Chapter 2.
EXERCISE 2.3

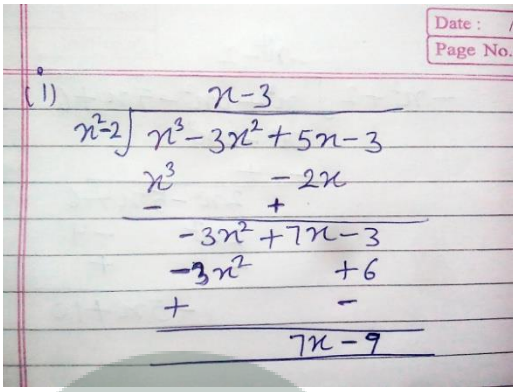
1. Divide the polynomial $p(x)$ by the polynomial $g(x)$ and find the quotient and remainder in each of the following :

(i) $p(x) = x^3 - 3x^2 + 5x - 3,\ g(x) = x^2 - 2$

Solution:

P(x) = x^3-3x^2+5x-3

g(x) = x^2-2



Quotient = $x - 3$

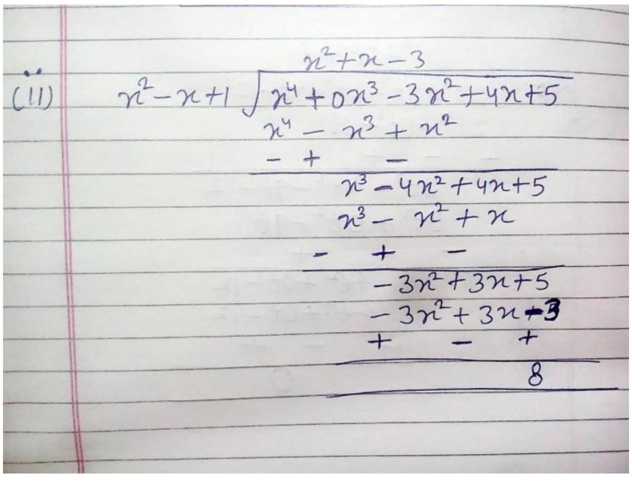
Remainder = $7x - 9$

(ii) $p(x) = x^4 - 3x^2 + 4x + 5,\ g(x) = x^2 + 1 - x$

Solution:

p(x) = x^4-3x^2+4x+5

g(x) = x^2+1-x



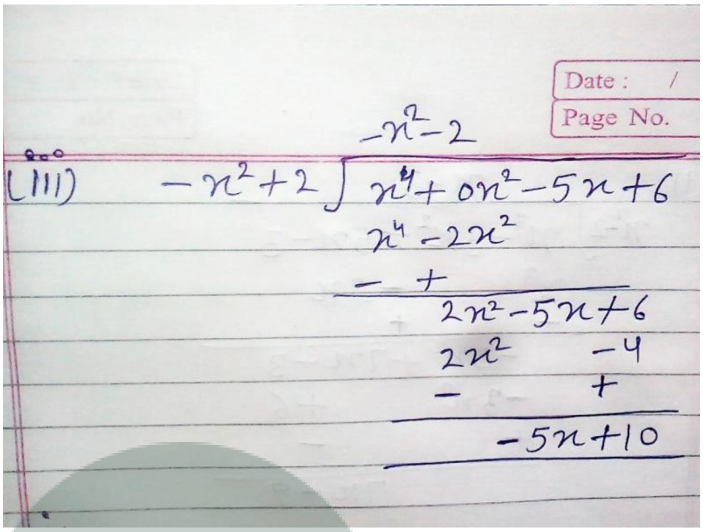
Quotient = $x^2 + x - 3$

Remainder = 8

(iii) $p(x) = x^4 - 5x + 6,\ g(x) = 2 - x^2$

Solution:

P(x) = x^4-5x+6
g(x) = 2-x^2



Quotient = $-x^2 - 2$

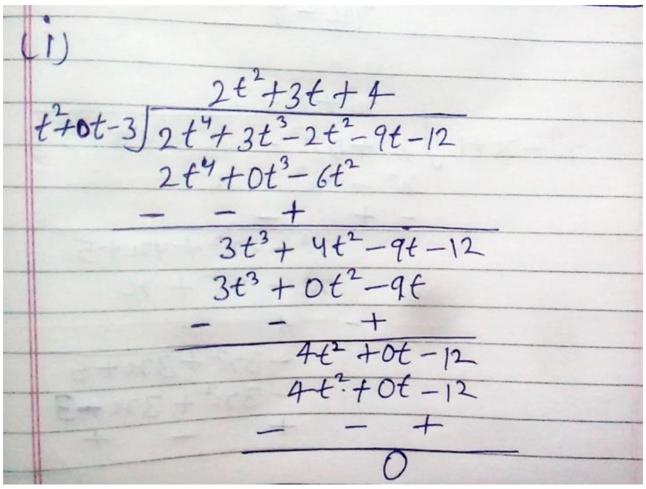
Remainder = $-5x + 10$

2. **Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:**

(i) $t^2 - 3, 2t^4 + 3t^3 - 2t^2 - 9t - 12$

Solution:

t^2-3 = t^2+0t-3

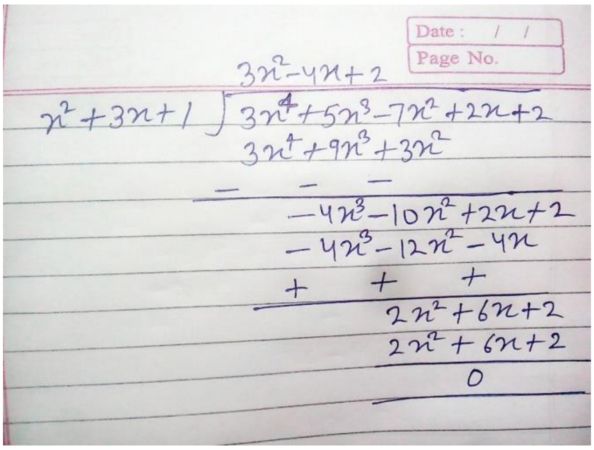


Since the remainder is 0,

Hence, $t^2 - 3$ is a factor of $2t^4+3t^3-2t^2-9t-12$.

(ii) $x^2+3x+1, 3x^4+5x^3-7x^2+2x+2$

Solution:

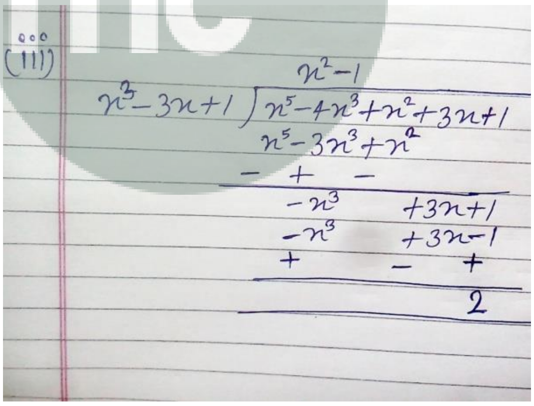


Since the remainder is 0,

Hence, x^2+3x+1 is a factor of $3x^4+5x^3-7x^2+2x+2$.

(iii) $x^3-3x+1, x^5-4x^3+x^2+3x+1$

Solution:



Since the remainder $\neq 0$,

Hence, x^3-3x+1 is not a factor of $x^5-4x^3+x^2+3x+1$.

3. Obtain all other zeroes of $3x^4+6x^3-2x^2-10x-5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Solution:

p(x) = $3x^4+6x^3-2x^2-10x-5$

Since the two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

$\therefore \left(x-\sqrt{\frac{5}{3}}\right)\left(x+\sqrt{\frac{5}{3}}\right)=\left(x^2-\frac{5}{3}\right)$ is a factor of $3x^4+6x^3-2x^2-10x-5$

