EXERCISE 5.3

1. Compute the indicated products:

(i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ -3 & 2 & 1 \end{bmatrix}$

(iii) $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$

Solution:

(i) Consider

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a\times a+b\times b & a\times(-b)+b\times a \\ (-b)\times a+a\times b & (-b)\times(-b)+a\times a \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a^2+b^2 & -ab+ab \\ -ab+ab & b^2+a^2 \end{bmatrix}$$

On simplification we get,

$$\Rightarrow \begin{bmatrix} a^2+b^2 & 0 \\ 0 & a^2+b^2 \end{bmatrix}$$

(ii) Consider

$$\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ -3 & 2 & -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1\times1+(-2\times(-3)) & 1\times2+(-2)\times2 & 1\times3+(-2)\times(-1) \\ 2\times1+3\times(-3) & 2\times2+3\times2 & 2\times3+3\times(-1) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1+6 & 2-4 & 3+2 \\ 2-9 & 4+6 & 6-3 \end{bmatrix}$$

On simplification we get,

$$\Rightarrow \begin{bmatrix} 7 & -2 & 5 \\ -7 & 10 & 3 \end{bmatrix}$$

(iii) Consider

$$\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2\times1+3\times0+4\times3 & 2\times(-3)+3\times2+4\times0 & 2\times5+3\times4+4\times5 \\ 3\times1+4\times0+5\times3 & 3\times(-3)+4\times2+5\times0 & 3\times5+4\times4+5\times5 \\ 4\times1+5\times0+6\times3 & 4\times(-3)+5\times2+6\times0 & 4\times5+5\times4+6\times5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2+0+12 & -6+6+0 & 10+12+20 \\ 3+0+15 & -9+8+0 & 15+16+25 \\ 4+0+18 & -12+10+0 & 20+20+30 \end{bmatrix}$$

On simplification we get,

$$\Rightarrow \begin{bmatrix} 14 & 0 & 42 \\ 18 & -1 & 56 \\ 22 & -2 & 70 \end{bmatrix}$$

2. Show that AB ≠ BA in each of the following cases:

$$(i)\ A = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} and\ B = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$

$$(ii)\ A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} and\ B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$(iii)\ A = \begin{bmatrix} 1 & 3 & 0 \\ 1 & 1 & 0 \\ 4 & 1 & 0 \end{bmatrix} and\ B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 5 & 1 \end{bmatrix}$$

Solution:

(i) Consider,

$$AB = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 10-3 & 5-4 \\ 12+21 & 6+28 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix} \text{..................(1)}$$

Again consider,

$$BA = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 10+6 & -2+7 \\ 15+24 & -3+28 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix} \text{------} (2)$$

From equation (1) and (2), it is clear that

AB ≠ BA

(ii) Consider,

$$AB = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} -1+0+0 & -2+1+0 & -3+0+0 \\ 0+0+1 & 0-1+1 & 0+0+0 \\ 2+0+4 & 4+3+4 & 6+0+0 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} -1 & -1 & -3 \\ 1 & 0 & 0 \\ 6 & 11 & 6 \end{bmatrix} \text{..............} (1)$$

Now again consider,

$$BA = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} -1+0+6 & 1-2+9 & 0+2+12 \\ 0+0+0 & 0-1+0 & 0+1+0 \\ -1+0+0 & 1-1+0 & 0+1+0 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix} \text{..............} (2)$$

From equation (1) and (2), it is clear that

AB ≠ BA

(iii) Consider,

$$AB = \begin{bmatrix} 1 & 3 & 0 \\ 1 & 1 & 0 \\ 4 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 5 & 1 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 0+3+0 & 1+0+0 & 0+0+0 \\ 0+1+0 & 1+0+0 & 0+0+0 \\ 0+1+0 & 4+0+0 & 0+0+0 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 3 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 4 & 0 \end{bmatrix} \text{..................}(1)$$

Now again consider,

$$BA = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 0 \\ 1 & 1 & 0 \\ 4 & 1 & 0 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 0+1+0 & 0+1+0 & 0+0+0 \\ 1+0+0 & 3+0+0 & 0+0+0 \\ 0+5+4 & 0+5+1 & 0+0+0 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} [illegible] \\ [illegible] \\ 9 & 6 & 0 \end{bmatrix} \text{..................}(2)$$

From equation (1) and (2), it is clear that
AB ≠ BA

3. Compute the products AB and BA whichever exists in each of the following cases:

$$(i)\ A = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$$

$$(ii)\ A = \begin{bmatrix} 3 & 2 \\ -1 & 0 \\ -1 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 & 5 & 6 \\ 0 & 1 & 2 \end{bmatrix}$$

$$(iii)\ A = \begin{bmatrix} 1 & -1 & 2 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 2 \end{bmatrix}$$

$$(iv)\ \begin{bmatrix} a & b \end{bmatrix}\begin{bmatrix} c \\ d \end{bmatrix} + \begin{bmatrix} a & b & c & d \end{bmatrix}\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

Solution:

(i) Consider,

$$AB = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 1-4 & 2-6 & 3-2 \\ 2+6 & 4+9 & 6+3 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix}$$

BA does not exist

Because the number of columns in B is greater than the rows in A

(ii) Consider,

$$AB = \begin{bmatrix} 3 & 2 \\ -1 & 0 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 4 & 5 & 6 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 12+0 & 15+2 & 18+4 \\ -4+0 & -5+0 & -6+0 \\ -4+0 & -5+1 & -6+2 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 12 & 17 & 22 \\ -4 & -5 & -6 \\ -4 & -4 & -4 \end{bmatrix}$$

