EXERCISE 4.12

Evaluate: Cos (sin $^{-1}$ 3/5 + sin^{-1} 5/13) Solution:

Given Cos (sin $^{-1}$ 3/5 + sin^{-1} 5/13)

We know that,

$$\sin^{-1}x + \sin^{-1}y = \sin^{-1}\left[x\sqrt{1-y^2} + y\sqrt{1-x^2}\right]$$

By substituting this formula we get,

$$= \cos\left(\sin^{-1}\left[\frac{3}{5}\sqrt{1-\left(\frac{5}{13}\right)^2} + \frac{5}{13}\sqrt{1-\left(\frac{3}{5}\right)^2}\right]\right)$$

$$= \cos\left(\sin^{-1}\left[\frac{3}{5}\times\frac{12}{13} + \frac{5}{13}\times\frac{4}{5}\right]\right)$$

$$= \cos\left(\sin^{-1}\left[\frac{56}{65}\right]\right)$$

Again, we know that

$$\sin^{-1}x = \cos^{-1}\sqrt{1-x^2}$$

Now substituting, we get

$$= \cos\left(\cos^{-1}\sqrt{1-\left(\frac{56}{65}\right)^2}\right)$$

$$= \cos\left(\cos^{-1}\sqrt{\left(\frac{33}{65}\right)^2}\right) = \cos\left(\cos^{-1}\left(\frac{33}{65}\right)\right)$$

$$= \frac{33}{65}$$

Hence, $\cos\left(\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{5}{13}\right) = \frac{33}{65}$