrix} 6 & 5 & -3 \\ 1 & 5 & -2 \\ 2 & 8 & 4 \end{vmatrix}$$

Solving determinant

$$\Rightarrow D_2 = 6[20 + 16] - 5[4 - 2(-2)] + (-3)[8 - 10]$$

$$\Rightarrow D_2 = 6[36] - 5(8) + (-3)(-2)$$

$$\Rightarrow D_2 = 182$$

And, Solve D_3 formed by replacing 1st column by B matrices

Here

$$B = \begin{vmatrix} 5 \\ 5 \\ 8 \end{vmatrix}$$

$$\Rightarrow D_3 = \begin{vmatrix} 6 & 1 & 5 \\ 1 & 3 & 5 \\ 2 & 1 & 8 \end{vmatrix}$$

Solving determinant, expanding along 1st Row

$$\Rightarrow D_3 = 6[24 - 5] - 1[8 - 10] + 5[1 - 6]$$

$$\Rightarrow D_3 = 6[19] - 1(-2) + 5(-5)$$

$$\Rightarrow D_3 = 114 + 2 - 25$$

$$\Rightarrow D_3 = 91$$

Thus by Cramer's Rule, we have

$$\Rightarrow x = \frac{D_1}{D}$$

$$\Rightarrow x = \frac{91}{91}$$

$$\Rightarrow x = 1$$

$$\Rightarrow y = \frac{D_2}{D}$$

$$\Rightarrow y = \frac{182}{91}$$

$$\Rightarrow y = 2$$

$$\Rightarrow z = \frac{D_3}{D}$$

$$\Rightarrow z = \frac{91}{91}$$

$$\Rightarrow z = 1$$

$$14. \quad x + y = 5$$

$$y + z = 3$$

$$x + z = 4$$

Solution:

$$\text{Given } x + y = 5$$

$$y + z = 3$$

$$x + z = 4$$

Let there be a system of n simultaneous linear equations and with n unknown given by

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

...

$$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \dots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$

Let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \dots, x_n = \frac{D_n}{D} \text{ Provided that } D \neq 0$$

Now, here we have

$$x + y = 5$$

$$y + z = 3$$

$$x + z = 4$$

So by comparing with theorem, now we have to find D , D_1 and D_2

$$\Rightarrow D = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix}$$

Solving determinant, expanding along 1st Row

$$\Rightarrow D = 1[1] - 1[-1] + 0[-1]$$

$$\Rightarrow D = 1 + 1 + 0$$

$$\Rightarrow D = 2$$

Again, Solve D_1 formed by replacing 1st column by B matrices

Here

$$B = \begin{vmatrix} 5 \\ 3 \\ 4 \end{vmatrix}$$

$$\Rightarrow D_1 = \begin{vmatrix} 5 & 1 & 0 \\ 3 & 1 & 1 \\ 4 & 0 & 1 \end{vmatrix}$$

Solving determinant, expanding along 1st Row

$$\Rightarrow D_1 = 5[1] - 1[(3)(1) - (4)(1)] + 0[0 - (4)(1)]$$

$$\Rightarrow D_1 = 5 - 1[3 - 4] + 0[-4]$$

$$\Rightarrow D_1 = 5 - 1[-1] + 0$$

$$\Rightarrow D_1 = 5 + 1 + 0$$

$$\Rightarrow D_1 = 6$$

Again, Solve D_2 formed by replacing 2nd column by B matrices

Here

$$B = \begin{vmatrix} 5 \\ 3 \\ 4 \end{vmatrix}$$

$$\Rightarrow D_2 = \begin{vmatrix} 1 & 5 & 0 \\ 0 & 3 & 1 \\ 1 & 4 & 1 \end{vmatrix}$$

Solving determinant

$$\Rightarrow D_2 = 1[3 - 4] - 5[-1] + 0[0 - 3]$$

$$\Rightarrow D_2 = 1[-1] + 5 + 0$$

$$\Rightarrow D_2 = 4$$

And, Solve D_3 formed by replacing 1st column by B matrices

Here

$$B = \begin{vmatrix} 5 \\ 3 \\ 4 \end{vmatrix}$$

$$\Rightarrow D_3 = \begin{vmatrix} 1 & 1 & 5 \\ 0 & 1 & 3 \\ 1 & 0 & 4 \end{vmatrix}$$