Again consider,

$$BA = \begin{bmatrix} 4 & 5 & 6 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ -1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 12-5-6 & 8+0+6 \\ 0-1-2 & 0+0+2 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 1 & 14 \\ -3 & 2 \end{bmatrix}$$

(iii) Consider,

$$AB = \begin{bmatrix} 1 & -1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 3 \\ 2 \end{bmatrix}$$

AB = [0 + (-1) + 6 + 6]
AB = 11
Again consider,

$$BA = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 & 3 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & -1 & 2 & 3 \\ 3 & -3 & 6 & 9 \\ 2 & -2 & 4 & 6 \end{bmatrix}$$

(iv) Consider,

$$\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix} + \begin{bmatrix} a & b & c & d \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$\Rightarrow [ac + bd] + [a^2 + b^2 + c^2 + d^2]$$

$$[a^2 + b^2 + c^2 + d^2 + ac + bd]$$

4. Show that AB ≠ BA in each of the following cases:

$$(i) A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & -1 & -1 \\ 3 & 0 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix}$$

$$(ii) A = \begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix}$$

Solution:

(i) Consider,

$$AB = \begin{bmatrix} 1 & 3 & -1 \\ 2 & -1 & -1 \\ 3 & 0 & -1 \end{bmatrix}\begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} -2-3+6 & 3+6-9 & -1-3+4 \\ -4+1+6 & 6-2-9 & -2+1+4 \\ -6-0+6 & 9+0-9 & -3-0+4 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 1 & 0 & 0 \\ 3 & -5 & 3 \\ 0 & 0 & 1 \end{bmatrix} \ldots\ldots\ldots\ldots(1)$$

Again consider,

$$BA = \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix}\begin{bmatrix} 1 & 3 & -1 \\ 2 & -1 & -1 \\ 3 & 0 & -1 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} -2+6-3 & -6-3+0 & 2-3+1 \\ -1+4-3 & -3-2+0 & 1-2+1 \\ -6+18-12 & -18-9+0 & 6-9+4 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & -27 & 1 \end{bmatrix} \ldots\ldots\ldots\ldots(2)$$

From equation (1) and (2), it is clear that

AB ≠ BA

(ii) Consider,

$$AB = \begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 10-12-1 & 20-16-3 & 10-8-2 \\ -11+15+0 & -22+20+0 & -11+10+0 \\ 9-15+1 & 18-20+3 & 9-10+2 \end{bmatrix}$$

$$AB = \begin{bmatrix} -3 & 1 & 0 \\ 4 & -2 & -1 \\ -5 & 1 & 1 \end{bmatrix} \text{......(1)}$$

Again consider,

$$BA = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 10 - 22 + 9 & -4 + 10 - 5 & -9 + 0 + 1 \\ 30 - 44 + 10 & -12 + 20 - 10 & -3 + 0 + 2 \\ 10 - 33 + 18 & -4 + 15 - 10 & -1 + 0 + 2 \end{bmatrix}$$

$$BA = \begin{bmatrix} -3 & 1 & 0 \\ 4 & -2 & -1 \\ -5 & 1 & 1 \end{bmatrix} \text{......(2)}$$

From equation (1) and (2) it is clear that,

$AB \neq BA$

5. Evaluate the following:

(i) $\left(\begin{bmatrix} 1 & 3 \\ -1 & -4 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} \right) \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 3 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 2 \end{bmatrix} \right)$

Solution:

(i) Given

$$\left(\begin{bmatrix} 1 & 3 \\ -1 & -4 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} \right) \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

First we have to add first two matrix,

$$\Rightarrow \left(\begin{bmatrix} 1+3 & 3-2 \\ -1-1 & -4+1 \end{bmatrix}\right)\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & 1 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4+2 & 12+4 & 20+6 \\ -2-6 & -6-12 & -10-18 \end{bmatrix}$$

On simplifying, we get

$$\Rightarrow \begin{bmatrix} 6 & 16 & 26 \\ -8 & -18 & -28 \end{bmatrix}$$

(ii) Given,

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

First we have to multiply first two given matrix,

$$\Rightarrow \begin{bmatrix} 1+4+0 & 0+0+3 & 2+2+6 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 5 & 3 & 10 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 10+12+60 \end{bmatrix}$$

= 82

(iii) Given

$$\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 3 \end{bmatrix}\left(\begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 2 \end{bmatrix}\right)$$

First we have subtract the matrix which is inside the bracket,

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1-0 & 0-1 & 2-2 \\ 2-1 & 0-0 & 1-2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1-1 & -1+0 & 0+1 \\ 0+2 & 0+0 & 0-2 \\ 2+3 & -2+0 & 0-3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & -1 & 1 \\ 2 & 0 & -2 \\ 5 & -2 & -3 \end{bmatrix}$$

6. If $\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and $\mathbf{C} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, show that $\mathbf{A}^2 = \mathbf{B}^2 = \mathbf{C}^2 = \mathbf{I}_2$

Solution:

Given

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \text{ and } C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

We know that,

$$A^2 = AA$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{.........(1)}$$

Again we know that,

$$B^2 = BB$$

$$\Rightarrow B^2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\Rightarrow B^2 = \begin{bmatrix} 1+0 & 0-0 \\ 0-0 & 0+1 \end{bmatrix}$$

$$\Rightarrow B^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{.........(2)}$$

Now, consider,

$C^2 = CC$

$\Rightarrow B^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$\Rightarrow B^2 = \begin{bmatrix} 0+1 & 0+0 \\ 0+0 & 1+0 \end{bmatrix}$

