

EXERCISE 4.1

Find the degree measure corresponding to the following radian measures (Use $\pi = 22/7$)

(i) $9\pi/5$ (ii) $-5\pi/6$ (iii) $(18\pi/5)^\circ$ (iv) $(-3)^\circ$ (v) 11° (vi) 1°

Solution:

We know that $\pi \text{ rad} = 180^\circ \Rightarrow 1 \text{ rad} = 180^\circ / \pi$

(i) $9\pi/5$

$$[(180/\pi) \times (9\pi/5)]^\circ$$

Substituting the value of $\pi = 22/7$

$$[180/22 \times 7 \times 9 \times 22/(7 \times 5)]$$

$$(36 \times 9)^\circ$$

$$324^\circ$$

\therefore Degree measure of $9\pi/5$ is 324°

(ii) $-5\pi/6$

$$[(180/\pi) \times (-5\pi/6)]^\circ$$

Substituting the value of $\pi = 22/7$

$$[180/22 \times 7 \times -5 \times 22/(7 \times 6)]$$

$$(30 \times -5)^\circ$$

$$- (150)^\circ$$

\therefore Degree measure of $-5\pi/6$ is -150°

(iii) $(18\pi/5)$

$$[(180/\pi) \times (18\pi/5)]^\circ$$

Substituting the value of $\pi = 22/7$

$$[180/22 \times 7 \times 18 \times 22/(7 \times 5)]$$

$$(36 \times 18)^\circ$$

$$648^\circ$$

\therefore Degree measure of $18\pi/5$ is 648°

(iv) $(-3)^\circ$

$$[(180/\pi) \times (-3)]^\circ$$

Substituting the value of $\pi = 22/7$

$$[180/22 \times 7 \times -3]^\circ$$

$$(-3780/22)^\circ$$

$$(-171 \frac{18}{22})^\circ$$

$$(-171^\circ (18/22 \times 60)')$$

$$(-171^\circ (49 \frac{1}{11})')$$

$$(-171^\circ 49' (1/11 \times 60)')$$

$$- (171^\circ 49' 5.45'')$$

$$\approx - (171^\circ 49' 5'')$$

\therefore Degree measure of $(-3)^\circ$ is $-171^\circ 49' 5''$

(v) 11°

$$(180/\pi \times 11)^\circ$$

Substituting the value of $\pi = 22/7$

$$(180/22 \times 7 \times 11)^\circ$$

$$(90 \times 7)^\circ$$

$$630^\circ$$

\therefore Degree measure of 11° is 630°

(vi) 1°

$$(180/\pi \times 1)^\circ$$

Substituting the value of $\pi = 22/7$

$$(180/22 \times 7 \times 1)^\circ$$

$$(1260/22)^\circ$$

$$(57 \frac{3}{11})^\circ$$

$$(57^\circ (3/11 \times 60)')$$

$$(57^\circ (16 \frac{4}{11})')$$

$$(57^\circ 16' (4/11 \times 60)')$$

$$(57^\circ 16' 21.81'')$$

$$\approx (57^\circ 16' 21'')$$

\therefore Degree measure of 1° is $57^\circ 16' 21''$

2. Find the radian measure corresponding to the following degree measures:

(i) 300° (ii) 35° (iii) -56° (iv) 135° (v) -300°

(vi) $7^\circ 30'$ (vii) $125^\circ 30'$ (viii) $-47^\circ 30'$

Solution:

We know that $180^\circ = \pi \text{ rad} \Rightarrow 1^\circ = \pi/180 \text{ rad}$