Therefore, we divide the given polynomial by $x^2-\frac{5}{3}$

Quotient: $3x^2+6x+3$

Divisor: $x^2+0x-\frac{5}{3}$

$3x^4+6x^3-2x^2-10x-5$

$3x^4+0x^3-5x^2$

$- \quad - \quad +$

$6x^3+3x^2-10x-5$

$6x^3+0x^2-10x$

$- \quad - \quad +$

$3x^2+0x-5$

$3x^2+0x-5$

$- \quad - \quad +$

0

$3x^4+6x^3-2x^2-10x-5 = \left(x^2-\frac{5}{3}\right)(3x^2+6x+3)$

$= 3\left(x^2-\frac{5}{3}\right)(x^2+2x+1)$

we factorize x^2+2x+1,

$= (x+1)^2$

Therefore, its zero is given by $x + 1 = 0$ $x = -1$

As it has the term $(x+1)^2$, therefore, there will be 2 zeroes at $x = -1$.

Hence, the zeroes of the given polynomial are $\sqrt{\frac{5}{3}}$, $-\sqrt{\frac{5}{3}}$, -1 and -1.

4. **On dividing** 3^3-3x^2+x+2 **by a polynomial** $g(x)$, **the quotient and remainder were** $x-2$ **and** $-2x+4$, **respectively. Find** $g(x)$.

Solution:

$p(x) = x^3-3x^2+x+2$ (dividend)

$g(x)$ = ? (Divisor)

Quotient = (x − 2)

Remainder = (− 2x + 4)

Dividend = Divisor × Quotient + Remainder

$x^3-3x^2+3x+2 = g(x)\times(x-2)+(-2x+4)$

$x^3-3x^2+3x+2+2x-4 = g(x)(x-2)$

$x^3-3x^2+3x-2 = g(x)(x-2)$

$g(x)$ is the quotient when we divide (x^3-3x^2+3x-2) by $(x-2)$

$$
\begin{array}{r|l}
 & x^2-x+1 \\
\hline
x-2 & x^3-3x^2+3x-2 \\
 & x^3-2x^2 \\
 & -\quad + \\
\hline
 & \quad -x^2+3x-2 \\
 & \quad -x^2+2x \\
 & \quad +\quad - \\
\hline
 & \qquad\quad x-2 \\
 & \qquad\quad x-2 \\
 & \qquad\quad -\quad + \\
\hline
 & \qquad\qquad 0
\end{array}
$$

$$\therefore g(x)=(x^2-x+1)$$

5. Give examples of polynomials $p(x), g(x), q(x)$ and $r(x)$, which satisfy the division algorithm and

(i) deg $p(x) = \deg q(x)$

(ii) deg $q(x) = \deg r(x)$

(iii) deg $r(x) = 0$

Solutions:

Degree of a polynomial is the highest power of the variable in the polynomial.

(i) deg $p(x)$ = deg $q(x)$

Degree of quotient will be equal to degree of dividend when divisor is constant.

Let us assume the division of $6x^2 + 2x + 2$ by 2.

Here, $p(x)$ = $6x^2 + 2x + 2$

$g(x)$ = 2

$q(x)$ = $3x^2 + x + 1$ and $r(x)$ = 0

Degree of $p(x)$ and $q(x)$ is the same i.e., 2. Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$6x^2 + 2x + 2 = 2(3x^2 + x + 1)$

= $6x^2 + 2x + 2$

Thus, the division algorithm is satisfied.

(ii) deg $q(x)$ = deg $r(x)$

Let us assume the division of $x^3 + x$ by x^2,

Here,

$p(x) = x^3 + x$

$g(x) = x^2$

$q(x) = x$ and $r(x) = x$

Clearly, the degree of $q(x)$ and $r(x)$ is the same i.e., 1. Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$ $x^3 + x$

$= (x^2) \times x + x$ $x^3 + x = x^3 + x$

Thus, the division algorithm is satisfied.

(iii)deg $r(x)$ = 0

Degree of remainder will be 0 when remainder comes to a constant.

Let us assume the division of $x^3 + 1$by x^2.

Here,

$p(x) = x^3 + 1$ $g(x) = x^2$

$q(x) = x$ and $r(x) = 1$

Clearly, the degree of $r(x)$ is 0. Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x) x^3 + 1$

$= (x^2) \times x + 1$ $x^3 + 1 = x^3 + 1$

Thus, the division algorithm is satisfied.