$\Rightarrow B^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$……(3)

We have,

$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$……………….(4)

Now, from equation (1), (2), (3) and (4), it is clear that $A^2 = B^2 = C^2 = I_2$

7. If $A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$, find $3A^2 - 2B + I$

Solution:

Given

$A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$

Consider,

$A^2 = AA$

$\Rightarrow A^2 = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 4-3 & -2-2 \\ 6+6 & -3+4 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 1 & -4 \\ 12 & 1 \end{bmatrix}$

Now we have to find,

$3A^2 - 2B + I$

$\Rightarrow 3A^2 - 2B + I = 3\begin{bmatrix} 1 & -4 \\ 12 & 1 \end{bmatrix} - 2\begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow 3A^2 - 2B + I = \begin{bmatrix} 3 & -12 \\ 36 & 3 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\Rightarrow 3A^2 - 2B + I = \begin{bmatrix} 3-0+1 & -12-8+0 \\ 36+2+0 & 3-14+1 \end{bmatrix}$$

$$\Rightarrow 3A^2 - 2B + I = \begin{bmatrix} 4 & -20 \\ 38 & -10 \end{bmatrix}$$

8. If $\mathbf{A} = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$, prove that $(\mathbf{A} - 2\mathbf{I})(\mathbf{A} - 3\mathbf{I}) = 0$

Solution:

Given

$$A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$$

Consider,

$$\Rightarrow (A-2I)(A-3I) = \left(\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}\right)\left(\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)$$

$$\Rightarrow (A-2I)(A-3I) = \left(\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\right)\left(\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\right)$$

$$\Rightarrow (A-2I)(A-3I) = \begin{bmatrix} 4-2 & 2-0 \\ -1-0 & 1-2 \end{bmatrix}\begin{bmatrix} 4-3 & 2-0 \\ -1-0 & 1-3 \end{bmatrix}$$

$$\Rightarrow (A-2I)(A-3I) = \begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$$

$$\Rightarrow (A-2I)(A-3I) = \begin{bmatrix} 2-2 & 4-4 \\ -1+1 & -2+2 \end{bmatrix}$$

$$\Rightarrow (A-2I)(A-3I) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow (A-2I)(A-3I) = 0$$

Hence the proof.

9. If $\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, show that $\mathbf{A}^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ and $\mathbf{A}^3 = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

Solution:

Given,

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

Consider,

$$A^2 = A\,A$$

$$A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1+0 & 1+1 \\ 0+0 & 0+1 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

Again consider,

$$A^3 = A^2 A$$

$$A^3 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 1+0 & 1+2 \\ 0+0 & 0+1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

Hence the proof.

10 If $\mathbf{A} = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$, show that $\mathbf{A}^2 = 0$

Solution:

Given,

$$A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$$

Consider,

$$A^2 = A\ A$$

$$\Rightarrow A^2 = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}\begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} a^2b^2 - a^2b^2 & ab^3 - ab^3 \\ -a^3b + a^3b & -a^2b^2 + a^2b^2 \end{bmatrix}$$

$\Rightarrow A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow A^2 = 0$

Hence the proof.

11. If $\mathbf{A} = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$, find $\mathbf{A}^2$

Solution:

Given,

$A = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

Consider,

$A^2 = A.A$

$\Rightarrow A^2 = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix} \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} \cos^2 2\theta - \sin^2 2\theta & \cos 2\theta \sin 2\theta + \sin 2\theta \cos 2\theta \\ -\sin 2\theta \cos 2\theta - \sin 2\theta \cos 2\theta & -\sin^2(2\theta) + \cos^2(2\theta) \end{bmatrix}$

We know that,

$\cos^2\theta - \sin^2\theta = \cos^2(2\theta)$

$\Rightarrow A^2 = \begin{bmatrix} \cos(2 \times 2\theta) & 2\sin 2\theta \cos 2\theta \\ -2\sin 2\theta \cos(2\theta) & \cos(2 \times 2\theta) \end{bmatrix}$

Again we have,

$\sin 2\theta = 2\sin\theta\cos\theta$

$\Rightarrow A^2 = \begin{bmatrix} \cos 4\theta & \sin(2 \times 2\theta) \\ -\sin(2 \times 2\theta) & \cos 4\theta \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{bmatrix}$

12. If $\mathbf{A} = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$ show that $\mathbf{AB} = \mathbf{BA} = \mathbf{0}_{3\times 3}$

Solution:

Given,

$$A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix} \; and \; B = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$$

Consider,

$$AB = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}\begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -2-3+5 & 6+9-15 & 5+15-20 \\ 1+4-5 & -3-12+15 & -5-15+20 \\ -1-3+4 & 3+9-12 & 5+15-20 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$AB = 0_{3\times3}$ ……(1)

Again consider,

$$BA = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} -2-3+5 & 3+12-15 & 5+15-20 \\ 2+3-5 & -3-12+15 & -5-15+20 \\ -2-3+5 & 3+9-12 & 5+15-20 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$BA = 0_{3\times3}$ ……(2)

From equation (1) and (2) $AB = BA = 0_{3\times3}$

$$13. If\ A = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}\ and\ B = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}\ show\ that\ AB = BA = 0_{3\times 3}$$

Solution:

Given

$$A = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}\ and\ B = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$$

Consider,

$$AB = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}\begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 0+abc-abc & 0+b^2c-b^2c & 0+bc^2-bc^2 \\ -a^2c+0+a^2c & -abc+0+abc & -ac^2+0+ac^2 \\ a^2b-a^2b+0 & ab^2-ab^2+0 & abc-abc+0 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow AB = O_{3\times 3} \ldots (1)$$

