

NCERT Solutions for Class-XII Maths

Chapter-5.2

Differentiate the function with respect to x in Exercises 1 to 8.

1. $\sin(x^2 + 5)$

1. Let $y = \sin(x^2 + 5)$

Therefore,

$$\begin{aligned}\frac{dy}{dx} &= \cos(x^2 + 5) \cdot \frac{d}{dx}(x^2 + 5) \\ &= \cos(x^2 + 5) \cdot 2x\end{aligned}$$

2. $\cos(\sin x)$

2. Let $y = \cos(\sin x)$

Therefore,

$$\begin{aligned}\frac{dy}{dx} &= -\sin(\sin x) \cdot \frac{d}{dx}(\sin x) \\ &= -\sin(\sin x) \cdot \cos x\end{aligned}$$

3. $\sin(ax + b)$

3. Let
 $y = \sin(ax + b)$

Therefore,

$$\begin{aligned}\frac{dy}{dx} &= \cos(ax + b) \cdot \frac{d}{dx}(ax + b) \\ &= \cos(ax + b) \cdot a\end{aligned}$$

4. $\sec(\tan(\sqrt{x}))$

4. Given: $\sec(\tan(\sqrt{x}))$

Let $y = \sec(\tan(\sqrt{x}))$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(\sec(\tan(\sqrt{x})))$$

$$= \sec(\tan(\sqrt{x})) \cdot \tan(\tan(\sqrt{x})) \left(\frac{d}{dx}(\tan \sqrt{x}) \right)$$

$$= \sec(\tan(\sqrt{x})) \cdot \tan(\tan(\sqrt{x})) \cdot \sec^2(\sqrt{x}) \cdot \frac{d}{dx}(\sqrt{x})$$

$$= \sec(\tan(\sqrt{x})) \cdot \tan(\tan(\sqrt{x})) \cdot \sec^2(\sqrt{x}) \cdot \frac{1}{2(\sqrt{x})}$$

$$= \frac{1}{2(\sqrt{x})} (\sec(\tan(\sqrt{x})) \cdot \tan(\tan(\sqrt{x})) \cdot \sec^2(\sqrt{x}))$$

5. $\frac{\sin(ax+b)}{\cos(cx+d)}$

5. Given: $\frac{\sin(ax+b)}{\cos(cx+d)}$

Let $y = \frac{\sin(ax+b)}{\cos(cx+d)}$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx} \left(\frac{\sin(ax+b)}{\cos(cx+d)} \right)$$

We know that $\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{d}{dx}(u) - u \frac{d}{dx}(v)}{v^2}$

$$= \frac{[\cos(cx+d) \cdot d(\sin(ax+b)) - \sin(ax+b) \cdot d(\cos(cx+d))]}{[\cos(cx+d)]^2}$$

$$= \frac{[\cos(cx+d) \cdot (\cos(ax+b) \cdot d(ax+b)) - \sin(ax+b) \cdot (-\sin(cx+d) d(cx+d))]}{[\cos(cx+d)]^2}$$

$$= \frac{[\cos(cx+d) \cdot (\cos(ax+b) \cdot (a) - \sin(ax+b) \cdot (-\sin(cx+d)(c))]}{[\cos(cx+d)]^2}$$

$$= \frac{[a \cos(cx+d) \cos(ax+b)]}{[\cos(cx+d)]^2} + \frac{[c \sin(cx+d) \sin(ax+b)]}{[\cos(cx+d)]^2}$$

$$= a \cos(ax+b) \sec^2(cx+d) + c \sin(ax+b) \tan(cx+d) \sec^2(cx+d)$$

6. $\cos x^3 \cdot \sin^2(x^5)$

6. Let $y = \cos x^3 \cdot \sin^2 x^5$

Therefore,

$$\frac{dy}{dx} = \cos x^3 \cdot \frac{d}{dx} \sin^2(x^5) + \sin^2(x^5) \cdot \frac{d}{dx} \cos x^3$$

$$= \cos x^2 \cdot 2 \sin x^5 \cos x^5 \cdot \frac{d}{dx} x^5 + \sin^2(x^5) [-\sin x^3] \cdot \frac{d}{dx} x^3$$

$$= \cos x^2 \cdot 2 \sin x^5 \cos x^5 \cdot 5x^4 - \sin^2(x^5) \sin x^3 \cdot 3x^2$$

7. $2\sqrt{\cot(x^2)}$

7. Let $y = 2\sqrt{\cot(x^2)}$

Therefore,

$$\begin{aligned}\frac{dy}{dx} &= 2 \cdot \frac{1}{2\sqrt{\cot(x^2)}} \cdot \frac{d}{dx}[\cot(x^2)] \\ &= \frac{1}{\sqrt{\cot(x^2)}} \cdot [-\operatorname{cosec} x^2] \cdot \frac{d}{dx} x^2 \\ &= \frac{1}{\sqrt{\cot(x^2)}} \cdot [-\operatorname{cosec} x^2] \cdot 2x\end{aligned}$$

