$\Rightarrow I = \frac{1}{9}\left[\frac{2}{5}(3x - 2)^{\frac{5}{2}} + \frac{4}{3}(3x - 2)^{\frac{3}{2}}\right] + c$

$\Rightarrow I = \frac{2}{45}(3x - 2)^{\frac{5}{2}} + \frac{4}{27}(3x - 2)^{\frac{3}{2}} + c$

$\Rightarrow I = \frac{2}{45}(3x - 2)^{\frac{5}{2}} + \frac{4}{27}(3x - 2)^{\frac{3}{2}} + c$

Ans) $\frac{2}{45}(3x - 2)^{\frac{5}{2}} + \frac{4}{27}(3x - 2)^{\frac{3}{2}} + c$

70. Question

Evaluate the following integrals:

$$\int \frac{dx}{x \cos^2(1 + \log x)}$$

Answer

To find: Value of $\int \frac{dx}{x\cos^2(1 + \log x)}$

Formula used: $\int \sec^2 x \, dx = \tan x + c$

We have, $I = \int \frac{dx}{x\cos^2(1 + \log x)}$ ... (i)

Let $1 + \log x = t$

$\Rightarrow \frac{1}{x} = \frac{dt}{dx}$

$\Rightarrow \frac{1}{x} dx = dt$

Putting this value in equation (i)

$I = \int \frac{dt}{\cos^2(t)}$ [$t = 1 + \log x$]

$\Rightarrow I = \int \sec^2 t \, dt$

$\Rightarrow I = \tan(t) + c$

$\Rightarrow I = \tan(1 + \log x) + c$

Ans) $\tan(1 + \log x) + c$

71. Question

Evaluate the following integrals:

$$\int x^2 \sin x^3 dx$$

Answer

To find: Value of $\int x^2 \sin x^3 \, dx$

Formula used: $\int \sin x \, dx = -\cos x + c$

We have, $I = \int x^2 \sin x^3 \, dx$ ... (i)

Let $x^3 = t$

$$\Rightarrow 3x^2 = \frac{dt}{dx}$$

$$\Rightarrow x^2 dx = \frac{dt}{3}$$

Putting this value in equation (i)

$$I = \int \sin t \frac{dt}{3} \ [\, t = x^3 \,]$$

$$\Rightarrow I = \frac{1}{3}\left[\int \sin t \, dt\right]$$

$$\Rightarrow I = \frac{1}{3}(-\cos t) + c$$

$$\Rightarrow I = \frac{1}{3}(-\cos x^3) + c$$

Ans) $\frac{-\cos x^3}{3} + c$

72. Question

Evaluate the following integrals:

$$\int (2x+4)\sqrt{x^2+4x+3}\, dx$$

Answer

To find: Value of $\int (2x+4)\sqrt{x^2+4x+3}\, dx$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int (2x+4)\sqrt{x^2+4x+3}\, dx$ ... (i)

Let $x^2 + 4x + 3 = t$

$$\Rightarrow (2x+4) = \frac{dt}{dx}$$

$$\Rightarrow (2x+4)dx = dt$$

Putting this value in equation (i)

$$I = \int \sqrt{t}\, dt \ [\, t = (2x+4) \,]$$

$$\Rightarrow I = \int t^{\frac{1}{2}} dx$$

$$\Rightarrow I = \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$\Rightarrow I = \frac{2}{3}\left[(t)^{\frac{3}{2}}\right] + c$$

$$\Rightarrow I = \frac{2}{3}\left[(x^2+4x+3)^{\frac{3}{2}}\right] + c$$

Ans) $\frac{2}{3}\left[(x^2+4x+3)^{\frac{3}{2}}\right]+c$

73. Question

Evaluate the following integrals:

$$\int \frac{\sin x}{(\sin x-\cos x)}dx$$

Answer

To find: Value of $\int \frac{\sin x}{(\sin x - \cos x)}dx$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int \frac{\sin x}{(\sin x - \cos x)}dx$ ... (i)

$$\Rightarrow I = \frac{1}{2}\int \frac{2\sin x}{(\sin x - \cos x)}dx$$

$$\Rightarrow I = \frac{1}{2}\int \frac{(\sin x + \cos x) + (\sin x - \cos x)}{(\sin x - \cos x)}dx$$

$$\Rightarrow I = \frac{1}{2}\int \frac{(\sin x + \cos x)}{(\sin x - \cos x)}dx + \frac{1}{2}\int \frac{(\sin x - \cos x)}{(\sin x - \cos x)}dx$$

Let $\sin x - \cos x = t$

$$\Rightarrow (\cos x + \sin x) = \frac{dt}{dx}$$

$$\Rightarrow (\cos x + \sin x)dx = dt$$

Putting this value in equation (i)

$$I = \frac{1}{2}\int \frac{dt}{t} + \frac{1}{2}\int dx$$

$$\Rightarrow I = \frac{1}{2}\log|\sin x - \cos x| + \frac{1}{2}x + c$$

$$\Rightarrow I = \frac{x}{2} + \frac{1}{2}\log|\sin x - \cos x| + c$$

Ans) $\frac{x}{2} + \frac{1}{2}\log|\sin x - \cos x| + c$

74. Question

Evaluate the following integrals:

$$\int \frac{dx}{(1-\tan x)}$$

Answer

To find: Value of $\int \frac{dx}{(1 - \tan x)}$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int \frac{dx}{(1 - \tan x)}$ ... (i)

$$\Rightarrow I = \int \frac{dx}{\left(1 - \frac{\sin x}{\cos x}\right)}$$

$$\Rightarrow I = \int \frac{dx}{\left(\frac{\cos x - \sin x}{\cos x}\right)}$$

$$\Rightarrow I = \frac{1}{2}\int \frac{2\cos x dx}{(\cos x - \sin x)}$$

$$I = \frac{1}{2}\int \frac{(\cos x + \sin x) + (\cos x - \sin x)dx}{(\cos x - \sin x)}$$

$$I = \frac{1}{2}\int \frac{(\cos x + \sin x)}{(\cos x - \sin x)}dx + \frac{1}{2}\int \frac{(\cos x - \sin x)}{(\cos x - \sin x)}dx$$

