

EXERCISE 19.3

$$1. \int (2x - 3)^5 + \sqrt{3x + 2} \, dx$$

Solution:

$$\text{Let } I = \int (2x - 3)^5 + \sqrt{3x + 2}$$

Then,

$$I = \int (2x - 3)^5 + (3x + 2)^{\frac{1}{2}}$$

Now by integrating the above equation, we get

$$= \frac{(2x-3)^{5+1}}{2(5+1)} + \frac{(3x+2)^{\frac{1}{2}+1}}{3(\frac{1}{2}+1)}$$

$$= \frac{(2x-3)^6}{2(6)} + \frac{(3x+2)^{\frac{3}{2}}}{3(\frac{3}{2})}$$

$$= \frac{(2x-3)^6}{12} + \frac{2(3x+2)^{\frac{3}{2}}}{9}$$

$$\text{Hence, } I = \frac{(2x-3)^6}{12} + \frac{2(3x+2)^{\frac{3}{2}}}{9} + C$$

$$2. \int \frac{1}{(7x - 5)^3} + \frac{1}{\sqrt{5x - 4}} \, dx$$

Solution:

$$\text{Let } I = \int \frac{1}{(7x-5)^3} + \frac{1}{\sqrt{5x-4}} \, dx \text{ then,}$$

$$I = \int (7x - 5)^{-3} + (5x - 4)^{-\frac{1}{2}}$$

Integrating the above equation, we get



$$= \frac{(7x-5)^{-3+1}}{7(-3+1)} + \frac{(5x-4)^{-\frac{1}{2}+1}}{5\left(-\frac{1}{2}+1\right)}$$

$$= \frac{(7x-5)^{-2}}{-14} + \frac{(5x-4)^{\frac{1}{2}}}{5\left(\frac{1}{2}\right)}$$

$$\text{Hence, } I = -\frac{1}{14}(7x-5)^{-2} + \frac{2}{5}\sqrt{5x-4} + C$$

$$3. \int \frac{1}{2-3x} + \frac{1}{\sqrt{3x-2}} dx$$

Solution:

$$\text{Let } I = \int \frac{1}{2-3x} + \frac{1}{\sqrt{3x-2}} dx$$

$$I = \int \frac{1}{2-3x} + \frac{1}{\sqrt{3x-2}} dx$$

$$\text{We know } \int \frac{1}{x} dx = \log|x|$$

By applying the above formula we get

$$= \frac{\log|2-3x|}{-3} + \frac{2}{3}(3x-2)^{\frac{1}{2}}$$

$$= -\frac{1}{3}\log|2x-3| + \frac{2}{3}\sqrt{3x-2} + C$$

$$4. \int \frac{x+3}{(x+1)^4} dx$$

Solution:

Let,

$$I = \int \frac{x+3}{(x+1)^4} dx$$

Splitting the above given equation

$$= \int \frac{x+1}{(x+1)^4} dx + \int \frac{2}{(x+1)^4} dx$$

$$= \int \frac{1}{(x+1)^3} dx + \int \frac{2}{(x+1)^4} dx$$

The above equation can be written as

$$= \int (x+1)^{-3} dx + \int 2(x+1)^{-4} dx$$

Integrating the above equation we get

$$= \frac{[x+1]^{-3+1}}{-3+1} + \frac{2[x+1]^{-4+1}}{-4+1}$$

$$= \frac{[x+1]^{-2}}{-2} + \frac{2[x+1]^{-3}}{-3}$$

Hence, $I = -\frac{1}{2(x+1)^2} - \frac{2}{3(x+1)^3} + C$

5. $\int \frac{1}{\sqrt{x+1} + \sqrt{x}} dx$

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Solution:

Let $I = \int \frac{1}{\sqrt{x+1} + \sqrt{x}} dx$

Now multiply with the conjugate, we get

$$= \int \frac{1}{\sqrt{x+1} + \sqrt{x}} \cdot \frac{\sqrt{x+1} - \sqrt{x}}{\sqrt{x+1} - \sqrt{x}} dx$$

$$= \int \frac{\sqrt{x+1} - \sqrt{x}}{x+1-x} dx$$

On simplification we get

$$= \int \sqrt{x+1} - \sqrt{x} dx$$

The above equation can be written as

$$= \int (x+1)^{\frac{1}{2}} - x^{\frac{1}{2}}$$

On integrating we get

$$= \frac{(x+1)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{3}{2}}}{\frac{3}{2}}$$

$$\text{Hence } I = \frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{2}{3}(x)^{\frac{3}{2}} + C$$