8. $\cos(\sqrt{x})$

8. Let $y = \cos(\sqrt{x})$

Therefore,

$$\begin{aligned}\frac{dy}{dx} &= \sin(\sqrt{x}) \cdot \frac{d}{dx} \sqrt{x} \\ &= -\sin(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}\end{aligned}$$

9. prove that the function f given by $f(x) = |x - 1|$, $x \in \mathbb{R}$, is not differentiable at $x = 1$

9. Given: $f(x) = |x - 1|$, $x \in \mathbb{R}$

because a function f is differentiable at a point $x=c$ in its domain if both its limits as:

$$\lim_{h \rightarrow 0^-} \frac{[f(c+h) - f(c)]}{h} \text{ and } \lim_{h \rightarrow 0^+} \frac{[f(c+h) - f(c)]}{h} \text{ are finite and equal.}$$

Now, to check the differentiability of the given function at $x=1$,

Let we consider the left hand limit of function f at $x=1$

$$\begin{aligned}\Rightarrow \lim_{h \rightarrow 0^-} \frac{[f(1+h) - f(1)]}{h} \\ &= \lim_{h \rightarrow 0^-} \frac{[|1+h-1| - |1-1|]}{h} \\ &= \lim_{h \rightarrow 0^-} \frac{[|h| - 0]}{h} \\ &= \lim_{h \rightarrow 0^-} \frac{[-h]}{h} \text{ because, } \{h < 0 \Rightarrow |h| = -h\} \\ &= -1\end{aligned}$$

Now, let we consider the right hand limit of function f at $x=1$

$$\Rightarrow \lim_{h \rightarrow 0^+} \frac{[f(1+h) - f(1)]}{h}$$

$$\begin{aligned}
&= \lim_{h \rightarrow 0^+} \frac{[|1+h-1| - |1-1|]}{h} \\
&= \lim_{h \rightarrow 0^+} \frac{[|h| - 0]}{h} \\
&= \lim_{h \rightarrow 0^+} \frac{[h]}{h} \text{ because, } \{ h > 0 \Rightarrow |h| = h \} \\
&= 1
\end{aligned}$$

Because, left hand limit is not equal to right hand limit of function f at $x=1$, so f is not differentiable at $x=1$.

10. Prove that the greatest integer function defined by $f(x) = [x]$, $0 < x < 3$, is not differentiable at $x = 1$ and $x = 2$.

10. Given: $f(x) = [x]$, $0 < x < 3$

because a function f is differentiable at a point $x=c$ in its domain if both its limits as:

$$\lim_{h \rightarrow 0^-} \frac{[f(c+h) - f(c)]}{h} \text{ and } \lim_{h \rightarrow 0^+} \frac{[f(c+h) - f(c)]}{h} \text{ are finite and equal.}$$

Now, to check the differentiability of the given function at $x=1$,

Let us consider the left-hand limit of function f at $x=1$

$$\begin{aligned}
&\Rightarrow \lim_{h \rightarrow 0^-} \frac{[f(1+h) - f(1)]}{h} \\
&= \lim_{h \rightarrow 0^-} \frac{[[1+h] - [1]]}{h} \\
&= \lim_{h \rightarrow 0^-} \frac{(1+h-1) - (1)}{h} \\
&= \lim_{h \rightarrow 0^-} \frac{[h-1]}{h} \text{ because, } \{ h < 0 \Rightarrow |h| = -h \} \\
&= -\frac{1}{0} = \infty
\end{aligned}$$

Let us consider the right hand limit of function f at $x=1$

$$\begin{aligned}
&\Rightarrow \lim_{h \rightarrow 0^+} \frac{[f(1+h) - f(1)]}{h} \\
&= \lim_{h \rightarrow 0^+} \frac{[1+h] - [1]}{h}
\end{aligned}$$

$$= \lim_{h \rightarrow 0^+} \frac{[1-1]}{h}$$

$$= \lim_{h \rightarrow 0^+} \frac{[0]}{h}$$

$$= 0$$

Because, left hand limit is not equal to right hand limit of function f at $x=1$, so f is not differentiable at $x=1$.

Let we consider the left hand limit of function f at $x=2$

$$\Rightarrow \lim_{h \rightarrow 0^-} \frac{[f(2+h) - f(2)]}{h}$$

$$= \lim_{h \rightarrow 0^-} \frac{[2+h] - [2]}{h}$$

$$= \lim_{h \rightarrow 0^-} \frac{(2+h-1) - (2)}{h}$$

$$= \lim_{h \rightarrow 0^-} \frac{(h+1-2)}{h}$$

$$= -\frac{1}{0} = \infty$$

Now, let we consider the right hand limit of function f at $x=2$

$$\Rightarrow \lim_{h \rightarrow 0^+} \frac{[f(2+h) - f(2)]}{h}$$

$$= \lim_{h \rightarrow 0^+} \frac{[2+h] - [2]}{h}$$

$$= \lim_{h \rightarrow 0^+} \frac{[2-2]}{h}$$

$$= \lim_{h \rightarrow 0^+} \frac{[0]}{h}$$

$$= 0$$

Because, left hand limit is not equal to right hand limit of function f at $x=2$, so f is not differentiable at $x=2$.



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