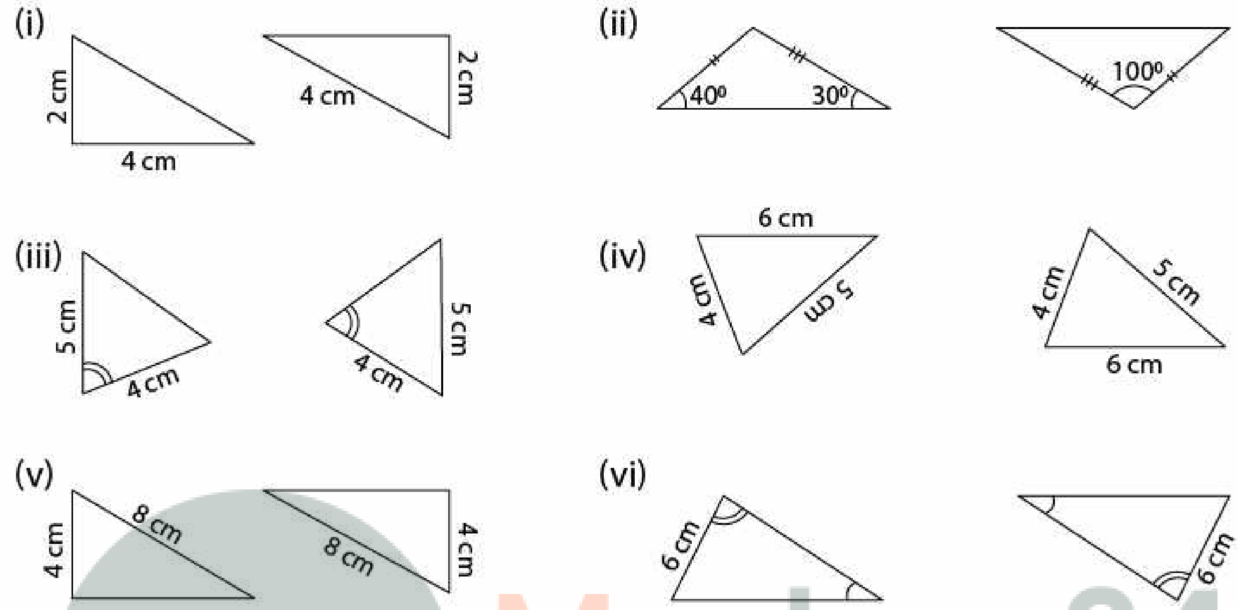
EXERCISE 19

State, whether the pairs of triangles given in the following figures are congruent or not:



(vii) ΔABC in which AB = 2 cm, BC = 3.5 cm and ∠C = 80^0 and, ΔDEF in which DE = 2cm, DF = 3.5 cm and ∠D = 80^0.

Solution:

(i) In the given figure, corresponding sides of the triangles are not equal.
Therefore, the given triangles are not congruent.

(ii) In the first triangle
Third angle = $180^0 – (40^0 + 30^0)$
By further calculation
= 180^0 - 70^0
So we get
= 110^0

In the two triangles, the sides and included angle of one are equal to the corresponding sides and included angle.
Therefore, the given triangles are congruent. (SAS axiom)

(iii) In the given figure, corresponding two sides are equal and the included angles are not equal.
Therefore, the given triangles are not congruent.

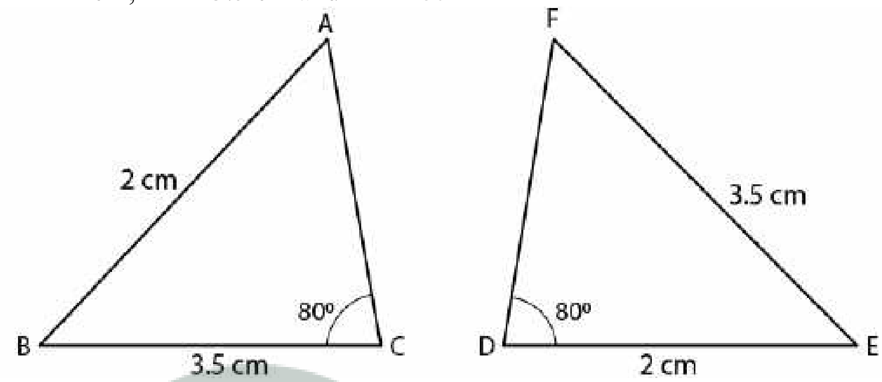
(iv) In the given figure, the corresponding three sides are equal.
Therefore, the given triangles are congruent. (SSS Axiom)