Again consider,

$$BA = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}\begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 0-abc+abc & a^2c+0-a^2c & -a^2b+a^2b+0 \\ 0-b^2c+b^2c & abc+0-abc & -ab^2+ab^2+0 \\ 0-bc^2+bc^2 & ac^2+0-ac^2 & -abc+abc+0 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow BA = O_{3\times 3} \ldots (2)$$

From equation (1) and (2) AB = BA = $0_{3\times 3}$

$$14. If\ A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}\ and\ B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}\ show\ that\ AB = A\ and\ BA = B.$$

Solution:

Given

$$A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}\ and\ B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

Now consider,

$$AB = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 4+3-5 & -4-9+10 & -8-12+15 \\ -2-4+5 & 2+12-10 & 4+16-15 \\ 2+3-4 & -2-9+18 & -4-12+12 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$$

Therefore AB = A

Again consider, BA we get,

$$BA = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 4+2-4 & -6-8+12 & -10-10+16 \\ -2-3+4 & 3+12-12 & 5+15-16 \\ 2+2-3 & -3-8+9 & -5-10+12 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

Hence BA = B

Hence the proof.

15. Let $A = \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & 5 & 5 \end{bmatrix}$ **and** $B = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$**, compute** $A^2 - B^2$**.**

Solution:

Given,

Consider,

$$A^2 = \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & 5 & 5 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & 5 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1+3+5 & -1-3-5 & 1+3-5 \\ -3-9+15 & 3+9+15 & -3-9+15 \\ -5+15+25 & 5-15+25 & -5+15+25 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} -1 & 9 & -1 \\ 3 & 27 & 3 \\ 35 & 15 & 35 \end{bmatrix} \text{.....(1)}$$

Now again consider, B^2

$$B^2 = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}\begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 0+4-3 & 0-12+12 & 0-12+12 \\ 0-3+3 & 4+9-12 & 3+9-12 \\ 0+4-4 & -4-12+16 & -3-12+16 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \ldots\ldots(2)$$

Now by subtracting equation (2) from equation (1) we get,

$$A^2 - B^2 = \begin{bmatrix} -1 & 9 & -1 \\ 3 & 27 & 3 \\ 135 & 15 & 35 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 9 & -1 \\ 3 & 26 & 3 \\ 135 & 15 & 34 \end{bmatrix}$$

16. For the following matrices verify the associativity of matrix multiplication i.e. (AB) C = A (BC)

$$(i)\ A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix},\ B = \begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 0 & 3 \end{bmatrix},\ and\ C = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$(ii)\ A = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix},\ B = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix},\ and\ C = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution:

(i) Given

$$A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix},\ B = \begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 0 & 3 \end{bmatrix},\ and\ C = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Consider,

$$(AB)C = \left(\begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 0 & 3 \end{bmatrix}\right)\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2+0 & 0+4+0 \\ -1+0+0 & 0+0+3 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 4 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1-4 \\ -1-3 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} -5 \\ -4 \end{bmatrix} \quad \ldots(1)$$

Now consider RHS,

$$A(BC) = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}\left(\begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1+0 \\ -1-2 \\ 0-3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ -3 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1-6+0 \\ -1+0-3 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} -5 \\ -4 \end{bmatrix} \quad \ldots(2)$$

From equation (1) and (2), it is clear that (AB) C = A (BC)

(ii) Given,

$$A = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix},\ B = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix},\ and\ C = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Consider the LHS,

$$(AB)C = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0+6 & -4+2-3 & 4+4+3 \\ 1+0+4 & -1+1-2 & 1+2+2 \\ 3+0+2 & -3+0-1 & 3+0+1 \end{bmatrix}\begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & -5 & 11 \\ 5 & -2 & 5 \\ 5 & -4 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 10-15+0 & 20+0+0 & -10-5+11 \\ 5-6+0 & 10+0+0 & -5-2+5 \\ 5-12+0 & 10+0+0 & -5-4+4 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} -5 & 20 & -4 \\ -1 & 10 & -2 \\ -7 & 10 & -5 \end{bmatrix} \ldots\ldots(1)$$

Now consider RHS,

$$A(BC) = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix}\left(\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1-3+0 & 2+0+0 & -1-1+1 \\ 0+3+0 & 0+0+0 & 0+1+2 \\ 2-3+0 & 4+0+0 & -2-1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & 2 & 1 \\ 3 & 0 & 3 \\ -1 & 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -8+6-3 & 8+0+12 & -4+6-6 \\ -2+3-2 & 2+0+8 & -1+3-4 \\ -6+0-1 & 6+0+4 & -3+0-2 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} -5 & 20 & -4 \\ -1 & 10 & -2 \\ -7 & 10 & -5 \end{bmatrix} \text{.....(2)}$$

From equation (1) and (2), it is clear that (AB) C = A (BC)

17. For the following matrices verify the distributivity of matrix multiplication over matrix addition i.e. A (B + C) = AB + AC.