6. $\int \frac{1}{\sqrt{2x+3} + \sqrt{2x-3}} dx$

Solution:

$$\text{Let } I = \int \frac{1}{\sqrt{2x+3} + \sqrt{2x-3}} dx$$

Now, multiply with the conjugate, we get

$$= \int \frac{1}{\sqrt{2x+3} + \sqrt{2x-3}} \times \frac{(\sqrt{2x+3} - \sqrt{2x-3})}{(\sqrt{2x+3} - \sqrt{2x-3})} dx$$

$$= \int \frac{(\sqrt{2x+3} - \sqrt{2x-3})}{(\sqrt{2x+3})^2 - (\sqrt{2x-3})^2} dx$$

$$= \int \frac{(\sqrt{2x+3} - \sqrt{2x-3})}{2x+3 - 2x+3} dx$$

On simplifying or computing we get

$$= \int \frac{\sqrt{2x+3}}{6} dx - \int \frac{\sqrt{2x-3}}{6} dx$$

Taking 1/6 as common

$$= \frac{1}{6} \int (2x+3)^{\frac{1}{2}} dx - \frac{1}{6} \int (2x-3)^{\frac{1}{2}} dx$$

On integrating we get

$$= \frac{1}{6} \left(\frac{2x+3}{2} \right)^{\frac{1}{2}+1} - \frac{1}{6} \left[\frac{2x-3}{2} \right]^{\frac{1}{2}+1}$$

$$= \frac{1}{6} \left(\frac{2x+3}{2 \times \frac{3}{2}} \right)^{\frac{3}{2}} - \frac{1}{6} \left(\frac{2x-3}{2 \times \frac{3}{2}} \right)^{\frac{3}{2}}$$

Hence, $I = \frac{1}{18} (2x+3)^{\frac{3}{2}} - \frac{1}{18} (2x-3)^{\frac{3}{2}} + C$

7. $\int \frac{2x}{(2x+1)^2} dx$

Solution:

Let $I = \int \frac{2x}{(2x+1)^2} dx$

Now by splitting the above equation we get

$$= \int \frac{2x+1}{(2x+1)^2} - \frac{1}{(2x+1)^2} dx$$

The above equation can be written as

$$= \int \frac{1}{(2x+1)} - (2x+1)^{-2} dx$$

On integrating we get

$$= \frac{1}{2} \log|2x+1| - \frac{(2x+1)^{-2+1}}{-2+1(2)}$$

$$= \frac{1}{2} \log|2x+1| - \frac{(2x+1)^{-1}}{-2}$$

Hence, $I = \frac{1}{2} \log|2x+1| + \frac{1}{2(2x+1)} + C$

8. $\int \frac{1}{\sqrt{x+a} + \sqrt{x+b}} dx$

Solution:

Let $I = \int \frac{1}{\sqrt{x+a} + \sqrt{x+b}} dx$

Now, multiply with conjugate, we get

$$= \int \frac{1}{\sqrt{x+a}+\sqrt{x+b}} \times \frac{(\sqrt{x+a}-\sqrt{x+b})}{\sqrt{x+a}-\sqrt{x+b}} dx$$

$$= \int \frac{(\sqrt{x+a}-\sqrt{x+b})}{(\sqrt{x+a})^2 - \sqrt{(x+b)}^2} dx$$

On computing, we get

$$= \int \frac{(\sqrt{x+a}-\sqrt{x+b})}{a-b} dx$$

On integrating the above equation we get

$$= \frac{1}{a-b} \left[\frac{2}{3} (x+a)^{\frac{3}{2}} - \frac{2}{3} (x+b)^{\frac{3}{2}} \right]$$

$$\text{Hence, } I = \frac{2}{3(a-b)} \left[(x+a)^{\frac{3}{2}} - (x+b)^{\frac{3}{2}} \right] + C$$