(v) In the right triangles, one side and diagonal of one triangle are equal to the corresponding side and diagonal of the other.
Therefore, the given triangles are congruent. (RHS Axiom)

(vi) In the given figure, two sides and one angle of one triangle are equal to the corresponding sides and one angle of the other.

Therefore, the given triangles are congruent. (SSA Axiom)

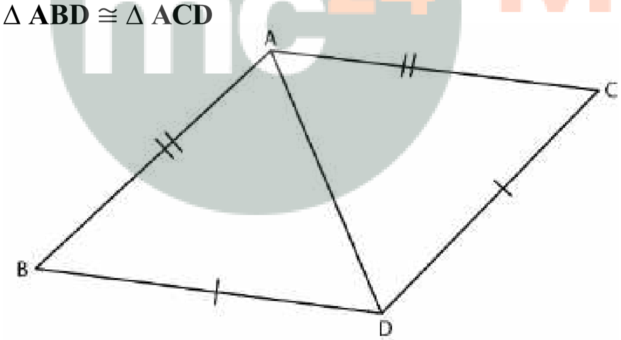
(vii) In Δ ABC,
It is given that
AB = 2cm, BC = 3.5 cm, $\angle C = 80^0$
In Δ DEF,
It is given that
DE = 2 cm, DF = 3.5 cm and $\angle D = 80^0$



We get to know that two corresponding sides are equal but the included angles are not equal.
Therefore, the triangles are not congruent.

1. In the given figure, prove that:

Δ ABD ≅ Δ ACD



Solution:

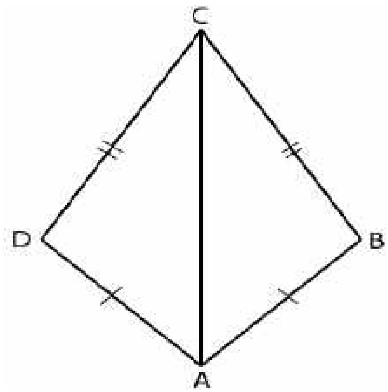
In Δ ABD and Δ ACD
AD = AD is common
It is given that
AB = AC and BD = DC
Here Δ ABD ≅ Δ ACD (SSS Axiom)
Therefore, it is proved.

2. Prove that:

(i) Δ ABC ≅ Δ ADC

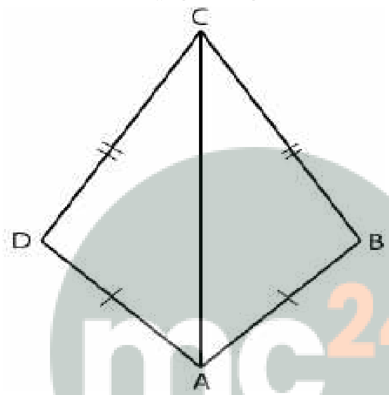
(ii) $\angle B = \angle D$

(iii) AC bisects angle DCB.



Solution:

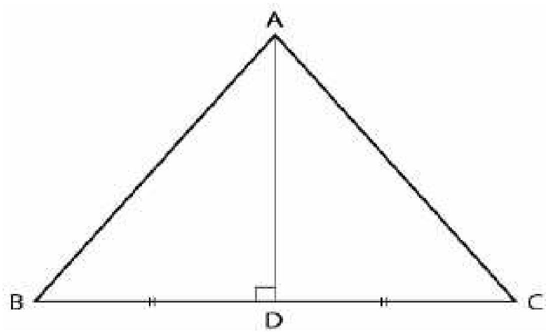
In the figure
AB = AD and CB = CD



In Δ ABC and Δ ADC
AC = AC is common
It is given that
AB = AD and CB = CD

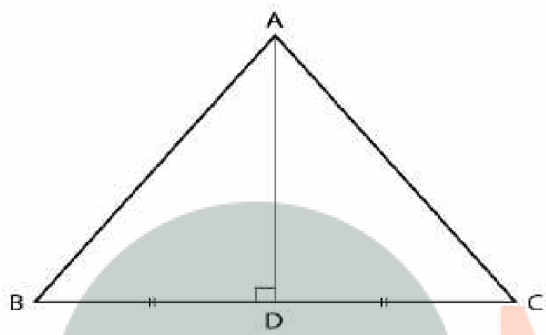
Here Δ ABC ≅ Δ ADC (SSS Axiom)
∠B = ∠D (c. p. c. t)
So we get
∠BCA = ∠DCA
Therefore, AC bisects ∠DCB.

3. Prove that:
(i) ΔABD ≡ ΔACD
(ii) ∠B = ∠C
(iii) ∠ADB = ∠ADC
(iv) ∠ADB = 90°



Solution:

From the figure
AD = AC and BD = CD



In ΔABD and ΔACD
AD = AD is common

(i) ΔABD ≡ ΔACD (SSS Axiom)

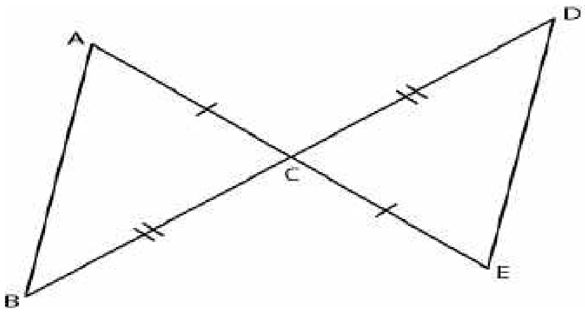
(ii) ∠B = ∠C (c. p. c. t)

(iii) ∠ADB = ∠ADC (c. p. c. t)

(iv) We know that
∠ADB + ∠ADC = 180^0 is a linear pair
Here ∠ADB = ∠ADC
So we get
∠ADB = $180^0/2$
∠ADB = 90°

4. In the given figure, prove that:

(i) Δ ACB ≅ Δ ECD
(ii) AB = ED

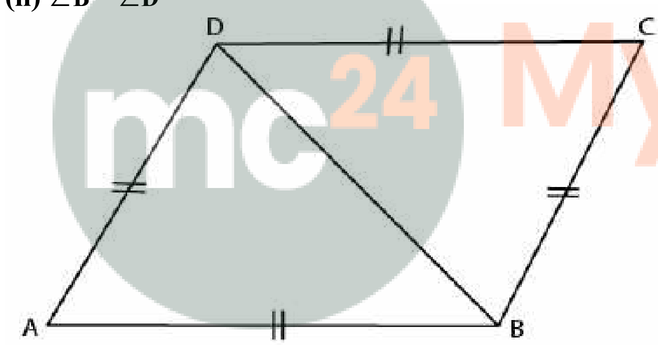


Solution:

(i) In Δ ACB and Δ ECD
It is given that AC = CE and BC = CD
∠ACB = ∠DCE are vertically opposite angles
Hence, Δ ACB ≅ Δ ECD (SAS Axiom)

(ii) Here AB = ED (c. p. c. t)
Therefore, it is proved.

5. Prove that
(i) Δ ABC ≅ Δ ADC
(ii) ∠B = ∠D

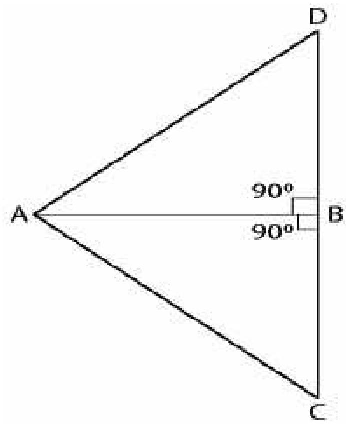


Solution:

(i) In Δ ABC and Δ ADC
It is given that
AB = DC and BC = AD
AC = AC is common
Hence, Δ ABC ≅ Δ ADC (SSS Axiom)

(ii) Here ∠B = ∠D (c. p. c. t)
Therefore, it is proved.