$(i)\ A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix},\ B = \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix},\ and\ C = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$

$(ii)\ A =$ [illegible] $,\ B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix},\ and\ C = \begin{bmatrix} 1 & \text{[illegible]} \\ 0 & 1 \end{bmatrix}$

Solution:

(i) Given

$$A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix},\ B = \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix},\ and\ C = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$$

Consider LHS,

$$A(B + C) = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \left(\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1+0 & 0+1 \\ 2+1 & 1-1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1-3 & 1+0 \\ 0+6 & 0+0 \end{bmatrix}$$

$$A(B + C) = \begin{bmatrix} -4 & 1 \\ 6 & 0 \end{bmatrix} \text{......(1)}$$

Now consider RHS,

$$AB + AC = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1-2 & 0-1 \\ 0+4 & 0+2 \end{bmatrix} + \begin{bmatrix} 0+-1 & 1+1 \\ 0+2 & 0-2 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -1 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 2 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -3-1 & -1+2 \\ 4+2 & 2-2 \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} -4 & 1 \\ 6 & 0 \end{bmatrix} \text{......(2)}$$

From equation (1) and (2), it is clear that A (B + C) = AB + AC

(ii) Given,

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, \text{ and } C = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

Consider the LHS

$$A(B + C) = \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}\left(\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 0+1 & 1-1 \\ 1+0 & 1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & 0+2 \\ 1+1 & 0+2 \\ -1+2 & 0+4 \end{bmatrix}$$

$$A(B+C) = \begin{bmatrix} 1 & -2 \\ 2 & 2 \\ 1 & 4 \end{bmatrix} \text{.....(1)}$$

Now consider RHS,

$$AB + AC = \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0-1 & 2-1 \\ 0+1 & 1+1 \\ 0+2 & -1+2 \end{bmatrix} + \begin{bmatrix} 2+0 & -2-1 \\ 1+0 & -1+1 \\ -1+0 & 1+2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 \\ 1 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -3 \\ 1 & 0 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1+2 & 1-3 \\ 1+1 & 2+0 \\ 2-1 & 1+3 \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} 1 & -2 \\ 2 & 2 \\ 1 & 4 \end{bmatrix} \text{.....(2)}$$

18. $If\ A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix},\ B = \begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix},\ and\ C = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix},$

verify that $A(B-C) = AB - AC$.

Solution:

Given,

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

Consider the LHS,

$$A(B-C) = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}\left(\begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 & -6 \\ -1 & 0 & 3 \\ -1 & 1 & 1 \end{bmatrix}$$

$$A(B-C) = \begin{bmatrix} 1 & -2 & -8 \\ -2 & 0 & -21 \\ 0 & 1 & 16 \end{bmatrix}$$

Now consider RHS

$$AB - AC = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 5 & -8 \\ 2 & 14 & -15 \\ -3 & -9 & 13 \end{bmatrix} - \begin{bmatrix} 1 & 7 & 0 \\ 4 & 14 & 6 \\ -3 & -10 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 & -8 \\ -2 & 0 & -21 \\ 0 & 1 & 16 \end{bmatrix}$$

From the above equations LHS = RHS
Therefore, A (B – C) = AB – AC.

19. Compute the elements a_{43} and a_{22} of the matrix:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 2 \\ 0 & 3 & 2 \\ 4 & 0 & 4 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -3 & 2 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 & 2 & -2 \\ 3 & -3 & 4 & -4 & 0 \end{bmatrix}$$

Solution:

Given

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 2 \\ 0 & 3 & 2 \\ 4 & 0 & 4 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -3 & 2 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 & 2 & -2 \\ 3 & -3 & 4 & -4 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} -3 & 2 \\ 12 & 4 \\ -1 & 12 \\ 24 & 8 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 & 2 & -2 \\ 3 & -3 & 4 & -4 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 6 & -9 & 11 & -14 & 6 \\ 12 & 0 & 4 & 8 & -24 \\ 36 & -37 & 49 & -50 & 2 \\ 24 & 0 & 8 & 16 & -48 \end{bmatrix}$$

From the above matrix, a_{43} = 8 and a_{22} = 0

$$20.\ If\ A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}\ and\ I\ is\ the\ identity\ matrix\ of\ order\ 3,\ that\ A^3 = pI + qA + rA^2$$

Solution:

Given

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}$$

Consider,

$A^2 = A\,A$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}$$

$$= \begin{bmatrix} 0+0+0 & 0+0+0 & 0+1+0 \\ 0+0+p & 0+0+q & 0+0+r \\ 0+0+pr & p+0+qr & 0+q+r^2 \end{bmatrix}$$

Again consider,

$A^3 = A^2 . A$

$$= \begin{bmatrix} 0+0+0 & 0+0+0 & 0+1+0 \\ 0+0+p & 0+0+q & 0+0+r \\ 0+0+pr & p+0+qr & 0+q+r^2 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}$$

$$= \begin{bmatrix} 0+0+p & 0+0+q & 0+0+r \\ 0+0+pr & p+0+qr & 0+q+r^2 \\ 0+0+pq+pr^2 & pr+0+q^2+qr^2 & 0+p+qr+qr+r^3 \end{bmatrix}$$

$$= \begin{bmatrix} p & q & r \\ pr & p+qr & q+r^2 \\ pq+pr^2 & pr+q^2+qr^2 & p+2qr+r^3 \end{bmatrix}$$

Now, consider the RHS

$pI + qA + rA^2$

$$= p\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + q\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix} + r\begin{bmatrix} 0 & 0 & 1 \\ p & q & r \\ pr & p+qr & q+r^2 \end{bmatrix}$$

$$= \begin{bmatrix} p & q & r \\ pr & p + qr & q + r^2 \\ pq + pr^2 & pr + q^2 + qr^2 & p + 2qr + r^3 \end{bmatrix}$$

Therefore, $A^3 = p\,I + q\,A + rA^2$

Hence the proof.

