EXERCISE 1(C)

State, with reason, which of the following are surds and which are not:

(i) $\sqrt{180}$
(ii) $\sqrt[4]{27}$
(iii) $\sqrt[5]{128}$
(iv) $\sqrt[3]{64}$
(v) $\sqrt[3]{23}.\sqrt[3]{40}$
(vi) $\sqrt[3]{-125}$
(vii) $\sqrt{\pi}$
(viii) $\sqrt{3+\sqrt{2}}$

Solution:

(i) $\sqrt{180} = \sqrt{(2 \times 2 \times 5 \times 3 \times 3)} = 6\sqrt{5}$
It is irrational
Therefore, $\sqrt{180}$ is a surd.

(ii) $\sqrt[4]{27} = \sqrt[4]{(3 \times 3 \times 3)}$
It is irrational
Therefore, $\sqrt[4]{27}$ is a surd

(iii)
$\sqrt[5]{128} = \sqrt[5]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2} = 2\sqrt[5]{4}$
It is irrational.
Therefore, $\sqrt[5]{128}$ is a surd

(iv) $\sqrt[3]{64} = \sqrt[3]{(4 \times 4 \times 4)} = 4$
It is rational
Therefore, $\sqrt[3]{64}$ is not a surd

(v) $\sqrt[3]{25}.\sqrt[3]{40} = \sqrt[3]{(25 \times 40)} = \sqrt[3]{(5 \times 5 \times 2 \times 2 \times 5 \times 2)} = 2 \times 5 = 10$
It is rational
Therefore, $\sqrt[3]{23}.\sqrt[3]{40}$ is not a surd

(vi) $\sqrt[3]{-125} = \sqrt[3]{(-5 \times -5 \times -5)} = -5$
It is rational
Therefore, $\sqrt[3]{-125}$ is not a surd

(vii) π is irrational.
Therefore, $\sqrt{\pi}$ is not a surd.

(viii) $3 + \sqrt{2}$ is irrational
Therefore, $\sqrt{3+\sqrt{2}}$ is not a surd

2. Write the lowest rationalizing factor of:
(i) $5\sqrt{2}$

(ii) $\sqrt{24}$
(iii) $\sqrt{5} - 3$
(iv) $7 - \sqrt{7}$
(v) $\sqrt{18} - \sqrt{50}$
(vi) $\sqrt{5} - \sqrt{2}$
(vii) $\sqrt{13} + 3$
(viii) $15 - 3\sqrt{2}$
(ix) $3\sqrt{2} + 2\sqrt{3}$
Solution:

(i) $5\sqrt{2}$
It can be written as
$5\sqrt{2} \times \sqrt{2} = 5 \times 2 = 10$
It is rational.
Therefore, lowest rationalizing factor is $\sqrt{2}$.

(ii) $\sqrt{24}$
It can be written as
$\sqrt{24} = \sqrt{(2 \text{ x } 2 \text{ x } 2 \text{ x } 3)} = 2\sqrt{6}$
Therefore, lowest rationalizing factor is $\sqrt{6}$.

(iii) $\sqrt{5} - 3$
It can be written as
$(\sqrt{5} - 3)(\sqrt{5} + 3) = (\sqrt{5})^2 - 3^2 = 5 - 9 = -4$
Therefore, lowest rationalizing factor is $(\sqrt{5} + 3)$.

(iv) $7 - \sqrt{7}$
It can be written as
$(7 - \sqrt{7})(7 + \sqrt{7}) = 49 - 7 = 42$
Therefore, lowest rationalizing factor is $(7 + \sqrt{7})$.

(v) $\sqrt{18} - \sqrt{50}$
It can be written as
$\sqrt{18} - \sqrt{50} = \sqrt{(2 \text{ x } 3 \text{ x } 3)} - \sqrt{(5 \text{ x } 5 \text{ x } 2)}$
$= 3\sqrt{2} - 5\sqrt{2}$
$= -2\sqrt{2}$
Therefore, lowest rationalizing factor is $\sqrt{2}$.

(vi) $\sqrt{5} - \sqrt{2}$
It can be written as
$(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2}) = (\sqrt{5})^2 - (\sqrt{2})^2 = 3$
Therefore, lowest rationalizing factor is $\sqrt{5} + \sqrt{2}$.