Let $(\cos x - \sin x) = t$

$$\Rightarrow (-\sin x - \cos x) = \frac{dt}{dx}$$

$$\Rightarrow (\sin x + \cos x)dx = -dt$$

Putting this value in equation (i)

$$I = -\frac{1}{2}\int \frac{dt}{(t)}\,dx + \frac{1}{2}\int dx$$

$$\Rightarrow I = -\frac{1}{2}\log|\cos x - \sin x| + \frac{1}{2}x + c$$

$$\Rightarrow I = \frac{1}{2}x - \frac{1}{2}\log|\sin x - \cos x| + c$$

Ans) $\frac{1}{2}x - \frac{1}{2}\log|\sin x - \cos x| + c$

75. Question

Evaluate the following integrals:

$$\int \frac{dx}{(1 - \cot x)}$$

Answer

To find: Value of $\int \frac{dx}{(1 - \cot x)}$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int \frac{dx}{(1 - \cot x)}$ ... (i)

$$\Rightarrow I = \int \frac{dx}{\left(1 - \frac{\cos x}{\sin x}\right)}$$

$$\Rightarrow I = \int \frac{dx}{\left(\frac{\sin x - \cos x}{\sin x}\right)}$$

$\Rightarrow I = \frac{1}{2}\int \frac{2\sin x dx}{(\sin x - \cos x)}$

$I = \frac{1}{2}\int \frac{(\sin x + \cos x) + (\sin x - \cos x)dx}{(\sin x - \cos x)}$

$I = \frac{1}{2}\int \frac{(\sin x + \cos x)}{(\sin x - \cos x)}dx + \frac{1}{2}\int \frac{(\sin x - \cos x)}{(\sin x - \cos x)}dx$

Let $(\sin x - \cos x) = t$

$\Rightarrow (\cos x + \sin x) = \frac{dt}{dx}$

$\Rightarrow (\cos x + \sin x)dx = dt$

Putting this value in equation (i)

$I = \frac{1}{2}\int \frac{dt}{(t)}dx + \frac{1}{2}\int dx$

$\Rightarrow I = \frac{1}{2}\log|\sin x - \cos x| + \frac{1}{2}x + c$

Ans) $\frac{1}{2}x + \frac{1}{2}\log|\sin x - \cos x| + c$

76. Question

Evaluate the following integrals:

$\int \frac{\cos 2x}{(\sin x + \cos x)}dx$

Answer

To find: Value of $\int \frac{\cos 2x}{(\sin x + \cos x)^2}dx$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int \frac{\cos 2x}{(\sin x + \cos x)^2}dx$ ... (i)

$\Rightarrow I = \int \frac{\cos^2 x - \sin^2 x}{(\sin x + \cos x)^2}dx$

$\Rightarrow I = \int \frac{(\cos x - \sin x)(\cos x + \sin x)}{(\sin x + \cos x)^2}dx$

$\Rightarrow I = \int \frac{(\cos x - \sin x)}{(\sin x + \cos x)}dx$

Let $(\cos x + \sin x) = t$

$\Rightarrow (-\sin x + \cos x) = \frac{dt}{dx}$

$\Rightarrow (\cos x - \sin x)dx = dt$

Putting this value in equation (i)

$I = \int \frac{dt}{t}$

$\Rightarrow I = \log|t| + c$

$\Rightarrow I = \log|\cos x + \sin x| + c$

Ans) $\log|\cos x + \sin x| + c$

77. Question

Evaluate the following integrals:

$$\int \frac{(\cos x - \sin x)}{(1 + \sin 2x)} dx$$

Answer

To find: Value of $\int \frac{(\cos x - \sin x)}{(1 + \sin 2x)} dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{(\cos x - \sin x)}{(1 + \sin 2x)} dx$ ... (i)

$$\Rightarrow I = \int \frac{\cos x - \sin x}{\cos^2 x + \sin^2 x + 2\sin x\cos x} dx$$

$$\Rightarrow I = \int \frac{(\cos x - \sin x)}{(\cos x + \sin x)^2} dx$$

Let $(\sin x + \cos x) = t$

$$\Rightarrow (\cos x - \sin x) = \frac{dt}{dx}$$

$\Rightarrow (\cos x - \sin x)dx = dt$

Putting this value in equation (i)

$$I = \int \frac{dt}{t^2}$$

$$\Rightarrow I = -\frac{1}{t} + c$$

$$\Rightarrow I = -\frac{1}{\sin x + \cos x} + c$$

Ans) $\frac{-1}{\sin x + \cos x} + c$

78. Question

Evaluate the following integrals:

$$\int \frac{(x+1)(x+\log x)^2}{x} dx$$

Answer

To find: Value of $\int \frac{(x+1)(x+\log x)^2}{x} dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{(x+1)(x+\log x)^2}{x} dx$ ... (i)

Let $(x + \log x) = t$

$\Rightarrow \left(1 + \frac{1}{x}\right) = \frac{dt}{dx}$

$\Rightarrow \left(\frac{x+1}{x}\right) = \frac{dt}{dx}$

Putting this value in equation (i)

$I = \int t^2 dt$

$\Rightarrow I = \frac{t^3}{3} + c$

$\Rightarrow I = \frac{(x+\log x)^3}{3} + c$

Ans) $\frac{(x+\log x)^3}{3} + c$

79. Question

Evaluate the following integrals:

$\int x \sin^3 x^2 \cos x^2 dx$

Answer

To find: Value of $\int x\sin^3 x^2 \cos x^2 dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int x\sin^3 x^2 \cos x^2 dx$ ... (i)