(i) 300°

$$(300 \times \pi/180) \text{ rad}$$

$$5\pi/3$$

\therefore Radian measure of 300° is $5\pi/3$

(ii) 35°

$$(35 \times \pi/180) \text{ rad}$$

$$7\pi/36$$

∴ Radian measure of 35° is $7\pi/36$

(iii) -56°

$$(-56 \times \pi/180) \text{ rad}$$

$$-14\pi/45$$

∴ Radian measure of -56° is $-14\pi/45$

(iv) 135°

$$(135 \times \pi/180) \text{ rad}$$

$$3\pi/4$$

∴ Radian measure of 135° is $3\pi/4$

(v) -300°

$$(-300 \times \pi/180) \text{ rad}$$

$$-5\pi/3$$

∴ Radian measure of -300° is $-5\pi/3$

(vi) $7^\circ 30'$

We know that, $30' = (1/2)^\circ$

$$7^\circ 30' = (7 \frac{1}{2})^\circ$$

$$= (15/2)^\circ$$

$$= (15/2 \times \pi/180) \text{ rad}$$

$$= \pi/24$$

∴ Radian measure of $7^\circ 30'$ is $\pi/24$

(vii) $125^\circ 30'$

We know that, $30' = (1/2)^\circ$

$$125^\circ 30' = (125 \frac{1}{2})^\circ$$

$$= (251/2)^\circ$$

$$= (251/2 \times \pi/180) \text{ rad}$$

$$= 251\pi/360$$

∴ Radian measure of $125^\circ 30'$ is $251\pi/360$

(viii) $-47^\circ 30'$

We know that, $30' = (1/2)^\circ$

$$-47^\circ 30' = - (47 \frac{1}{2})^\circ$$

$$= - (95/2)^\circ$$

$$= - (95/2 \times \pi/180) \text{ rad}$$

$$= - 19\pi/72$$

\therefore Radian measure of $-47^\circ 30'$ is $-19\pi/72$

3. The difference between the two acute angles of a right-angled triangle is $2\pi/5$ radians. Express the angles in degrees.

Solution:

Given the difference between the two acute angles of a right-angled triangle is $2\pi/5$ radians.

We know that $\pi \text{ rad} = 180^\circ \Rightarrow 1 \text{ rad} = 180^\circ / \pi$

Given:

$$2\pi/5$$

$$(2\pi/5 \times 180 / \pi)^\circ$$

Substituting the value of $\pi = 22/7$

$$(2 \times 22 / (7 \times 5) \times 180 / 22 \times 7)$$

$$(2/5 \times 180)^\circ$$

$$72^\circ$$

Let one acute angle be x° and the other acute angle be $90^\circ - x^\circ$.

Then,

$$x^\circ - (90^\circ - x^\circ) = 72^\circ$$

$$2x^\circ - 90^\circ = 72^\circ$$

$$2x^\circ = 72^\circ + 90^\circ$$

$$2x^\circ = 162^\circ$$

$$x^\circ = 162^\circ / 2$$

$$x^\circ = 81^\circ \text{ and}$$

$$90^\circ - x^\circ = 90^\circ - 81^\circ$$

$$= 9^\circ$$

\therefore The angles are 81° and 9°

4. One angle of a triangle is $2/3x$ grades, and another is $3/2x$ degrees while the third is $\pi x/75$ radians. Express all the angles in degrees.

Solution:

Given:

One angle of a triangle is $2x/3$ grades and another is $3x/2$ degree while the third is $\pi x/75$ radians.

We know that, $1 \text{ grad} = (9/10)^\circ$

$$2/3x \text{ grad} = (9/10) (2/3x)^\circ$$

$$= 3/5x^\circ$$

We know that, $\pi \text{ rad} = 180^\circ \Rightarrow 1 \text{ rad} = 180^\circ / \pi$

Given: $\pi x/75$

$$\left(\frac{\pi x}{75} \times \frac{180}{\pi}\right)^\circ$$

$$\left(\frac{12}{5}x\right)^\circ$$

We know that, the sum of the angles of a triangle is 180° .