(vii) $\sqrt{13} + 3$
It can be written as
$(\sqrt{13} + 3)(\sqrt{13} - 3) = (\sqrt{13})^2 - 3^2 = 13 - 9 = 4$
Therefore, lowest rationalizing factor is $\sqrt{13} - 3$.

(viii) $15 - 3\sqrt{2}$
It can be written as
$15 - 3\sqrt{2} = 3 (5 - \sqrt{2})$
By further simplification
$= 3 (5 - \sqrt{2}) (5 + \sqrt{2})$
$= 3 [5^2 - (\sqrt{2})^2]$
So, we get
$= 3 \times [25 - 2]$
$= 3 \times 23$
$= 69$
Therefore, lowest rationalizing factor is $(5 + \sqrt{2})$.

(ix) $3\sqrt{2} + 2\sqrt{3}$
It can be written as
$3\sqrt{2} + 2\sqrt{3} = (3\sqrt{2} + 2\sqrt{3}) (3\sqrt{2} - 2\sqrt{3})$
By further calculation
$= (3\sqrt{2})^2 - (2\sqrt{3})^2$
So, we get
$= 9 \times 2 - 4 \times 3$
$= 18 - 12$
$= 6$
Therefore, lowest rationalizing factor is $3\sqrt{2} - 2\sqrt{3}$.

3. Rationalize the denominators of:

Solution:

(i) $(3/\sqrt{5}) \times (\sqrt{5}/\sqrt{5}) = 3\sqrt{5}/5$

(ii) $(2\sqrt{3}/\sqrt{5}) \times (\sqrt{5}/\sqrt{5}) = 2\sqrt{15}/5$

(iii)

$$\frac{1}{\sqrt{3}-\sqrt{2}} \times \left(\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}\right)$$

It can be written as

$$= \frac{\sqrt{3}+\sqrt{2}}{\left(\sqrt{3}\right)^2 - \left(\sqrt{2}\right)^2}$$

So we get

$$= \frac{\sqrt{3}+\sqrt{2}}{3-2}$$

$$= \sqrt{3}+\sqrt{2}$$

$$\frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

It can be written as

$$= \frac{\left(\sqrt{3}+1\right)^2}{\left(\sqrt{3}\right)^2 - (1)^2}$$

$$= \frac{3+1+2\sqrt{3}}{3-1}$$

$$= \frac{4+2\sqrt{3}}{2}$$

So we get

$$= \frac{2\left(2+\sqrt{3}\right)}{2}$$

$$= 2+\sqrt{3}$$

(vii)

$$\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

It can be written as

$$= \frac{\left(\sqrt{3}-\sqrt{2}\right)^2}{\left(\sqrt{3}\right)^2 - \left(\sqrt{2}\right)^2}$$

So we get

$$= \frac{3+2-2\sqrt{6}}{3-2}$$

$= 5 - 2\sqrt{6}$

(viii)

$$\frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}} \times \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}-\sqrt{5}}$$

It can be written as

$$= \frac{6+5-2\sqrt{30}}{\left(\sqrt{6}\right)^2 - \left(\sqrt{5}\right)^2}$$

So we get

$$= \frac{11-2\sqrt{30}}{6-5}$$

$$= 11-2\sqrt{30}$$

(ix)

$$\frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}-3\sqrt{2}} \times \frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}+3\sqrt{2}}$$

It can be written as

$$= \frac{\left(2\sqrt{5}+3\sqrt{2}\right)^2}{\left(2\sqrt{5}\right)^2-\left(3\sqrt{2}\right)^2}$$

$$= \frac{4\times5+9\times2+12\sqrt{10}}{20-18}$$

So we get

$$= \frac{20+18+12\sqrt{10}}{2}$$

$$= \frac{38+12\sqrt{10}}{2}$$

$$= \frac{\cancel{2}\left(19+6\sqrt{10}\right)}{\cancel{2}}$$

$$= 19+6\sqrt{10}$$

4. Find the values of 'a' and 'b' in each of the following:

(i) $\frac{2+\sqrt{3}}{2-\sqrt{3}} = a + b\sqrt{3}$

(ii) $\frac{\sqrt{7}-2}{\sqrt{7}+2} = a\sqrt{7} + b$

(iii) $\frac{3}{\sqrt{3}-\sqrt{2}} = a\sqrt{3} + b\sqrt{2}$

(iv) $\frac{5+3\sqrt{2}}{5-3\sqrt{2}} = a + b\sqrt{2}$

Solution:

(i)

$$\frac{2+\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = a+b\sqrt{3}$$

It can be written as

$$\frac{\left(2+\sqrt{3}\right)^2}{(2)^2-\left(\sqrt{3}\right)^2} = a+b\sqrt{3}$$

$$\frac{4+3+4\sqrt{3}}{4-3} = a+b\sqrt{3}$$

So we get

$$7+4\sqrt{3} = a+b\sqrt{3}$$

$$a=7,\ b=4$$

(ii)

$$\frac{\sqrt{7}-2}{\sqrt{7}+2}\times\frac{\sqrt{7}-2}{\sqrt{7}-2}=a\sqrt{7}+b$$

It can be written as

$$\frac{\left(\sqrt{7}-2\right)^2}{\left(\sqrt{7}\right)^2-(2)^2}=a\sqrt{7}+b$$

$$\frac{7+4-4\sqrt{7}}{7-4}=a\sqrt{7}+b$$

So we get

$$\frac{11-4\sqrt{7}}{3}=a\sqrt{7}+b$$

$$a=\frac{-4}{3},\ b=\frac{11}{3}$$

(iii)

$$\frac{3}{\sqrt{3}-\sqrt{2}}\times\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}=a\sqrt{3}-b\sqrt{2}$$

It can be written as

$$\frac{3\left(\sqrt{3}+\sqrt{2}\right)}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}=a\sqrt{3}-b\sqrt{2}$$

$$\frac{3\left(\sqrt{3}+\sqrt{2}\right)}{3-2}=a\sqrt{3}-b\sqrt{2}$$

So we get

$$\left(3\sqrt{3}+3\sqrt{2}\right)=a\sqrt{3}-b\sqrt{2}$$

$$a=3, b=-3$$

(iv)

$$\frac{5+3\sqrt{2}}{5-3\sqrt{2}}\times\frac{5+3\sqrt{2}}{5+3\sqrt{2}}=a+b\sqrt{2}$$

It can be written as

$$\frac{\left(5+3\sqrt{2}\right)^2}{(5)^2-\left(3\sqrt{2}\right)^2}=a+b\sqrt{2}$$

$$\frac{25+18+30\sqrt{2}}{25-18}=a+b\sqrt{2}$$

So we get

$$\frac{43+30\sqrt{2}}{7}=a+b\sqrt{2}$$

$$a=\frac{43}{7},\quad b=\frac{30}{7}$$

5. Simplify:

(i) $\frac{22}{2\sqrt{3}+1}+\frac{17}{2\sqrt{3}-1}$

(ii) $\frac{\sqrt{2}}{\sqrt{6}-\sqrt{2}}+\frac{\sqrt{3}}{\sqrt{6}+\sqrt{2}}$

Solution:

(i)

$$\frac{22}{2\sqrt{3}+1}+\frac{17}{2\sqrt{3}-1}$$

It can be written as

$$=\frac{22\left(2\sqrt{3}-1\right)+17\left(2\sqrt{3}+1\right)}{\left(2\sqrt{3}+1\right)\left(2\sqrt{3}-1\right)}$$

By further calculation

$$=\frac{44\sqrt{3}-22+34\sqrt{3}+17}{\left(2\sqrt{3}\right)^2-1}$$

So we get

$$=\frac{78\sqrt{3}-5}{12-1}$$

$$=\frac{78\sqrt{3}-5}{11}$$

(ii)

$$\frac{\sqrt{2}}{\sqrt{6}-2}-\frac{\sqrt{3}}{\sqrt{6}+\sqrt{2}}$$

It can be written as

$$=\frac{\sqrt{2}\left(\sqrt{6}+\sqrt{2}\right)-\sqrt{3}\left(\sqrt{6}-\sqrt{2}\right)}{\left(\sqrt{6}\right)^2-\left(\sqrt{2}\right)^2}$$

By further calculation

$$=\frac{\sqrt{12}+2-\sqrt{18}+\sqrt{6}}{6-2}$$

So we get

$$=\frac{2\sqrt{3}+2-3\sqrt{2}+\sqrt{6}}{4}$$

6. If $x = \frac{\sqrt{5}-2}{\sqrt{5}+2}$ and $y = \frac{\sqrt{5}+2}{\sqrt{5}-2}$; Find:

(i) x^2

(ii) y^2

(iii) xy

(iv) $x^2 + y^2 = xy$

Solution:

(i)

$$x^2 = \left(\frac{\sqrt{5}-2}{\sqrt{5}+2}\right)^2$$

It can be written as

$$= \frac{5+4-4\sqrt{5}}{5+4+4\sqrt{5}} = \frac{9-4\sqrt{5}}{9+4\sqrt{5}}$$

By further calculation

$$= \frac{9-4\sqrt{5}}{9+4\sqrt{5}} \times \left(\frac{9-4\sqrt{5}}{9-4\sqrt{5}}\right) = \frac{\left(9-4\sqrt{5}\right)^2}{(9)^2-\left(4\sqrt{5}\right)^2}$$

So we get

$$= \frac{81+80-72\sqrt{5}}{81-80} = 161-72\sqrt{5}$$

(ii)

$$y^2 = \left(\frac{\sqrt{5}+2}{\sqrt{5}-2}\right)^2$$

It can be written as

$$= \frac{5+4+4\sqrt{5}}{5+4-4\sqrt{5}} = \frac{9+4\sqrt{5}}{9-4\sqrt{5}}$$

Bv further calculation

$$= \frac{9+4\sqrt{5}}{9-4\sqrt{5}} \times \frac{9+4\sqrt{5}}{9+4\sqrt{5}} = \frac{\left(9+4\sqrt{5}\right)^2}{(9)^2-\left(4\sqrt{5}\right)^2}$$

So we get

$$= \frac{81+80+72\sqrt{5}}{81-80} = 161+72\sqrt{5}$$

(iii) We know that

$$xy = \frac{(\sqrt{5}-2)}{(\sqrt{5}+2)}\frac{(\sqrt{5}+2)}{(\sqrt{5}-2)} = 1$$

(iv) $x^2 + y^2 = xy$
By substituting the values
$= 161 - 72\sqrt{5} + 161 + 72\sqrt{5} + 1$
So, we get
$= 322 + 1$
$= 323$

7. If $m = 1/(3 - 2\sqrt{2})$ **and** $n = 1/(3 + 2\sqrt{2})$**, find:**
(i) m^2
(ii) n^2
(iii) mn
Solution:

(i)

$$m = \frac{1}{3-2\sqrt{2}}$$

It can be written as

$$= \frac{1}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}}$$

By further calculation

$$= \frac{3+2\sqrt{2}}{(3)^2-(2\sqrt{2})^2}$$

So we get

$$= \frac{3+2\sqrt{2}}{9-8}$$

$$= 3+2\sqrt{2}$$

Here

$$m^2 = (3+2\sqrt{2})^2$$

Expanding using the formula

$$= (3)^2 + 2\times 3\times 2\sqrt{2} + (2\sqrt{2})^2$$

$$= 9 + 12\sqrt{2} + 8$$

$$= 17 + 12\sqrt{2}$$

(ii)

$$n = \frac{1}{3+2\sqrt{2}}$$

It can be written as

$$= \frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$$

By further calculation

$$= \frac{3-2\sqrt{2}}{(3)^2-(2\sqrt{2})^2}$$

So we get

$$= \frac{3+2\sqrt{2}}{9-8}$$

$$= 3-2\sqrt{2}$$

Here

$$n^2 = (3-2\sqrt{2})^2$$

Expanding using the formula

$$= (3)^2 - 2\times 3\times 2\sqrt{2} + (2\sqrt{2})^2$$

$$= 9 - 12\sqrt{2} + 8$$

$$= 17 - 12\sqrt{2}$$

(iii) We know that

$mn = (3 + \sqrt{2})(3 - \sqrt{2})$

By further calculation, we get

$mn = 3^2 - (2\sqrt{2})^2$

So, we get

$= 9 - 8$

$= 1$

8. If $x = 2\sqrt{3} + 2\sqrt{2}$, find:

(i) $1/x$

(ii) $x + 1/x$

(iii) $(x + 1/x)^2$

Solution:

(i)

$$\frac{1}{x} = \frac{1}{2\sqrt{3}+2\sqrt{2}} \times \frac{2\sqrt{3}-\sqrt{2}}{2\sqrt{3}-2\sqrt{2}}$$