6. In the given figure, prove that:
BD = BC.

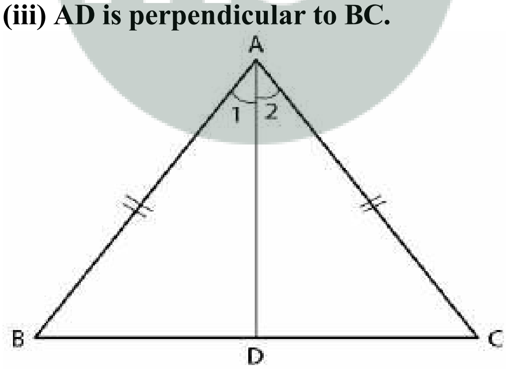


Solution:

In right Δ ABD and Δ ABC
AB = AB is common
It is given that
AD = AC
Hence, Δ ABD ≅ Δ ABC (RHS Axiom)

Here BD = BC (c. p. c. t)
Therefore, it is proved.

7. In the given figure, ∠1 = ∠2 and AB = AC.
Prove that:
(i) ∠B = ∠ C
(ii) BD = DC
(iii) AD is perpendicular to BC.



Solution:

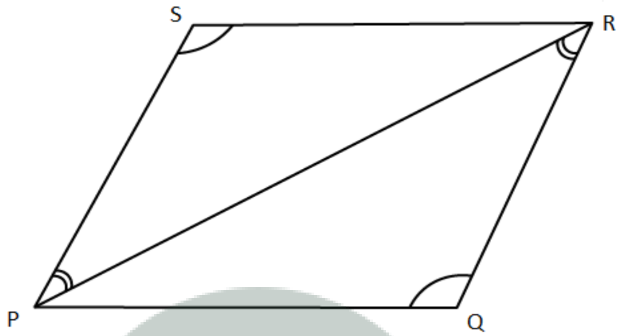
In Δ ADB and Δ ADC
It is given that
AB = AC and ∠1 = ∠2
AD = AD is common
Hence, Δ ADB ≅ Δ ADC (SAS Axiom)

(i) ∠B = ∠C (c. p. c. t)

(ii) BD = DC (c. p. c. t)

(iii) ∠ADB = ∠ADC (c. p. c. t)
We know that
∠ADB + ∠ADC = 180^0 is a linear pair
So we get
∠ADB = ∠ADC = 90^0
Here, AD is perpendicular to BC
Therefore, it is proved.

8. In the given figure, prove that:
(i) PQ = RS
(ii) PS = QR



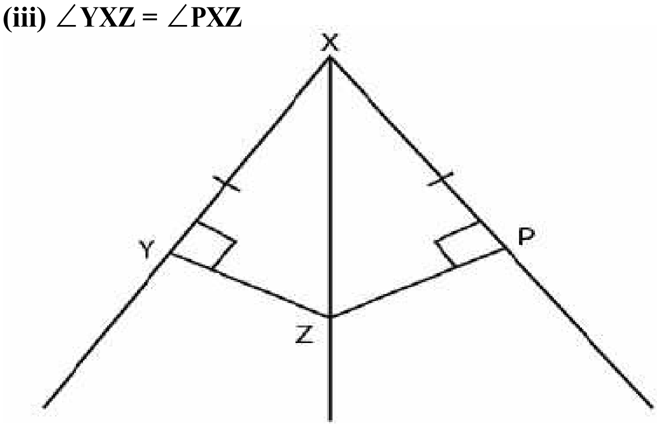
Solution:

In Δ PQR and Δ PSR
PR = PR is common
It is given that
∠PRQ = ∠RPS and ∠PQR = ∠PSR
Δ PQR ≅ Δ PSR (AAS Axiom)

(i) PQ = RS (c. p. c. t)

(ii) QR = PS or PS = QR (c. p. c. t)
Therefore, it is proved.