21. If ω is a complex cube root of unity, show that

$$\left(\begin{bmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{bmatrix} + \begin{bmatrix} \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \\ \omega & \omega^2 & 1 \end{bmatrix}\right)\begin{bmatrix} 1 \\ \omega \\ \omega^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Solution:

Given

$$\left(\begin{bmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{bmatrix} + \begin{bmatrix} \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \\ \omega & \omega^2 & 1 \end{bmatrix}\right)\begin{bmatrix} 1 \\ \omega \\ \omega^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

It is also given that ω is a complex cube root of unity.

Consider the LHS

$$= \begin{bmatrix} 1+\omega & \omega+\omega^2 & \omega^2+1 \\ \omega+\omega^2 & \omega^2+1 & 1+\omega \\ \omega^2+\omega & 1+\omega^2 & \omega+1 \end{bmatrix}\begin{bmatrix} 1 \\ \omega \\ \omega^2 \end{bmatrix}$$

We know that $1 + \omega + \omega^2 = 0$ and $\omega^3 = 1$

$$= \begin{bmatrix} -\omega^2 & -1 & -\omega \\ -1 & -\omega & -\omega^2 \\ -1 & -\omega & -\omega^2 \end{bmatrix}\begin{bmatrix} 1 \\ \omega \\ \omega^2 \end{bmatrix}$$

Now by simplifying we get,

$$= \begin{bmatrix} -\omega^2 & -\omega & -\omega^3 \\ -1 & -\omega^2 & -\omega^4 \\ -1 & -\omega^2 & -\omega^4 \end{bmatrix}$$

Again by substituting $1 + \omega + \omega^2 = 0$ and $\omega^3 = 1$ in above matrix we get,

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Therefore LHS = RHS

Hence the proof.

$$22. If\ A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix},\ show\ that\ A^2 = A$$

Solution:

Given,

$$A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$$

Consider A^2

$$A^2 = A.A$$

$$= \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 4+3-5 & \text{[illegible]}\,15 & -10\,\text{[illegible]}\,20 \\ \text{[illegible]} & \text{[illegible]}-15 & 5+20\,\text{[illegible]} \\ 2+3-\text{[illegible]} & \text{[illegible]}-12+12 & -5-15+16 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix} = A$$

Therefore $A^2 = A$

$$23. If\ A = \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix},\ show\ that\ A^2 = I_3$$

Solution:

Given

$$A = \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$$

Consider A^2,

$A^2 = A.A$

$$= \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix} \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 16-3-12 & -4+0+4 & 16+4+12 \\ 12+0-12 & -3+0+4 & -12+0+12 \\ 12-3-9 & -3+0+3 & -12+4+9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3$$

Hence $A^2 = I_3$

24. (i) If $\begin{bmatrix} 1 & 1 & x \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0$, find x.

(ii) If $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$, find x.

Solution:

(i) Given

$$\begin{bmatrix} 1 & 1 & x \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2x+0 & x+0+2 & 2+1+0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0$$

$$= \begin{bmatrix} 2x+4 & x+2 & 2x+4 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0$$

= [2x + 1 + 2 + x + 3] = 0

= [3x + 6] = 0

= 3x = -6

x = -6/3
x = -2

(ii) Given,

$$\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2-6 & -6+12 \\ 5-14 & -15+28 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -4 & 6 \\ -9 & 13 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$$

On comparing the above matrix we get,
x = 13

25. *If* $\begin{bmatrix} x & 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 2 \\ 0 & 2 & -4 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0$, *find x.*

Solution:
Given,

$$\begin{bmatrix} x & 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 2 \\ 0 & 2 & -4 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x+4+0 & x+0+2 & 2x+8-4 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 2x+4 & x+2 & 2x+4 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0$$

⇒ [(2x + 4) x + 4 (x + 2) - 1(2x + 4)] = 0
⇒ $2x^2 + 4x + 4x + 8 - 2x - 4 = 0$
⇒ $2x^2 + 6x + 4 = 0$
⇒ $2x^2 + 2x + 4x + 4 = 0$
⇒ 2x (x + 1) + 4 (x + 1) = 0
⇒ (x + 1) (2x + 4) = 0
⇒ x = -1 or x = -2

Hence, x = -1 or x = -2

26. *If* $\begin{bmatrix}1 & -1 & x\end{bmatrix}\begin{bmatrix}0 & 1 & -1\\2 & 1 & 3\\1 & 1 & 1\end{bmatrix}\begin{bmatrix}0\\1\\1\end{bmatrix} = 0$, *find x.*

Solution:

Given

$$\begin{bmatrix}1 & -1 & x\end{bmatrix}\begin{bmatrix}0 & 1 & -1\\2 & 1 & 3\\1 & 1 & 1\end{bmatrix}\begin{bmatrix}0\\1\\1\end{bmatrix} = 0$$

By multiplying we get,

$$\Rightarrow \begin{bmatrix}0-2+x & x & (-1)-3+x\end{bmatrix}\begin{bmatrix}0\\1\\1\end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix}x-2 & x & x-4\end{bmatrix}\begin{bmatrix}0\\1\\1\end{bmatrix} = 0$$

$$[(x-2)\times 0 + x\times 1 + (x-4)\times 1] = 0$$

$$\Rightarrow x + x - 4 = 0$$

$$\Rightarrow 2x = 4 \Rightarrow x = 2$$

27. *If* $A = \begin{bmatrix}3 & -2\\4 & -2\end{bmatrix}$ *and* $I = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$, *then prove that* $A^2 - A + 2I = 0$.