Let $(\sin x^2) = t$

$\Rightarrow (\sin x^2 . 2x) = \frac{dt}{dx}$

$\Rightarrow (\sin x^2 . x) dx = \frac{dt}{2}$

Putting this value in equation (i)

$I = \int t^3 \frac{dt}{2}$

$I = \frac{1}{2} \int t^3 dt$

$\Rightarrow I = \frac{1}{2}\frac{t^4}{4} + c$

$\Rightarrow I = \frac{t^4}{8} + c$

$\Rightarrow I = \frac{\sin^4 x^2}{8} + c$

Ans) $\frac{\sin^4 x^2}{8} + c$

80. Question

Evaluate the following integrals:

$$\int \frac{\sec^2 x}{\sqrt{1 - \tan^2 x}} dx$$

Answer

To find: Value of $\int \frac{\sec^2 x}{\sqrt{1 - \tan^2 x}} dx$

Formula used: $\int \frac{1}{\sqrt{1 - x^2}} dx = \sin^{-1} x + c$

We have, $I = \int \frac{\sec^2 x}{\sqrt{1 - \tan^2 x}} dx$ ... (i)

Let $(\tan x) = t$

$\Rightarrow (\sec^2 x) = \frac{dt}{dx}$

$\Rightarrow (\sec^2 x)dx = dt$

Putting this value in equation (i)

$I = \int \frac{dt}{\sqrt{1 - t^2}}$

$\Rightarrow I = \sin^{-1}(t) + c$

$\Rightarrow I = \sin^{-1}(\tan x) + c$

Ans) $\sin^{-1}(\tan x) + c$

81. Question

Evaluate the following integrals:

$$\int e^{-x} \text{cosec}^2 \left(2e^{-x} + 5\right) dx$$

Answer

To find: Value of $\int e^{-x} \text{cosec}^2(2e^{-x} + 5)\, dx$

Formula used: $\int \text{cosec}^2 x\, dx = -\cot x + c$

We have, $I = \int e^{-x} \text{cosec}^2(2e^{-x} + 5)\, dx$ ... (i)

Let $(2e^{-x} + 5) = t$

$\Rightarrow (2e^{-x}(-1)) = \frac{dt}{dx}$

$\Rightarrow (e^{-x})dx = \frac{dt}{-2}$

Putting this value in equation (i)

$I = \int \text{cosec}^2(t) \frac{dt}{-2}$

$I = \frac{1}{-2}\int \text{cosec}^2(t)\, dt$

$\Rightarrow I = \frac{1}{-2}(-\cot t) + c$

$\Rightarrow I = \frac{1}{2}\cot(2e^{-x} + 5) + c$

Ans) $\frac{1}{2}\cot(2e^{-x} + 5) + c$

82. Question

Evaluate the following integrals:

$\int 2x\sec^3\left(x^2+3\right)\tan\left(x^2+3\right)dx$

Answer

To find: Value of $\int 2x\sec^3(x^2+3)\ \tan(x^2+3)dx$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int 2x\sec^2(x^2+3)\sec(x^2+3)\ \tan(x^2+3)dx$ ... (i)

Let $\sec(x^2+3) = t$

$\Rightarrow \sec(x^2+3) = \frac{dt}{dx}$

$\Rightarrow \sec(x^2+3)\tan(x^2+3).2x = \frac{dt}{dx}$

$\Rightarrow \sec(x^2+3)\tan(x^2+3).2x = \frac{dt}{dx}$

Putting this value in equation (i)

$I = \int t^2\, dt$

$\Rightarrow I = \frac{t^3}{3} + c$

$\Rightarrow I = \frac{\sec^3(x^2+3)}{3} + c$

Ans) $\frac{\sec^3(x^2+3)}{3} + c$

83. Question

Evaluate the following integrals:

$\int \frac{\sin 2x}{(a+b\cos x)^2}dx$

Answer

To find: Value of $\int \frac{\sin 2x}{(a+b\cos x)^2}dx$

Formula used: (i) $\int \frac{1}{x}dx = \log|x| + c$

(ii) $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int \frac{\sin 2x}{(a+b\cos x)^2}dx$ ... (i)

$$I = \int \frac{2\sin x\cos x}{(a+b\cos x)^2}dx$$

Let $(a + b\cos x) = t$

$$\Rightarrow (\cos x) = \frac{t-a}{b}$$

$$\Rightarrow (\sin x)dx = \frac{dt}{-b}$$

Putting this value in equation (i)

$$I = \frac{2}{-b^2}\int \frac{t-a}{t^2}\,dt$$

$$I = \frac{2}{-b^2}\left[\int \frac{t}{t^2}\,dt - \int \frac{a}{t^2}\,dt\right]$$

$$I = \frac{2}{-b^2}\left[\int \frac{1}{t}\,dt - a\int \frac{1}{t^2}\,dt\right]$$

$$I = \frac{2}{-b^2}\left[\log|t| - a\left(-\frac{1}{t}\right) + c\right]$$

$$I = -\frac{2}{b^2}\left[\log|a + b\cos x| + \left(\frac{a}{a+b\cos x}\right)\right] + c$$

Ans) $-\frac{2}{b^2}\left[\log|a + b\cos x| + \left(\frac{a}{a+b\cos x}\right)\right] + c$

84. Question

Evaluate the following integrals:

$$\int \frac{dx}{(3-5x)}$$

Answer

To find: Value of $\int \frac{dx}{(3-5x)}$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int \frac{dx}{(3-5x)}$ ... (i)

Let $(3 - 5x) = t$

$$\Rightarrow (-5) = \frac{dt}{dx}$$

$\Rightarrow dx = \frac{dt}{-5}$

Putting this value in equation (i)

$I = \int \frac{1}{t} \frac{dt}{-5}$

$I = \frac{1}{-5} \int \frac{dt}{t}$

$\Rightarrow I = \frac{1}{-5} \log|t| + c$

$\Rightarrow I = -\frac{1}{5} \log|3 - 5x| + c$

Ans) $-\frac{1}{5} \log|3 - 5x| + c$

85. Question

Evaluate the following integrals:

$\int \sqrt{1+x}\, dx$

Answer

To find: Value of $\int \sqrt{1+x}\, dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \sqrt{1+x}\, dx$ ... (i)

Let $(1 + x) = t$

$\Rightarrow dx = dt$

Putting this value in equation (i)

$I = \int \sqrt{t}\, dt$

$I = \int t^{\frac{1}{2}} dt$

$\Rightarrow I = \frac{2}{3}(1+x)^{\frac{3}{2}} + c$

Ans) $\frac{2}{3}(1+x)^{\frac{3}{2}} + c$

86. Question

Evaluate the following integrals:

$\int x^2 e^{x^3} \cos\left(e^{x^3}\right) dx$

Answer

To find: Value of $\int x^2 e^{x^3} \cos(e^{x^3}) dx$

Formula used: $\int \cos x\, dx = \sin x + c$

We have, $I = \int x^2 e^{x^3} \cos(e^{x^3}) dx$ ... (i)

Let $e^{x^3} = t$

$\Rightarrow e^{x^3}.3x^2 = \frac{dt}{dx}$

$\Rightarrow e^{x^3}.x^2.dx = \frac{dt}{3}$

Putting this value in equation (i)

$I = \int \cos(t) \frac{dt}{3}$

$I = \frac{\sin(t)}{3} + c$

$I = \frac{\sin\left(e^{x^3}\right)}{3} + c$

Ans) $\frac{\sin\left(e^{x^3}\right)}{3} + c$

87. Question

Evaluate the following integrals:

$\int \frac{e^{m \tan^{-1} x}}{\left(1+x^2\right)} dx$

Answer

To find: Value of $\int \frac{e^{m\tan^{-1}x} dx}{(1+x^2)}$

Formula used: $\int e^t dx = e^t + c$

We have, $I = \int \frac{e^{m\tan^{-1}x} dx}{(1+x^2)}$ ... (i)

Let $(m\tan^{-1}x) = t$

$\Rightarrow m\left(\frac{1}{1+x^2}\right) = \frac{dt}{dx}$

$\Rightarrow \left(\frac{1}{1+x^2}\right)dx = \frac{dt}{m}$

Putting this value in equation (i)

$I = \int e^t \frac{dt}{m}$

$\Rightarrow I = \frac{e^t}{m} + c$

$\Rightarrow I = \frac{e^{m\tan^{-1}x}}{m} + c$

Ans) $\frac{e^{m\tan^{-1}x}}{m} + c$

88. Question

Evaluate the following integrals:

$$\int \frac{(x+1)e^x}{\cos^2(xe^x)}dx$$

Answer

To find: Value of $\int \frac{(x+1)e^x\,dx}{\cos^2(xe^x)}$

Formula used: $\int \sec^2 x\,dx = \tan x + c$

We have, $I = \int \frac{(x+1)e^x\,dx}{\cos^2(xe^x)}$ ... (i)

Let $(xe^x) = t$

$\Rightarrow xe^x + e^x.1 = \frac{dt}{dx}$

$\Rightarrow e^x(x+1) = \frac{dt}{dx}$

Putting this value in equation (i)

$I = \int \frac{dt}{\cos^2(t)}$

$\Rightarrow I = \int \sec^2(t)\,dt$

$\Rightarrow I = \tan(t) + c$

$\Rightarrow I = \tan(xe^x) + c$

Ans) $\tan(xe^x) + c$

89. Question

Evaluate the following integrals:

$$\int \frac{e^{\sqrt{x}}\cos\left(e^{\sqrt{x}}\right)}{\sqrt{x}}dx$$

Answer

To find: Value of $\int \frac{e^{\sqrt{x}}\cos\left(e^{\sqrt{x}}\right)dx}{\sqrt{x}}$

Formula used: $\int \cos x\,dx = \sin x + c$

We have, $I = \int \frac{e^{\sqrt{x}}\cos\left(e^{\sqrt{x}}\right)dx}{\sqrt{x}}$ ... (i)

Let $\left(e^{\sqrt{x}}\right) = t$

$\Rightarrow e^{\sqrt{x}}\frac{1}{2\sqrt{x}} = \frac{dt}{dx}$

$\Rightarrow \frac{e^{\sqrt{x}}}{\sqrt{x}}dx = 2dt$

Putting this value in equation (i)

$I = \int \cos(t)\, 2dt$

$I = 2\sin\left(e^{\sqrt{x}}\right) + c$

Ans) $2\sin\left(e^{\sqrt{x}}\right) + c$

90. Question

Evaluate the following integrals:

$\int \sqrt{e^x - 1}\,dx$

Answer

To find: Value of $\int \sqrt{e^x - 1}\ dx$

Formula used: $\int \frac{1}{x^2+1}dx = \tan^{-1}x + c$

We have, $I = \int \sqrt{e^x - 1}\ dx \ldots$ (i)

Let $(e^x - 1) = t^2$

$\Rightarrow e^x - 1 = t^2$

$\Rightarrow e^x = t^2 + 1$

$\Rightarrow e^x = \frac{2tdt}{dx}$

$\Rightarrow dx = \frac{2tdt}{e^x}$

$\Rightarrow dx = \frac{2t}{t^2+1}dt$

Putting this value in equation (i)

$I = \int \sqrt{t^2}\,\frac{2t}{t^2+1}dt$

$\Rightarrow I = \int \frac{2t^2}{t^2+1}dt$

$\Rightarrow I = 2\int \frac{t^2+1-1}{t^2+1}dt$

$\Rightarrow I = 2\int \left(1 - \frac{1}{t^2+1}\right)dt$

$\Rightarrow I = 2\,[\,t - \tan^{-1}t] + c$

$\Rightarrow I = 2\,[\,\sqrt{e^x-1} - \tan^{-1}\sqrt{e^x-1}] + c$

Ans) $2\,[\,\sqrt{e^x-1} - \tan^{-1}\sqrt{e^x-1}] + c$

91. Question

Evaluate the following integrals:

Answer

To find: Value of $\int \frac{dx}{(x-\sqrt{x})}$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int \frac{dx}{(x-\sqrt{x})}$ ... (i)

$$\Rightarrow I = \int \frac{dx}{\sqrt{x}(\sqrt{x}-1)}$$

Let $(\sqrt{x}-1) = t$

$$\Rightarrow \frac{1}{2\sqrt{x}} = \frac{dt}{dx}$$

$$\Rightarrow \frac{1}{\sqrt{x}}dx = \frac{dt}{2}$$

Putting this value in equation (i)

$$I = \int \frac{1}{t}\frac{dt}{2}$$

$$I = \frac{1}{2}\log|t| + c$$

$$I = \frac{1}{2}\log|\sqrt{x}-1| + c$$

Ans) $\frac{1}{2}\log|\sqrt{x}-1| + c$

92. Question

Evaluate the following integrals:

$$\int \frac{\sec^2\left(2\tan^{-1}x\right)}{\left(1+x^2\right)}dx$$

Answer

To find: Value of $\int \frac{\sec^2(2\tan^{-1}x)}{(1+x^2)}dx$

Formula used: $\int \sec^2 x\,dx = \tan x + c$

We have, $I = \int \frac{\sec^2(2\tan^{-1}x)}{(1+x^2)}dx$ ... (i)

Let $2\tan^{-1}x = t$

$$\Rightarrow \frac{2}{1+x^2} = \frac{dt}{dx}$$

$$\Rightarrow \frac{1}{1+x^2}dx = \frac{dt}{2}$$

Putting this value in equation (i)

$I = \int \sec^2(t)\frac{dt}{2}$

$I = \frac{1}{2}\tan(t) + c$

$I = \frac{1}{2}\tan(2\tan^{-1}x) + c$

Ans) $\frac{1}{2}\tan(2\tan^{-1}x) + c$

93. Question

Evaluate the following integrals:

$$\left(\frac{1+\sin 2x}{x+\sin^2 x}\right)dx$$

Answer

To find: Value of $\int \left(\frac{1+\sin 2x}{x+\sin^2 x}\right)dx$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int \left(\frac{1+\sin 2x}{x+\sin^2 x}\right)dx$ ... (i)

Let $x + \sin^2 x = t$

$\Rightarrow 1 + 2\sin x.\cos x = \frac{dt}{dx}$

$\Rightarrow (1 + \sin 2x)\,dx = dt$

Putting this value in equation (i)

$I = \int \frac{dt}{t}$

$I = \log|t| + c$

$I = \log\left|x + \sin^2 x\right| + c$

Ans) $\log\left|x + \sin^2 x\right| + c$

94. Question

Evaluate the following integrals:

$$\int\left(\frac{1-\tan x}{x+\log\cos x}\right)dx$$

Answer

To find: Value of $\int \left(\frac{1-\tan x}{x+\log(\cos x)}\right)dx$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int \left(\frac{1-\tan x}{x+\log(\cos x)}\right)dx$ ... (i)

Let $x + \log(\cos x) = t$

$$\Rightarrow 1 + \frac{1.(-\sin x)}{\cos x} = \frac{dt}{dx}$$

$$\Rightarrow 1 - \tan x = \frac{dt}{dx}$$

$$\Rightarrow (1 - \tan x)dx = dt$$

Putting this value in equation (i)

$$I = \int \frac{dt}{t}$$

$$I = \log|t| + c$$

$$I = \log|x + \log(\cos x)| + c$$

Ans) $\log|x + \log(\cos x)| + c$

95. Question

Evaluate the following integrals:

$$\int \frac{(1 + \cot x)}{(x + \log \sin x)} dx$$

Answer

To find: Value of $\int \left(\frac{1 + \cot x}{x + \log(\sin x)}\right) dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \left(\frac{1 + \cot x}{x + \log(\sin x)}\right) dx$ ... (i)

Let $x + \log(\sin x) = t$

$$\Rightarrow 1 + \frac{1.(\cos x)}{\sin x} = \frac{dt}{dx}$$

$$\Rightarrow 1 + \cot x = \frac{dt}{dx}$$

$$\Rightarrow (1 + \cot x)dx = dt$$

Putting this value in equation (i)

$$I = \int \frac{dt}{t}$$

$$I = \log|x + \log(\sin x)| + c$$

$$I = \log|x + \log(\sin x)| + c$$

Ans) $\log|x + \log(\sin x)| + c$

96. Question

Evaluate the following integrals:

$$\int \frac{\tan x \sec^2 x}{(1 - \tan^2 x)} dx$$

Answer

To find: Value of $\int \frac{\tan x \sec^2 x}{(1 - \tan^2 x)} dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \frac{\tan x \sec^2 x}{(1 - \tan^2 x)} dx$ ... (i)

Let $1 - \tan^2 x = t$

$\Rightarrow 0 - 2.\tan x.\sec^2 x = \frac{dt}{dx}$

$\Rightarrow (\tan x.\sec^2 x)dx = \frac{dt}{-2}$

$\Rightarrow (1 + \cot x)dx = dt$

Putting this value in equation (i)

$I = \int \frac{1}{t} \frac{dt}{(-2)}$

$I = \frac{1}{2} \log|t| + c$

$I = \frac{1}{2} \log|1 - \tan^2 x| + c$

Ans) $\frac{1}{2} \log|1 - \tan^2 x| + c$

97. Question

Evaluate the following integrals:

$\int \frac{\sin(2\tan^{-1} x)}{(1 + x^2)} dx$

Answer

To find: Value of $\int \frac{\sin(2\tan^{-1} x)}{(1 + x^2)} dx$

Formula used: $\int \sin x \, dx = \cos x + c$

We have, $I = \int \frac{\sin(2\tan^{-1} x)}{(1 + x^2)} dx$ ... (i)