By further calculation

$$= \frac{2\sqrt{3}-2\sqrt{2}}{12-8}$$

So we get

$$= \frac{2\left(\sqrt{3}-\sqrt{2}\right)}{42}$$

$$= \frac{\sqrt{3}-\sqrt{2}}{2}$$

(ii)

$$x+\frac{1}{x}=2\sqrt{3}+2\sqrt{2}+\frac{\sqrt{3}-\sqrt{2}}{2}$$

By further calculation

$$=2\left(\sqrt{3}+\sqrt{2}\right)+\frac{\left(\sqrt{3}-\sqrt{2}\right)}{2}$$

$$=\frac{4\left(\sqrt{3}+\sqrt{2}\right)+\left(\sqrt{3}-\sqrt{2}\right)}{2}$$

So we get

$$=\frac{4\sqrt{3}+4\sqrt{2}+\sqrt{3}-\sqrt{2}}{2}$$

$$=\frac{5\sqrt{3}+3\sqrt{2}}{2}$$

(iii)

$$\left(x+\frac{1}{x}\right)^2=\left(\frac{5\sqrt{3}+3\sqrt{2}}{2}\right)^2$$

By further calculation

$$=\frac{75+18+30\sqrt{6}}{4}$$

So we get

$$=\frac{93+30\sqrt{6}}{4}$$

9. If $x = 1 - \sqrt{2}$, find the value of $(x + 1/x)^3$.
Solution:

It is given that
$x = 1 - \sqrt{2}$
We should find the value of $(x + 1/x)^3$
So, $x = 1 - \sqrt{2}$, we get

$$\frac{1}{x}=\frac{1}{1-\sqrt{2}}\times\frac{1+\sqrt{2}}{1+\sqrt{2}}$$

Using the formula $(a - b)(a + b) = a^2 - b^2$

$$\frac{1}{x} = \frac{1 + \sqrt{2}}{1^2 - (\sqrt{2})^2}$$

$$\frac{1}{x} = \frac{1 + \sqrt{2}}{1 - 2}$$

$$\frac{1}{x} = \frac{1 + \sqrt{2}}{-1}$$

$$\frac{1}{x} = -(1 + \sqrt{2}) \ldots (1)$$

Here

$(x - 1/x) = (1 - \sqrt{2}) - (-(1 + \sqrt{2}))$

$= 1 - \sqrt{2} + 1 + \sqrt{2}$

$= 2$

By cubing on both sides, we get

$(x - 1/x)^3 = 2^3$

$= 8$

10. If $x = 5 - 2\sqrt{6}$, find: $x^2 + 1/x^2$

Solution:

It is given that

$x = 5 - 2\sqrt{6}$

We should find the value of $(x^2 + 1/x^2)$

So, $x = 5 - 2\sqrt{6}$, we get

$$\frac{1}{x} = \frac{1}{5 - 2\sqrt{6}} \times \frac{5 + 2\sqrt{6}}{5 + 2\sqrt{6}}$$

$$\frac{1}{x} = \frac{5 + 2\sqrt{6}}{(5 - 2\sqrt{6})(5 + 2\sqrt{6})}$$

Using the formula $(a - b)(a + b) = a^2 - b^2$

$$\frac{1}{x} = \frac{5 + 2\sqrt{6}}{5^2 - (2\sqrt{6})^2}$$

$$\frac{1}{x} = \frac{5 + 2\sqrt{6}}{25 - 24}$$

$$\frac{1}{x} = 5 + 2\sqrt{6} \ldots (1)$$

Here,

$(x - 1/x) = (5 - 2\sqrt{6}) - (5 + 2\sqrt{6})$

$= 5 - 2\sqrt{6} - 5 - 2\sqrt{6}$

$= -4\sqrt{6} \ldots (2)$

Now,

Consider $(x - 1/x)^2$
Using the equation $(a - b)^2 = a^2 + b^2 - 2ab$

$(x - 1/x)^2 = x^2 + 1/x^2 - 2(x)(1/x)$
$(x - 1/x)^2 = x^2 + 1/x^2 - 2$
$(x - 1/x)^2 + 2 = x^2 + 1/x^2$ ... (3)
From equations (2) and (3), we get
$x^2 + 1/x^2 = (-4\sqrt{6})^2 + 2$
$= 96 + 2$
$= 98$

11. Show that:

$$\frac{1}{3-2\sqrt{2}} - \frac{1}{2\sqrt{2}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2} = 5$$