Solution:

Given

$$A = \begin{bmatrix}3 & -2\\4 & -2\end{bmatrix} \text{ and } I = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

Now we have to prove $A^2 - A + 2I = 0$

Now, we will find the matrix for A^2, we get

$$A^2 = A \times A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 3 \times 3 + (-2 \times 4) & 3 \times (-2) + (-2 \times -2) \\ 4 \times 3 + (-2 \times 4) & 4 \times (-2) + (-2 \times -2) \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} \ldots\ldots\ldots\ldots (i)$$

Now, we will find the matrix for 2I, we get

$$2I = 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow 2I = \begin{bmatrix} 2 \times 1 & 2 \times 0 \\ 2 \times 0 & 2 \times 1 \end{bmatrix}$$

$$\Rightarrow 2I = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \ldots\ldots\ldots\ldots (ii)$$

$A^2 - A + 2I$

Substitute corresponding values from eqn (i) and eqn (ii), we get

$$\Rightarrow = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow = \begin{bmatrix} 1-3+2 & -2-(-2)+0 \\ 4-4+0 & -4-(-2)+2 \end{bmatrix}$$

$$\Rightarrow = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

Therefore,

$A^2 - A + 2I = 0$

Hence proved

28. *If* $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ *and* $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, *then prove that* $A^2 = 5A + \lambda I$.

Solution:

Given

$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Now, we have to find A^2,

$A^2 = A \times A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 3\times3 + (1\times-1) & 3\times1 + 1\times2 \\ (-1\times3) + 2\times(-1) & (-1\times1) + 2\times2 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$………(i)

Now, we will find the matrix for 5A, we get

$5A = 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$\Rightarrow 5A = \begin{bmatrix} 5\times3 & 5\times1 \\ 5\times(-1) & 5\times2 \end{bmatrix}$

$\Rightarrow 5A = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$………(ii)

So,

$A^2 = 5A + \lambda I$

Substitute corresponding values from eqn (i) and eqn (ii), we get

$\Rightarrow \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \lambda\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$

$$\begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} 15 + \lambda & 5 + 0 \\ -5 + 0 & 10 + \lambda \end{bmatrix}$$

And to satisfy the above condition of equality, the corresponding entries of the matrices should be equal,

Hence,

$8 = 15 + \lambda \Rightarrow \lambda = -7$

$3 = 10 + \lambda \Rightarrow \lambda = -7$

20. $If\ A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ *show that* $A^2 - 5A + 7I_2 = 0.$

Solution:

Given

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

I_2 is an identity matrix of size 2, so

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

To show that

$A^2 - 5A + 7I_2 = 0$

Now, we will find the matrix for A^2, we get

$$A^2 = A \times A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 3 \times 3 + (1 \times -1) & 3 \times 1 + 1 \times 2 \\ (-1 \times 3) + 2 \times (-1) & (-1 \times 1) + 2 \times 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 9 - 1 & 3 + 2 \\ -3 - 2 & -1 + 4 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} \text{-------(1)}$$

Now, we will find the matrix for 5A, we get

$5A = 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$\Rightarrow 5A = \begin{bmatrix} 5 \times 3 & 5 \times 1 \\ 5 \times (-1) & 5 \times 2 \end{bmatrix}$

$\Rightarrow 5A = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$ ---------(ii)

Now,

$7I_2 = 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$ ……..(iii)

So,

$A^2 - 5A + 7I_2$

Substitute corresponding values from eqn (i), (ii) and (iii), we get

$\Rightarrow = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$\Rightarrow = \begin{bmatrix} 8 - 15 + 7 & 5 - 5 + 0 \\ -5 - (-5) + 0 & 3 - 10 + 7 \end{bmatrix}$

$\Rightarrow = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

Hence the proof.

30. *If* $A = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$ *show that* $A^2 - 2A + 3I_2 = 0.$

Solution:

Given

$A = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$

I_2 is an identity matrix of size 2, so

$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Now we have to show,

$A^2 - 2A + 3I_2 = 0$

Now, we will find the matrix for A^2, we get

$$A^2 = A \times A = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 2\times 2 + (3\times -1) & 2\times 3 + 3\times 0 \\ (-1\times 2) + 0\times(-1) & (-1\times 3) + 0\times 0 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 4-3 & 6+0 \\ -2+0 & -3+0 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix} \ldots\ldots\ldots (i)$$

Now, we will find the matrix for 2A, we get

$$2A = 2\begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$$

$$\Rightarrow 2A = \begin{bmatrix} 2\times 2 & 2\times 3 \\ 2\times(-1) & 2\times 0 \end{bmatrix}$$

$$\Rightarrow 2A = \begin{bmatrix} 4 & 6 \\ -2 & 0 \end{bmatrix} \ldots\ldots (ii)$$

Now,

$$3I_2 = 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \ldots\ldots (iii)$$

So,

$A^2 - 2A + 3I_2$

Substitute corresponding values from eqn (i), (ii) and (iii), we get

$$\Rightarrow = \begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix} - \begin{bmatrix} 4 & 6 \\ -2 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$\Rightarrow = \begin{bmatrix} 1-4+3 & 6-6+0 \\ -2-(-2)+0 & -3-0+3 \end{bmatrix}$$

$$\Rightarrow = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

Hence the proof.

31. *Show that the matrix* $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ *satisfies the equation* $A^3 - 4A^2 + A = 0$.