Let $2\tan^{-1} x = t$

$\Rightarrow 2 \frac{1}{1 + x^2} = \frac{dt}{dx}$

$\Rightarrow \frac{dx}{1 + x^2} = \frac{dt}{2}$

$\Rightarrow (1 + \cot x)dx = dt$

Putting this value in equation (i)

$$I = \int \sin(t)\frac{dt}{(2)}$$

$$I = -\frac{1}{2}\cos(t) + c$$

$$I = -\frac{1}{2}\cos(2\tan^{-1}x) + c$$

Ans) $-\frac{1}{2}\cos(2\tan^{-1}x) + c$

98. Question

Evaluate the following integrals:

$$\int \frac{dx}{\left(x^{1/2} + x^{1/3}\right)}$$

Answer

To find: Value of $\int \frac{dx}{\left(x^{\frac{1}{2}} + x^{\frac{1}{3}}\right)}$

Formula used: (i) $\int \frac{1}{x}dx = \log|x| + c$

(ii) $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int \frac{dx}{\left(x^{\frac{1}{2}} + x^{\frac{1}{3}}\right)}$ ... (i)

Let $x = t^6$

$\Rightarrow x^{\frac{1}{6}} = t$

$\Rightarrow 6t^5\,dt = dx$

Putting this value in equation (i)

$$I = \int \frac{6t^5\,dt}{(t^3 + t^2)}$$

$$I = \int \frac{6t^5\,dt}{t^2(t+1)}$$

$$I = 6\int \frac{t^3\,dt}{(t+1)}$$

$$I = 6\int \frac{t^3 + 1 - 1}{(t+1)}dt$$

$$I = 6\int \frac{(t+1)(t^2 - t + 1)}{(t+1)}dt - \int \frac{1}{(t+1)}dt$$

$$I = 6\left[\frac{t^3}{3} - \frac{t^2}{2} + t - \log|t+1|\right] + c$$

$$I = \left[2t^3 - 3t^2 + 6t - 6\log|t+1|\right] + c$$

$$I = \left[2\left(x^{\frac{1}{6}}\right)^3 - 3\left(x^{\frac{1}{6}}\right)^2 + 6\left(x^{\frac{1}{6}}\right) - 6\log\left|\left(x^{\frac{1}{6}}\right) + 1\right|\right] + c$$

$$I = \left[2\sqrt{x} - 3\left(x^{\frac{1}{3}}\right) + 6\left(x^{\frac{1}{6}}\right) - 6\log\left|\left(x^{\frac{1}{6}}\right) + 1\right|\right] + c$$

Ans) $\left[2\sqrt{x} - 3\left(x^{\frac{1}{3}}\right) + 6\left(x^{\frac{1}{6}}\right) - 6\log\left|\left(x^{\frac{1}{6}}\right) + 1\right|\right] + c$

99. Question

Evaluate the following integrals:

$$\int\left(\sin^{-1}x\right)^2 dx$$

Answer

To find: Value of $\int(\sin^{-1}x)^2 dx$

Formula used: $\int \sin x\, dx = \cos x + c$

We have, $I = \int(\sin^{-1}x)^2 dx$ ... (i)

Let $\sin^{-1}x = t$, $x = \sin t$,

$\Rightarrow \cos t = \sqrt{1 - x^2}$

$\Rightarrow \frac{1}{\sqrt{1 - x^2}} = \frac{dt}{dx}$

$\Rightarrow \sqrt{1 - x^2}\,dt = dx$

$\Rightarrow \sqrt{1 - (\sin t)^2}\,dt = dx$

$\Rightarrow \sqrt{1 - \sin^2 t}\,dt = dx$

$\Rightarrow \cos t\,dt = dx$

Putting this value in equation (i)

$$I = \int t^2 \cos t\ dt$$

$$I = \int t^2 \cos t\ dt - \int\left[\frac{d(t^2)}{dt}\int \cos t\ dt\right]dt$$

$$I = t^2 \sin t - \int[2t.\sin t]dt$$

$$I = t^2 \sin t - 2\left\{\int t[\sin t]dt - \int\left[\frac{dt}{dt}\int \sin t\, dt\right]dt\right\}$$

$$I = t^2 \sin t - 2\left[-t\cos t + \int 1.\cos t\, dt\right]$$

$$I = t^2 \sin t + 2t\cos t - 2\sin t + c$$

$$I = (\sin^{-1}x)^2 x + 2(\sin^{-1}x)\sqrt{1 - x^2} - 2x + c$$

Ans) $(\sin^{-1}x)^2 x + 2(\sin^{-1}x)\sqrt{1-x^2} - 2x + c$

100. Question

Evaluate the following integrals:

$$\int \frac{2x\tan^{-1}x^2}{\left(1+x^4\right)}dx$$

Answer

To find: Value of $\int \frac{2x\tan^{-1}(x^2)}{(1+x^4)}dx$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int \frac{2x\tan^{-1}(x^2)}{(1+x^4)}dx$ ... (i)

Let $\tan^{-1}(x^2) = t$

$$\Rightarrow \frac{1}{1+(x^2)^2}.\,2x = \frac{dt}{dx}$$

$$\Rightarrow \frac{2x}{1+x^4}dx = dt$$

Putting this value in equation (i)

$$I = \int t.\,dt$$

$$I = \frac{t^2}{2} + c$$

$$I = \frac{\{\tan^{-1}(x^2)\}^2}{2} + c$$

Ans) $\frac{\{\tan^{-1}(x^2)\}^2}{2} + c$

101. Question

Evaluate the following integrals:

$$\int \frac{\left(x^2+1\right)}{\left(x^4+1\right)}dx$$

Answer

To find: Value of $\int \frac{(x^2+1)}{(x^4+1)}dx$

Formula used: $\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

We have, $I = \int \frac{(x^2+1)}{(x^4+1)}dx$ ... (i)