Solving determinant, expanding along 1st Row

$$\Rightarrow D_3 = 1[4 - 0] - 1[0 - 3] + 5[0 - 1]$$

$$\Rightarrow D_3 = 1[4] - 1[-3] + 5(-1)$$

$$\Rightarrow D_3 = 4 + 3 - 5$$

$$\Rightarrow D_3 = 2$$

Thus by Cramer's Rule, we have

$$\Rightarrow x = \frac{D_1}{D}$$

$$\Rightarrow x = \frac{6}{2}$$

$$\Rightarrow x = 3$$

$$\Rightarrow y = \frac{D_2}{D}$$

$$\Rightarrow y = \frac{4}{2}$$

$$\Rightarrow y = 2$$

$$\Rightarrow z = \frac{D_3}{D}$$

$$\Rightarrow z = \frac{2}{2}$$

$$\Rightarrow z = 1$$

$$15. 2y - 3z = 0$$

$$x + 3y = -4$$

$$3x + 4y = 3$$

Solution:

Given

$$2y - 3z = 0$$

$$x + 3y = -4$$

$$3x + 4y = 3$$

Let there be a system of n simultaneous linear equations and with n unknown given by

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

...

$$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$

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Let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \dots, x_n = \frac{D_n}{D} \text{ Provided that } D \neq 0$$

Now, here we have

$$2y - 3z = 0$$

$$x + 3y = -4$$

$$3x + 4y = 3$$

So by comparing with theorem, now we have to find D , D_1 and D_2

$$\Rightarrow D = \begin{vmatrix} 0 & 2 & -3 \\ 1 & 3 & 0 \\ 3 & 4 & 0 \end{vmatrix}$$

Solving determinant, expanding along 1st Row

$$\Rightarrow D = 0[0] - 2[(0)(1) - 0] - 3[1(4) - 3(3)]$$

$$\Rightarrow D = 0 - 0 - 3[4 - 9]$$

$$\Rightarrow D = 0 - 0 + 15$$

$$\Rightarrow D = 15$$

Again, Solve D_1 formed by replacing 1st column by B matrices

Here

$$B = \begin{vmatrix} 0 \\ -4 \\ 3 \end{vmatrix}$$

$$\Rightarrow D_1 = \begin{vmatrix} 0 & 2 & -3 \\ -4 & 3 & 0 \\ 3 & 4 & 0 \end{vmatrix}$$

Solving determinant, expanding along 1st Row

$$\Rightarrow D_1 = 0[0] - 2[(0)(-4) - 0] - 3[4(-4) - 3(3)]$$

$$\Rightarrow D_1 = 0 - 0 - 3[-16 - 9]$$

$$\Rightarrow D_1 = 0 - 0 - 3(-25)$$

$$\Rightarrow D_1 = 0 - 0 + 75$$

$$\Rightarrow D_1 = 75$$

Again, Solve D_2 formed by replacing 2nd column by B matrices

Here

$$B = \begin{vmatrix} 0 \\ -4 \\ 3 \end{vmatrix}$$

$$\Rightarrow D_2 = \begin{vmatrix} 0 & 0 & -3 \\ 1 & -4 & 0 \\ 3 & 3 & 0 \end{vmatrix}$$

Solving determinant

$$\Rightarrow D_2 = 0[0] - 0[(0)(1) - 0] - 3[1(3) - 3(-4)]$$

$$\Rightarrow D_2 = 0 - 0 + (-3)(3 + 12)$$

$$\Rightarrow D_2 = -45$$

And, Solve D_3 formed by replacing 3rd column by B matrices

Here

$$B = \begin{vmatrix} 0 \\ -4 \\ 3 \end{vmatrix}$$

$$\Rightarrow D_3 = \begin{vmatrix} 0 & 2 & 0 \\ 1 & 3 & -4 \\ 3 & 4 & 3 \end{vmatrix}$$

Solving determinant, expanding along 1st Row

$$\Rightarrow D_3 = 0[9 - (-4) 4] - 2[(3) (1) - (-4) (3)] + 0[1 (4) - 3 (3)]$$

$$\Rightarrow D_3 = 0[25] - 2(3 + 12) + 0(4 - 9)$$

$$\Rightarrow D_3 = 0 - 30 + 0$$

$$\Rightarrow D_3 = -30$$

Thus by Cramer's Rule, we have

$$\Rightarrow x = \frac{D_1}{D}$$

$$\Rightarrow x = \frac{75}{15}$$

$$\Rightarrow x = 5$$

$$\Rightarrow y = \frac{D_2}{D}$$

$$\Rightarrow y = \frac{-45}{15}$$

$$\Rightarrow y = -3$$

$$\Rightarrow z = \frac{D_3}{D}$$

$$\Rightarrow z = \frac{-30}{15}$$

$$\Rightarrow z = -2$$

$$16. 5x - 7y + z = 11$$

$$6x - 8y - z = 15$$

$$3x + 2y - 6z = 7$$

Solution:

Given

$$5x - 7y + z = 11$$

$$6x - 8y - z = 15$$

$$3x + 2y - 6z = 7$$

Let there be a system of n simultaneous linear equations and with n unknown given by

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

⋮

$$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$

Let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \dots, x_n = \frac{D_n}{D} \text{ Provided that } D \neq 0$$

Now, here we have

$$5x - 7y + z = 11$$

$$6x - 8y - z = 15$$

$$3x + 2y - 6z = 7$$

So by comparing with theorem, now we have to find D , D_1 and D_2

$$\Rightarrow D = \begin{vmatrix} 5 & -7 & 1 \\ 6 & -8 & -1 \\ 3 & 2 & -6 \end{vmatrix}$$

Solving determinant, expanding along 1st Row

$$\Rightarrow D = 5[(-8)(-6) - (-1)(2)] - 7[(-6)(6) - 3(-1)] + 1[2(6) - 3(-8)]$$

$$\Rightarrow D = 5[48 + 2] - 7[-36 + 3] + 1[12 + 24]$$

$$\Rightarrow D = 250 - 231 + 36$$

$$\Rightarrow D = 55$$

Again, Solve D_1 formed by replacing 1st column by B matrices

Here

$$B = \begin{vmatrix} 11 \\ 15 \\ 7 \end{vmatrix}$$

$$\Rightarrow D_1 = \begin{vmatrix} 11 & -7 & 1 \\ 15 & -8 & -1 \\ 7 & 2 & -6 \end{vmatrix}$$

Solving determinant, expanding along 1st Row

$$\Rightarrow D_1 = 11[(-8)(-6) - (2)(-1)] - (-7)[(15)(-6) - (-1)(7)] + 1[(15)(2) - (7)(-8)]$$

$$\Rightarrow D_1 = 11[48 + 2] + 7[-90 + 7] + 1[30 + 56]$$

$$\Rightarrow D_1 = 11[50] + 7[-83] + 86$$

$$\Rightarrow D_1 = 550 - 581 + 86$$

$$\Rightarrow D_1 = 55$$

Again, Solve D_2 formed by replacing 2nd column by B matrices

Here

$$B = \begin{vmatrix} 11 \\ 15 \\ 7 \end{vmatrix}$$

$$\Rightarrow D_2 = \begin{vmatrix} 5 & 11 & 1 \\ 6 & 15 & -1 \\ 3 & 7 & -6 \end{vmatrix}$$

Solving determinant, expanding along 1st Row

$$\Rightarrow D_2 = 5[(15)(-6) - (7)(-1)] - 11[(6)(-6) - (-1)(3)] + 1[(6)(7) - (15)(3)]$$

$$\Rightarrow D_2 = 5[-90 + 7] - 11[-36 + 3] + 1[42 - 45]$$

$$\Rightarrow D_2 = 5[-83] - 11(-33) - 3$$

$$\Rightarrow D_2 = -415 + 363 - 3$$

$$\Rightarrow D_2 = -55$$

And, Solve D_3 formed by replacing 3rd column by B matrices

Here

$$B = \begin{vmatrix} 11 \\ 15 \\ 7 \end{vmatrix}$$

$$\Rightarrow D_3 = \begin{vmatrix} 5 & -7 & 11 \\ 6 & -8 & 15 \\ 3 & 2 & 7 \end{vmatrix}$$

Solving determinant, expanding along 1st Row

$$\Rightarrow D_3 = 5[(-8)(7) - (15)(2)] - (-7)[(6)(7) - (15)(3)] + 11[(6)(2) - (-8)(3)]$$

$$\Rightarrow D_3 = 5[-56 - 30] - (-7)[42 - 45] + 11[12 + 24]$$

$$\Rightarrow D_3 = 5[-86] + 7[-3] + 11[36]$$

$$\Rightarrow D_3 = -430 - 21 + 396$$

$$\Rightarrow D_3 = -55$$

Thus by Cramer's Rule, we have

$$\Rightarrow X = \frac{D_1}{D}$$

$$\Rightarrow X = \frac{55}{55}$$

$$\Rightarrow X = 1$$

$$\Rightarrow Y = \frac{D_2}{D}$$

$$\Rightarrow Y = \frac{-55}{55}$$

$$\Rightarrow Y = -1$$

$$\Rightarrow Z = \frac{D_3}{D}$$

$$\Rightarrow Z = \frac{-55}{55}$$

$$\Rightarrow Z = -1$$