$$\frac{3}{5}x^\circ + \frac{3}{2}x^\circ + \frac{12}{5}x^\circ = 180^\circ$$

$$(6+15+24)/10x^\circ = 180^\circ$$

Upon cross-multiplication we get,

$$45x^\circ = 180^\circ \times 10^\circ$$

$$= 1800^\circ$$

$$x^\circ = 1800^\circ/45^\circ$$

$$= 40^\circ$$

\therefore The angles of the triangle are:

$$\frac{3}{5}x^\circ = \frac{3}{5} \times 40^\circ = 24^\circ$$

$$\frac{3}{2}x^\circ = \frac{3}{2} \times 40^\circ = 60^\circ$$

$$\frac{12}{5}x^\circ = \frac{12}{5} \times 40^\circ = 96^\circ$$

5. Find the magnitude, in radians and degrees, of the interior angle of a regular:

(i) Pentagon (ii) Octagon (iii) Heptagon (iv) Duodecagon.

Solution:

We know that the sum of the interior angles of a polygon = $(n - 2) \pi$

And each angle of polygon = sum of interior angles of polygon / number of sides

Now, let us calculate the magnitude of

(i) Pentagon

Number of sides in pentagon = 5

Sum of interior angles of pentagon = $(5 - 2) \pi = 3\pi$

\therefore Each angle of pentagon = $3\pi/5 \times 180^\circ/\pi = 108^\circ$

(ii) Octagon

Number of sides in octagon = 8

Sum of interior angles of octagon = $(8 - 2) \pi = 6\pi$

\therefore Each angle of octagon = $6\pi/8 \times 180^\circ/\pi = 135^\circ$

(iii) Heptagon

Number of sides in heptagon = 7

Sum of interior angles of heptagon = $(7 - 2) \pi = 5\pi$

\therefore Each angle of heptagon = $5\pi/7 \times 180^\circ/\pi = 900^\circ/7 = 128^\circ 34' 17''$

(iv) Duodecagon

Number of sides in duodecagon = 12

Sum of interior angles of duodecagon = $(12 - 2) \pi = 10\pi$
 \therefore Each angle of duodecagon = $10\pi/12 \times 180^\circ/\pi = 150^\circ$

6. The angles of a quadrilateral are in A.P., and the greatest angle is 120° . Express the angles in radians.

Solution:

Let the angles of quadrilateral be $(a - 3d)^\circ$, $(a - d)^\circ$, $(a + d)^\circ$ and $(a + 3d)^\circ$.

We know that, the sum of angles of a quadrilateral is 360° .

$$a - 3d + a - d + a + d + a + 3d = 360^\circ$$

$$4a = 360^\circ$$

$$a = 360/4$$

$$= 90^\circ$$

Given:

The greatest angle = 120°

$$a + 3d = 120^\circ$$

$$90^\circ + 3d = 120^\circ$$

$$3d = 120^\circ - 90^\circ$$

$$3d = 30^\circ$$

$$d = 30^\circ/3$$

$$= 10^\circ$$

\therefore The angles are:

$$(a - 3d)^\circ = 90^\circ - 30^\circ = 60^\circ$$

$$(a - d)^\circ = 90^\circ - 10^\circ = 80^\circ$$

$$(a + d)^\circ = 90^\circ + 10^\circ = 100^\circ$$

$$(a + 3d)^\circ = 120^\circ$$

Angles of quadrilateral in radians:

$$(60 \times \pi/180) \text{ rad} = \pi/3$$

$$(80 \times \pi/180) \text{ rad} = 4\pi/9$$

$$(100 \times \pi/180) \text{ rad} = 5\pi/9$$

$$(120 \times \pi/180) \text{ rad} = 2\pi/3$$

7. The angles of a triangle are in A.P., and the number of degrees in the least angle is to the number of degrees in the mean angle as 1:120. Find the angle in radians.

Solution:

Let the angles of the triangle be $(a - d)^\circ$, a° and $(a + d)^\circ$.