Dividing Numerator and Denominator by x^2,

$$I = \int \frac{\left(1 + \frac{1}{x^2}\right)}{\left(x^2 + \frac{1}{x^2} + 2 - 2\right)} dx$$

$$I = \int \frac{\left(1 + \frac{1}{x^2}\right)}{\left(x^2 - 2.x.\frac{1}{x} + \left(\frac{1}{x}\right)^2 + 2\right)} dx$$

$$I = \int \frac{\left(1 + \frac{1}{x^2}\right)}{\left(\left(x - \frac{1}{x}\right)^2 + (\sqrt{2})^2\right)} dx$$

Let $x - \frac{1}{x} = t$

$$\Rightarrow \left(1 + \frac{1}{x^2}\right) dx = dt$$

Putting this value in equation (i)

$$I = \int \frac{1}{(t)^2 + (\sqrt{2})^2} dt$$

$$I = \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{t}{\sqrt{2}}\right) + c$$

$$I = \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x - \frac{1}{x}}{\sqrt{2}}\right) + c$$

$$I = \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x^2 - 1}{\sqrt{2}x}\right) + c$$

Ans) $\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x^2 - 1}{\sqrt{2}x}\right) + c$

102. Question

Evaluate the following integrals:

$$\int \frac{(\sin x + \cos x)}{\sqrt{\sin 2x}} dx$$

Answer

To find: Value of $\int \frac{(\sin x + \cos x)}{\sqrt{\sin 2x}} dx$

Formula used: $\int \frac{1}{\sqrt{1 - x^2}} dx = \sin^{-1} x + c$

We have, $I = \int \frac{(\sin x + \cos x)}{\sqrt{\sin 2x}} dx$ ... (i)

Let (sinx – cosx) = t

$$\Rightarrow (\cos x + \sin x) = \frac{dt}{dx}$$

$$\Rightarrow (\cos x + \sin x)\, dx = dt$$

$\Rightarrow t^2 = \sin^2 x - 2\sin x.\cos x + \cos^2 x$

$\Rightarrow t^2 = 1 - 2\sin x.\cos x$

$\Rightarrow 2\sin x.\cos x = 1 - t^2$

$\Rightarrow \sin 2x = 1 - t^2$

Putting this value in equation (i)

$$\Rightarrow I = \int \frac{dt}{\sqrt{1-t^2}}$$

$I = \sin^{-1} t$

$I = \sin^{-1}(\sin x - \cos x)$

Let $\sin^{-1}(\sin x - \cos x) = \theta$

$\Rightarrow I = \sin^{-1}(\sin x - \cos x) = \theta$ ... (ii)

$\Rightarrow \sin\theta = \sin x - \cos x$

Now if $\sin\theta = \sin x - \cos x$

Then $\cos\theta = \sqrt{1 - (\sin x - \cos x)^2}$

$\Rightarrow \cos\theta = \sqrt{1 - (\sin^2 x - 2\sin x.\cos x + \cos^2 x)}$

$\Rightarrow \cos\theta = \sqrt{1 - (1 - 2\sin x.\cos x)}$

$\Rightarrow \cos\theta = \sqrt{2\sin x.\cos x)}$

Now $\tan\theta = \frac{\sin\theta}{\cos\theta}$

Now $\tan\theta = \frac{\sin x - \cos x}{\sqrt{2\sin x.\cos x)}}$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{\sin x - \cos x}{\sqrt{2\sin x.\cos x)}}\right)$$

Comparing the value θ from eqn. (ii)

$$I = \theta = \tan^{-1}\left(\frac{\sin x - \cos x}{\sqrt{2\sin x.\cos x)}}\right)$$

Dividing Numerator and denominator from cosx

$$I = \theta = \tan^{-1}\left(\frac{\tan x - 1}{\sqrt{2\tan x)}}\right)$$

Ans.) $\tan^{-1}\left(\frac{\tan x - 1}{\sqrt{2\tan x)}}\right)$

Objective Questions I

1. Question

Mark (√) against the correct answer in each of the following:

$$\int (2x+3)^5 \, dx = ?$$

A. $\frac{(2x+3)^6}{6}+C$

B. $\frac{(2x+3)^4}{8}+C$

C. $\frac{(2x+3)^6}{12}+C$

D. none of these

Answer

Given = $\int(2x+3)^5$

Let, 2x + 3 = z

⇒ 2dx = dz

So,

$\int(2x+3)^5 dx$

$=\int\frac{z^5}{2}dz$

$=\frac{1}{2}\frac{z^6}{6}+c$ where c is the integrating constant.

$=\frac{z^6}{12}+c$

$=\frac{(2x+3)^6}{12}+c$

2. Question

Mark (√) against the correct answer in each of the following:

$\int(3-5x)^7 dx = ?$

A. $-5(3-5x)^6 + C$

B. $\frac{(3-5x)^8}{-40}+C$

C. $\frac{-5(3-5x)^8}{8}+C$

D. none of these

Answer

Given = $\int(3-5x)^7$

Let, 3 - 5x = z

⇒ -5dx = dz

So,

$\int (3-5x)^7 dx$

$= -\int \frac{z^7}{5} dz$

$= -\frac{1}{5}\frac{z^8}{8} + c$ where c is the integrating constant.