9. In the given figure, prove that:
(i) Δ XYZ ≅ Δ XPZ
(ii) YZ = PZ
(iii) ∠YXZ = ∠PXZ



Solution:

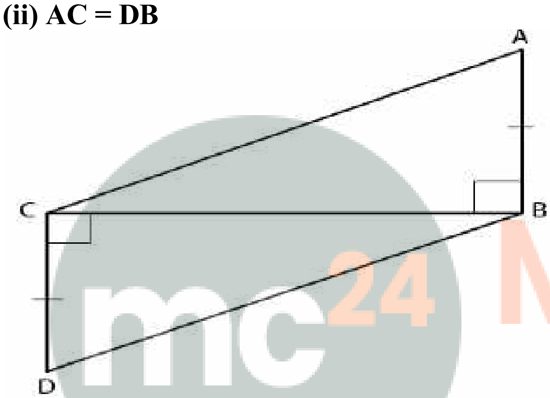
In Δ XYZ and Δ XPZ
It is given that
XY = XP
XZ = XZ is common

(i) Δ XYZ ≅ Δ XPZ (RHS Axiom)

(ii) YZ = PZ (c. p. c. t)

(iii) ∠YXZ = ∠PXZ (c. p. c. t)
Therefore, it is proved.

10. In the given figure, prove that:
(i) Δ ABC ≅ Δ DCB
(ii) AC = DB



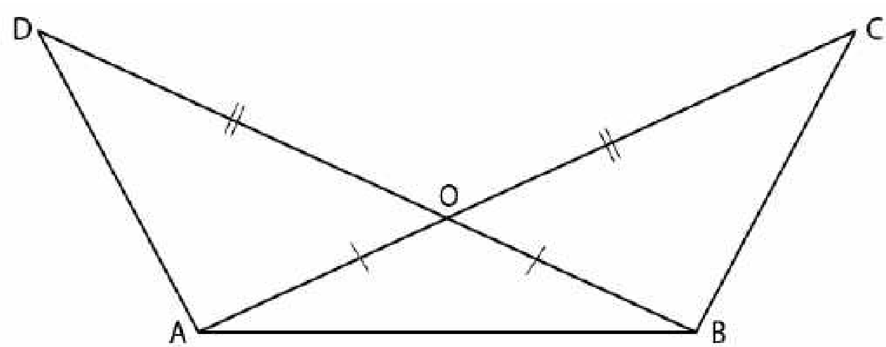
Solution:

In Δ ABC and Δ DCB
CB = CB is common
∠ABC = ∠BCD = 90^0
It is given that
AB = CD

(i) Δ ABC ≅ Δ DCB (SAS Axiom)

(ii) AC = DB (c. p. c. t)
Therefore, it is proved.

11. In the given figure, prove that:
(i) Δ AOD ≅ Δ BOC
(ii) AD = BC
(iii) ∠ADB = ∠ACB
(iv) ΔADB ≅ ΔBCA



Solution:

In Δ AOD and Δ BOC
It is given that
OA = OB and OD = OC
∠AOD = ∠BOC are vertically opposite angles

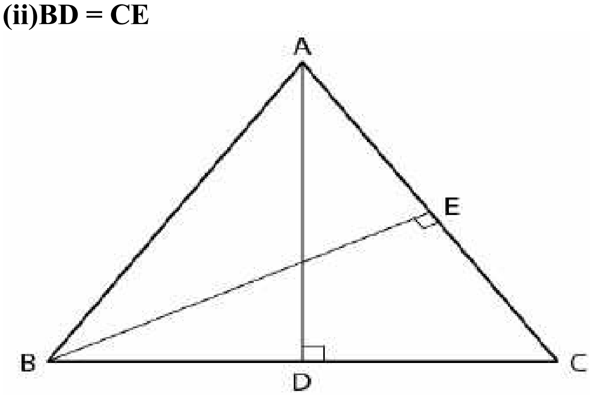
(i) Δ AOD ≅ Δ BOC (SAS Axiom)

(ii) AD = BC (c. p. c. t)

(iii) ∠ADB = ∠ACB (c. p. c. t)

(iv) ΔADB ≅ ΔBCA
It is given that
ΔADB = ΔBCA
AB = AB is common
Here Δ AOB ≅ Δ BCA
Therefore, it is proved.