9. $\int \sin x \sqrt{1 + \cos 2x} dx$

Solution:

$$\text{Let } I = \int \sin x \sqrt{1 + \cos 2x} dx$$

$$= \int \sin x \sqrt{1 + \cos 2x} dx$$

By substituting the formula, we get

$$= \int \sin x \sqrt{2 \cos^2 x} dx$$

$$= \int \sin x \sqrt{2} \cos x dx$$

$$= \sqrt{2} \int \sin x \cos x dx$$

Now, multiply and Divide by 2 we get,

$$= \frac{\sqrt{2}}{2} \int 2 \sin x \cos x dx$$

$$= \frac{\sqrt{2}}{2} \int \sin 2x \, dx$$

On integrating

$$= \frac{\sqrt{2} - \cos 2x}{2}$$

$$\text{Hence, } I = -\frac{1}{2\sqrt{2}} \cos 2x + C$$



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EXERCISE 19.4

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$$1. \int \frac{x^2 + 5x + 2}{x + 2} dx$$

Solution:

Given

$$\int \frac{x^2 + 5x + 2}{x + 2} dx$$

By performing long division of the given equation we get

$$\text{Quotient} = x + 3$$

$$\text{Remainder} = -4$$

∴ We can write the above equation as

$$\Rightarrow x + 3 - \frac{4}{x+2}$$

∴ The above equation becomes

$$\Rightarrow \int x + 3 - \frac{4}{x+2} dx$$

By splitting

$$\Rightarrow \int x dx + 3 \int dx - 4 \int \frac{1}{x+2} dx$$

$$\text{We know } \int x dx = \frac{x^n}{n+1}; \int \frac{1}{x} dx = \ln x$$

$$\Rightarrow \frac{x^2}{2} + 3x - 4 \ln(x + 2) + c. \text{ (Where } c \text{ is some arbitrary constant)}$$

$$= \frac{x^2}{2} + 3x - 4 \log |x + 2| + c$$

$$2. \int \frac{x^3}{x - 2} dx$$

Solution:

Given

$$\int \frac{x^3}{x-2} dx$$

By performing long division of the given equation we get

$$\text{Quotient} = x^2 + 2x + 4$$

$$\text{Remainder} = 8$$

∴ We can write the above equation as

$$\Rightarrow x^2 + 2x + 4 + \frac{8}{x-2}$$

∴ The above equation becomes

$$\Rightarrow \int x^2 + 2x + 4 + \frac{8}{x-2} dx$$

$$\Rightarrow \int x^2 dx + 2 \int x dx + 4 \int dx + 8 \int \frac{1}{x-2} dx$$

We know $\int x^n dx = \frac{x^{n+1}}{n+1}$; $\int \frac{1}{x} dx = \ln x$

$$\Rightarrow \frac{x^3}{3} + 2 \frac{x^2}{2} + 4x + 8 \ln(x-2) + c$$

$$\Rightarrow \frac{x^3}{3} + x^2 + 4x + 8 \ln(x-2) + c \quad \text{(Where } c \text{ is some arbitrary constant)}$$

$$= \frac{x^3}{3} + x^2 + 4x + 8 \log |x-2| + c$$

3. $\int \frac{x^2 + x + 5}{3x + 2} dx$

Solution:

Given

$$\int \frac{x^2 + x + 5}{3x + 2} dx$$

By doing long division of the given equation we get

$$\text{Quotient} = \frac{x}{3} + \frac{1}{9}$$

$$\text{Remainder} = \frac{43}{9}$$

∴ We can write the above equation as

$$\Rightarrow \frac{x}{3} + \frac{1}{9} + \frac{43}{9} \left(\frac{1}{3x+2} \right)$$

∴ The above equation becomes

$$\Rightarrow \int \left(\frac{x}{3} + \frac{1}{9} + \frac{43}{9} \left(\frac{1}{3x+2} \right) \right) dx$$

$$\Rightarrow \frac{1}{3} \int x dx + \frac{1}{9} \int dx + \frac{43}{9} \int \frac{1}{3x+2} dx$$

We know $\int x^n dx = \frac{x^{n+1}}{n+1}$; $\int \frac{1}{x} dx = \ln x$

$$\Rightarrow \frac{1}{3} \times \frac{x^2}{2} + \frac{1}{9} \times x + \frac{43}{9} \times \frac{1}{3} \ln(3x+2) + c$$

$$= \frac{x^2}{6} + \frac{1}{9}x + \frac{43}{27} \log |3x+2| + c \quad (\text{Where } c \text{ is some arbitrary constant})$$