$= -\frac{z^8}{40} + c$

$= -\frac{(3-5x)^8}{40} + c$

3. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{1}{(2-3x)^4} dx = ?$

A. $\frac{1}{15(2-3x)^5} + C$

B. $\frac{1}{-12(2-3x)^3} + C$

C. $\frac{1}{9(2-3x)^3} + C$

D. none of these

Answer

Given = $\int \frac{1}{(2-3x)^4}$

Let, 2 - 3x = z

⇒ -3dx = dz

So,

$$\int \frac{1}{(2-3x)^4} dx$$

$$= \int \frac{1}{z^4}\left(\frac{dz}{-3}\right)$$

$$= -\frac{1}{3}\int \frac{dz}{z^4}$$

where c is the integrating constant.

$$= -\frac{1}{3}\int z^{-4} dz$$

$$= -\frac{1}{3}\frac{z^{-3}}{-3} + c$$

$$= \frac{1}{9(2-3x)^3} + c$$

4. Question

Mark (√) against the correct answer in each of the following:

$$\int \sqrt{ax+b}\, dx = ?$$

A. $\frac{2(ax+b)^{3/2}}{3a} + C$ B. $\frac{3(ax+b)^{3/2}}{2a} + C$

C. $\frac{1}{2\sqrt{ax+b}} + C$

D. none of these

Answer

Given = $\int \sqrt{ax+b}$

Let, ax + b = z^2

⇒ adx = 2zdz

So,

$$\int \sqrt{ax+b} dx$$

$$= \int z \frac{2zdz}{a}$$

$$= \frac{2}{a}\int z^2 dz$$

where c is the integrating constant.

$$= \frac{2}{a}\frac{z^3}{3} + c$$

$$= \frac{2}{3a} z^3 + c$$

$$= \frac{2(ax+b)^{3/2}}{3a} + c$$

5. Question

Mark (√) against the correct answer in each of the following:

$\int \sec^2(7-4x)dx = ?$

A. $\frac{1}{4}\tan(7-4x)+C$

B. $\frac{-1}{4}\tan(7-4x)+C$

C. 4 tan (7 - 4x) + C

D. - 4 tan (7 - 4x) + C

Answer

Given = $\int \sec^2(7-4x)$

Let, 7 - 4x = z

⇒ -4dx = dz

So,

$\int \sec^2(7-4x)dx$

$= \int \sec^2 z \frac{dz}{-4}$

$= -\frac{1}{4}\int \sec^2 zdz$ where c is the integrating constant.

$= -\frac{1}{4}\tan z + c$

$= -\frac{1}{4}\tan(7-4x) + c$

6. Question

Mark (√) against the correct answer in each of the following:

$\int \cos 3x \, dx = ?$

A. $-\frac{1}{3}\sin 3x + C$

B. $\frac{1}{3}\sin 3x + C$

C. 3 sin 3x + C

D. -3 sin 3x + C

Answer

Given = $\int \cos 3x$

So, $\int \cos 3x dx = \frac{\sin 3x}{3} + c$ where c is the integrating constant.

7. Question

Mark (√) against the correct answer in each of the following:

$\int e^{(5-3x)}dx = ?$

A. $-3e^{(5-3x)} + C$

B. $\frac{1}{3}e^{(5-3x)} + C$

C. $\frac{e^{(5-3x)}}{-3} + C$

D. none of these

Answer

Given = $\int e^{(5-3x)}$

Let, 5 - 3x = z

⇒ -3dx = dz

So,

$\int e^{(5-3x)}dx$

$= \int e^z \frac{dz}{-3}$

$= -\frac{1}{3}\int e^z dz$ where c is the integrating constant.

$= -\frac{1}{3}e^z + c$

$= -\frac{1}{3}e^{(5-3x)} + c$

8. Question

Mark (√) against the correct answer in each of the following:

$\int e^{(3x+4)}dx = ?$

A. $\frac{3}{(\log 2)}.2^{(3x+4)} + C$

B. $\frac{2^{(3x+4)}}{3(\log 2) + C}$

C. $\frac{2^{(3x+4)}}{2(\log 3)} + C$

D. none of these

Answer

Given = $\int e^{(3x+4)}$

Let, 3x + 4 = z

⇒ 3dx = dz

So,

$$\int e^{(3x+4)}dx$$

$$= \int e^{z}\frac{dz}{3}$$

$$= \frac{1}{3}\int e^{z}dz$$

$$= \frac{1}{3}e^{z} + c$$

$$= \frac{1}{3}e^{(3x+4)} + c$$

where c is the integrating constant.

9. Question

Mark (√) against the correct answer in each of the following:

$$\int \tan^{2}\frac{x}{2}dx = ?$$

A. $\tan\frac{x}{2} - x + C$

B. $\tan\frac{x}{2} + x + C$

C. $2\tan\frac{x}{2} + x + C$

D. $2\tan\frac{x}{2} - x + C$

Answer

Given = $\int \tan^{2}\frac{x}{2}$

Let, $\frac{x}{2} = z$

⇒ dx = 2dz

So,

$$\int \tan^2 \frac{x}{2} dx$$
$$= 2\int \tan^2 z dz$$
$$= 2\int \frac{\sin^2 z}{\cos^2 z} dz$$
$$= 2\int \frac{1-\cos^2 z}{\cos^2 z} dz$$
$$= 2\int (\sec^2 z - 1) dz$$
$$= 2\left[\tan z - z\right] + c$$
$= 2[\tan \frac{x}{2} - \frac{x}{2}] + c$ where c is the integrating constant.

10. Question

Mark (√) against the correct answer in each of the following:

$$\int \sqrt{1-\cos x}\, dx = ?$$

A. $-\sqrt{2}\cos\frac{x}{2} + C$

B. $-2\sqrt{2}\cos\frac{x}{2} + C$

C. $\frac{-1}{2}\cos\frac{x}{2} + C$

D. $\frac{-1}{\sqrt{2}}\cos\frac{x}{2} + C$

Answer

Given $= \int \sqrt{1-\cos x}$

So,

$$\int \sqrt{1-\cos x} dx$$
$$= \int \sqrt{1-\cos x}\, \frac{\sqrt{1+\cos x}}{\sqrt{1+\cos x}} dx$$
$$= \int \frac{\sqrt{1-\cos^2 x}}{\sqrt{1+\cos x}} dx$$
$$= \int \frac{\sin x}{\sqrt{1+\cos x}} dx$$

Let $1 + \cos x = u^2$

So, $-\sin x dx = 2u du$

$$-\int \frac{2u}{u} du = -2\int du = -2u + c = -2\sqrt{1+\cos x} + c$$