12. ABC is an equilateral triangle, AD and BE are perpendiculars to BC and AC respectively. Prove that:
(i) AD = BE
(ii)BD = CE



Solution:

In Δ ABC
AB = BC = CA
We know that
AD is perpendicular to BC and BE is perpendicular to AC

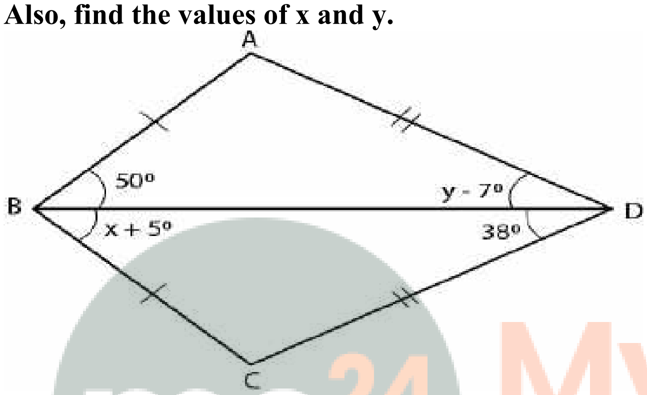
In Δ ADC and Δ BEC
$\angle ADC = \angle BEC = 90^0$
$\angle ACD = \angle BCE$ is common
AC = BC are the sides of an equilateral triangle
Δ ADC ≅ Δ BEC (AAS Axiom)

(i) AD = BE (c. p. c. t)

(ii) BD = CE (c. p. c. t)

Therefore, it is proved.

13. Use the informations given in the following figure to prove triangles ABD and CBD are congruent. Also, find the values of x and y.



Solution:

In the figure
AB = BC and AD = DC
$\angle ABD = 50^0$, $\angle ADB = y - 7^0$
$\angle CBD = x + 5^0$, $\angle CDB = 38^0$

In Δ ABD and Δ CBD
BD = BD is common
It is given that
AB = BC and AD = CD
Here Δ ABD ≅ Δ CBD (SSS Axiom)

$\angle ABD = \angle CBD$
So we get
$50 = x + 5$
$x = 50 – 5 = 45^0$

$\angle ADB = \angle CDB$
$y – 7 = 38$
$y = 38 + 7 = 45^0$

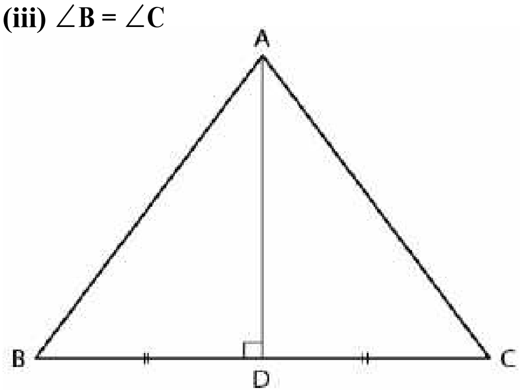
Therefore, $x = 45^0$ and $y = 45^0$.

14. The given figure shows a triangle ABC in which AD is perpendicular to side BC and BD = CD. Prove

that:

(i) Δ ABD ≅ Δ ACD
(ii) AB = AC

(iii) ∠B = ∠C



Solution:

(i) In Δ ABC
AD is perpendicular to BC
BD = CD
In Δ ABD and Δ ACD
AD = AD is common
$\angle ADB = \angle ADC = 90^0$
It is given that
BD = CD
Δ ABD ≅ Δ CAD (SAS Axiom)

(ii) AB = AC (c. p. c. t)

(iii) ∠B = ∠C as Δ ADB ≅ Δ ADC
Therefore, it is proved.