Solution:

Consider

$$\text{L.H.S.} = \frac{1}{3-2\sqrt{2}} - \frac{1}{2\sqrt{2}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}$$

It can be written as

$$= \frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}$$

$$= \frac{1}{3-\sqrt{8}} \times \frac{3+\sqrt{8}}{3+\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} \times \frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}+\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}$$
$$- \frac{1}{\sqrt{6}-\sqrt{5}} \times \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}} + \frac{1}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2}$$

Using the formula $a^2 - b^2 = (a + b)(a - b)$

$$= \frac{3+\sqrt{8}}{(3)^2-(\sqrt{8})^2} - \frac{\sqrt{8}+\sqrt{7}}{(\sqrt{8})^2-(\sqrt{7})^2} + \frac{\sqrt{7}+\sqrt{6}}{(\sqrt{7})^2-(\sqrt{6})^2} - \frac{\sqrt{6}+\sqrt{5}}{(\sqrt{6})^2-(\sqrt{5})^2} + \frac{\sqrt{5}+2}{(\sqrt{5})^2-(2)^2}$$

$$= \frac{3+\sqrt{8}}{9-8} - \frac{\sqrt{8}+\sqrt{7}}{8-7} + \frac{\sqrt{7}+\sqrt{6}}{7-6} - \frac{\sqrt{6}+\sqrt{5}}{6-5} + \frac{\sqrt{5}+2}{5-4}$$

So, we get
$= 3 + \sqrt{8} - \sqrt{8} - \sqrt{7} + \sqrt{7} + \sqrt{6} - \sqrt{6} - \sqrt{5} + \sqrt{5} + 2$
$= 3 + 2$
$= 5$
= R.H.S.

12. Rationalize the denominator of:

$$\frac{1}{\sqrt{3}-\sqrt{2}+1}$$

Solution:

We know that,

$$\frac{1}{\sqrt{3}-\sqrt{2}+1}$$

$$= \frac{1}{\left(\sqrt{3}-\sqrt{2}\right)+1} \times \frac{\left(\sqrt{3}-\sqrt{2}\right)-1}{\left(\sqrt{3}-\sqrt{2}\right)-1}$$

Using the formula $a^2 - b^2 = (a + b)(a - b)$

$$= \frac{\sqrt{3}-\sqrt{2}-1}{\left(\sqrt{3}-\sqrt{2}\right)^2-(1)^2}$$

Using the formula $(a - b)^2 = a^2 + b^2 - 2ab$

$$= \frac{\sqrt{3}-\sqrt{2}-1}{\left(\sqrt{3}\right)^2-2\sqrt{6}+\left(\sqrt{2}\right)^2-1}$$

$$= \frac{\sqrt{3}-\sqrt{2}-1}{3-2\sqrt{6}+2-1}$$

$$= \frac{\sqrt{3}-\sqrt{2}-1}{4-2\sqrt{6}}$$

It can be written as

$$= \frac{\left(\sqrt{3}-\sqrt{2}\right)-1}{2\left(2-\sqrt{6}\right)}$$

$$= \frac{\sqrt{3}-\sqrt{2}-1}{2\left(2-\sqrt{6}\right)} \times \frac{2+\sqrt{6}}{2+\sqrt{6}}$$

Using the formula $a^2 - b^2 = (a + b)(a - b)$

$$= \frac{2\sqrt{3}-2\sqrt{2}-2+\sqrt{18}-\sqrt{12}-\sqrt{6}}{2\left[(2)^2-\left(\sqrt{6}\right)^2\right]}$$

$$= \frac{2\sqrt{3}-2\sqrt{2}-2+3\sqrt{2}-2\sqrt{3}-\sqrt{6}}{2(4-6)}$$

So, we get

$$= \frac{\sqrt{2}-2-\sqrt{6}}{2(-2)}$$

$$= \frac{\sqrt{2}-2-\sqrt{6}}{-4}$$

$$= \frac{1}{4}\left(2+\sqrt{6}-\sqrt{2}\right)$$

13. If $\sqrt{2} = 1.4$ and $\sqrt{3} = 1.7$, find the value of each of the following, correct to one decimal place:

(i) $1/(\sqrt{3} - \sqrt{2})$

(ii) $1/(3 + 2\sqrt{2})$

(iii) $(2 - \sqrt{3})/\sqrt{3}$
Solution:

(i)
$$\frac{1}{\sqrt{3}-\sqrt{2}}$$
$$= \frac{1}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$
It can be written as
$$= \frac{\sqrt{3}+\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})^2}$$
$$= \frac{\sqrt{3}+\sqrt{2}}{3-2}$$
So, we get
$= \sqrt{3} + \sqrt{2}$
$= 1.7 + 1.4$
$= 3.1$

(ii)
$$\frac{1}{3+2\sqrt{2}}$$
$$= \frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$$
It can be written as
$$= \frac{3-2\sqrt{2}}{(3)^2-(2\sqrt{2})^2}$$
$$= \frac{3-2\sqrt{2}}{9-8}$$
So, we get
$= 3 - 2\sqrt{2}$
$= 3 – 2(1.4)$
$= 3 – 2.8$
$= 0.2$

(iii)
$$\frac{2-\sqrt{3}}{\sqrt{3}}$$
It can be written as
$$\frac{2-\sqrt{3}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$
By further calculation

$$\frac{2\sqrt{3}-3}{3} = \frac{(2 \times 1.7)-3}{3}$$

So, we get
(3.4 - 3)/3 = 0.4/3
= 0.133333...
≈ 0.1

14. Evaluate:
(4 - √5)/(4 + √5) + (4 + √5)/(4 - √5)
Solution:

We have,

$$\frac{4-\sqrt{5}}{4+\sqrt{5}} + \frac{4+\sqrt{5}}{4-\sqrt{5}}$$

$$= \frac{4-\sqrt{5}}{4+\sqrt{5}} \times \frac{4-\sqrt{5}}{4-\sqrt{5}} + \frac{4+\sqrt{5}}{4-\sqrt{5}} \times \frac{4+\sqrt{5}}{4+\sqrt{5}}$$

Using the formula $(a^2 - b^2) = (a + b)(a - b)$

$$= \frac{(4-\sqrt{5})^2}{4^2-(\sqrt{5})^2} + \frac{(4+\sqrt{5})^2}{4^2-(\sqrt{5})^2}$$

$$= \frac{16+5-8\sqrt{5}}{16-5} + \frac{16+5+8\sqrt{5}}{16-5}$$

By further calculation

$$= \frac{21-8\sqrt{5}}{11} + \frac{21+8\sqrt{5}}{11}$$

$$= \frac{21-8\sqrt{5}+21+8\sqrt{5}}{11}$$

$$= \frac{42}{11}$$

$$= 3\frac{9}{11}$$

15. If (2 + √5)/(2 - √5) = x and (2 - √5)/(2 + √5) = y; find the value of $x^2 - y^2$.
Solution:

We have,

$$x = \frac{2+\sqrt{5}}{2-\sqrt{5}}$$

$$= \frac{2+\sqrt{5}}{2-\sqrt{5}} \times \frac{2+\sqrt{5}}{2+\sqrt{5}}$$

Using the formula $a^2 - b^2 = (a + b)(a - b)$

$$= \frac{(2+\sqrt{5})^2}{2^2 - (\sqrt{5})^2}$$

$$= \frac{4 + 4\sqrt{5} + 5}{4 - 5}$$

So, we get

$$= \frac{9 + 4\sqrt{5}}{-1}$$

$$= -9 - 4\sqrt{5}$$

Similarly,

$$y = \frac{2-\sqrt{5}}{2+\sqrt{5}}$$

$$= \frac{2-\sqrt{5}}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$$

Using the formula $a^2 - b^2 = (a + b)(a - b)$

$$= \frac{(2-\sqrt{5})^2}{2^2 - (\sqrt{5})^2}$$

By further calculation

$$= \frac{4 - 4\sqrt{5} + 5}{4 - 5}$$

$$= \frac{9 - 4\sqrt{5}}{-1}$$

$$= -9 + 4\sqrt{5}$$

Here,

$x^2 - y^2 = (-9 - 4\sqrt{5})^2 - (-9 + 4\sqrt{5})^2$

Expanding using the formula, we get

$= 81 + 72\sqrt{5} + 80 - (81 - 72\sqrt{5} + 80)$

$= 81 + 72\sqrt{5} + 80 - 81 + 72\sqrt{5} - 80$

$= 144\sqrt{5}$