We know that, the sum of the angles of a triangle is 180° .

$$a - d + a + a + d = 180^\circ$$

$$3a = 180^\circ$$

$$a = 60^\circ$$

Given:

Number of degrees in the least angle / Number of degrees in the mean angle = $1/120$

$$(a-d)/a = 1/120$$

$$(60-d)/60 = 1/120$$

$$(60-d)/1 = 1/2$$

$$120-2d = 1$$

$$2d = 119$$

$$d = 119/2$$

$$= 59.5$$

∴ The angles are:

$$(a - d)^\circ = 60^\circ - 59.5^\circ = 0.5^\circ$$

$$a^\circ = 60^\circ$$

$$(a + d)^\circ = 60^\circ + 59.5^\circ = 119.5^\circ$$

Angles of triangle in radians:

$$(0.5 \times \pi/180) \text{ rad} = \pi/360$$

$$(60 \times \pi/180) \text{ rad} = \pi/3$$

$$(119.5 \times \pi/180) \text{ rad} = 239\pi/360$$

8. The angle in one regular polygon is to that in another as 3:2 and the number of sides in first is twice that in the second. Determine the number of sides of two polygons.

Solution:

Let the number of sides in the first polygon be $2x$ and

The number of sides in the second polygon be x .

We know that, angle of an n -sided regular polygon = $[(n-2)/n] \pi$ radian

The angle of the first polygon = $[(2x-2)/2x] \pi = [(x-1)/x] \pi$ radian

The angle of the second polygon = $[(x-2)/x] \pi$ radian

Thus,

$$[(x-1)/x] \pi / [(x-2)/x] \pi = 3/2$$

$$(x-1)/(x-2) = 3/2$$

Upon cross-multiplication we get,

$$2x - 2 = 3x - 6$$

$$3x - 2x = 6 - 2$$

$$x = 4$$

∴ Number of sides in the first polygon = $2x = 2(4) = 8$

Number of sides in the second polygon = $x = 4$

9. The angles of a triangle are in A.P. such that the greatest is 5 times the least. Find the angles in radians.

Solution:

Let the angles of the triangle be $(a - d)^\circ$, a° and $(a + d)^\circ$.

We know that, the sum of angles of triangle is 180° .

$$a - d + a + a + d = 180^\circ$$

$$3a = 180^\circ$$

$$a = 180^\circ/3$$

$$= 60^\circ$$

Given:

Greatest angle = $5 \times$ least angle

Upon cross-multiplication,

Greatest angle / least angle = 5

$$(a+d)/(a-d) = 5$$

$$(60+d)/(60-d) = 5$$

By cross-multiplying we get,

$$60 + d = 300 - 5d$$

$$6d = 240$$

$$d = 240/6$$

$$= 40$$

Hence, angles are:

$$(a - d)^\circ = 60^\circ - 40^\circ = 20^\circ$$

$$a^\circ = 60^\circ$$

$$(a + d)^\circ = 60^\circ + 40^\circ = 100^\circ$$

\therefore Angles of triangle in radians:

$$(20 \times \pi/180) \text{ rad} = \pi/9$$

$$(60 \times \pi/180) \text{ rad} = \pi/3$$

$$(100 \times \pi/180) \text{ rad} = 5\pi/9$$

10. The number of sides of two regular polygons is 5:4 and the difference between their angles is 9° . Find the number of sides of the polygons.

Solution:

Let the number of sides in the first polygon be $5x$ and

The number of sides in the second polygon be $4x$.

We know that, angle of an n -sided regular polygon = $[(n-2)/n] \pi$ radian

The angle of the first polygon = $[(5x-2)/5x] 180^\circ$

The angle of the second polygon = $[(4x-1)/4x] 180^\circ$

Thus,

$$[(5x-2)/5x] 180^\circ - [(4x-1)/4x] 180^\circ = 9$$

$$180^\circ [(4(5x-2) - 5(4x-1))/20x] = 9$$

Upon cross-multiplication we get,

$$(20x - 8 - 20x + 10)/20x = 9/180$$

$$2/20x = 1/20$$

$$2/x = 1$$

$$x = 2$$

\therefore Number of sides in the first polygon = $5x = 5(2) = 10$

Number of sides in the second polygon = $4x = 4(2) = 8$



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