Solution:

Given

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

To show that $A^3 - 4A^2 + A = 0$

Now, we will find the matrix for A^2, we get

$$A^2 = (A \times A) = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 2\times 2 + (3\times 1) & 2\times 3 + 3\times 2 \\ 1\times 2 + 2\times 1 & 1\times 3 + 2\times 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 4 + 3 & 6 + 6 \\ 2 + 2 & 3 + 4 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} \ldots\ldots (i)$$

Now, we will find the matrix for A^3, we get

$$A^3 = A^2 \times A = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 7\times 2 + 12\times 1 & 7\times 3 + 12\times 2 \\ 4\times 2 + 7\times 1 & 4\times 3 + 7\times 2 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 14 + 12 & 21 + 24 \\ 8 + 7 & 12 + 14 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} \ldots\ldots (ii)$$

So,

$A^3 - 4A^2 + A$

Substitute corresponding values from eqn (i) and (ii), we get

$$\Rightarrow = \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} - 4\begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$\Rightarrow = \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} - \begin{bmatrix} 4 \times 7 & 4 \times 12 \\ 4 \times 4 & 4 \times 7 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

$\Rightarrow = \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} - \begin{bmatrix} 28 & 48 \\ 16 & 28 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

$\Rightarrow = \begin{bmatrix} 26 - 28 + 2 & 45 - 48 + 3 \\ 15 - 16 + 1 & 26 - 28 + 2 \end{bmatrix}$

$\Rightarrow = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

Therefore,

$A^3 - 4A^2 + A = 0$

Hence matrix A satisfies the given equation.

32. *Show that the matrix* $A = \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$ *satisfies the equation* $A^2 - 12A - I = 0$.

Solution:

Given

$A = \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$

I is an identity matrix so $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

To show that $A^2 - 12A - I = 0$

Now, we will find the matrix for A^2, we get

$A^2 = A \times A = \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix} \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 5 \times 5 + 3 \times 12 & 5 \times 3 + 3 \times 7 \\ 12 \times 5 + 7 \times 12 & 12 \times 3 + 7 \times 7 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 25 + 36 & 15 + 21 \\ 60 + 84 & 36 + 49 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 61 & 36 \\ 144 & 85 \end{bmatrix}$ ……… (I)

Now, we will find the matrix for 12A, we get

$$12A = 12\begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$$

$$\Rightarrow 12A = \begin{bmatrix} 12 \times 5 & 12 \times 3 \\ 12 \times 12 & 12 \times 7 \end{bmatrix}$$

$$\Rightarrow 12A = \begin{bmatrix} 60 & 36 \\ 144 & 84 \end{bmatrix} \ldots\ldots\ldots\ldots(ii)$$

So,

$A^2 - 12A - I$

Substitute corresponding values from eqn (i) and (ii), we get

$$\Rightarrow = \begin{bmatrix} 61 & 36 \\ 144 & 85 \end{bmatrix} - \begin{bmatrix} 60 & 36 \\ 144 & 84 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow = \begin{bmatrix} 61 - 60 - 1 & 36 - 36 - 0 \\ 144 - 144 - 0 & 85 - 84 - 1 \end{bmatrix}$$

$$\Rightarrow = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$$

Therefore,

$A^2 - 12A - I = 0$

Hence matrix A is the root of the given equation.

33. If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$ find $A^2 - 5A - 14I$.

Solution:

Given

$$A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

I is identity matrix so

$$14I = 14\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

To find $A^2 - 5A - 14I$

Now, we will find the matrix for A^2, we get

$$A^2 = A \times A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 3 \times 3 + (-5 \times -4) & 3 \times (-5) + (-5 \times 2) \\ (-4 \times 3) + (2 \times -4) & (-4 \times -5) + 2 \times 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 9 + 20 & -15 - 10 \\ -12 - 8 & 20 + 4 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} \text{.........(i)}$$

Now, we will find the matrix for 5A, we get

$$5A = 5\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

$$\Rightarrow 5A = \begin{bmatrix} 5 \times 3 & 5 \times (-5) \\ 5 \times (-4) & 5 \times 2 \end{bmatrix}$$

$$\Rightarrow 5A = \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} \text{.........(ii)}$$

So,

$A^2 - 5A - 14I$

Substitute corresponding values from eqn (i) and (ii), we get

$$\Rightarrow = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

$$\Rightarrow = \begin{bmatrix} 29 - 15 - 14 & -25 + 25 - 0 \\ -20 + 20 - 0 & 24 - 10 - 14 \end{bmatrix}$$

$$\Rightarrow = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

34. *If* $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ *show that* $A^2 - 5A + 7I = 0$. *Use this to find* A^4.

Solution:

Given

$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

I is identity matrix so

$7I = 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

To show that $A^2 - 5A + 7I = 0$

Now, we will find the matrix for A^2, we get

$A^2 = A \times A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 3 \times 3 + (1 \times -1) & 3 \times 1 + 1 \times 2 \\ (-1 \times 3) + (2 \times -1) & (-1 \times 1) + 2 \times 2 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 9 - 1 & 3 + 2 \\ -3 - 2 & -1 + 4 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$ ---------(i)

Now, we will find the matrix for 5A, we get

$5A = 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$\Rightarrow 5A = \begin{bmatrix} 5 \times 3 & 5 \times 1 \\ 5 \times (-1) & 5 \times 2 \end{bmatrix}$

$\Rightarrow 5A = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$ ---------(ii)

So,

$A^2 - 5A + 7I$

Substitute corresponding values from eqn (i) and (ii), we get

$\Rightarrow = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} - \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$\Rightarrow = \begin{bmatrix} 8-15-7 & 5-5-0 \\ -5+5-0 & 3-10-7 \end{bmatrix}$

$\Rightarrow = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

Therefore

$A^2 - 5A + 7I = 0$

Hence proved

We will find A^4

$A^2 - 5A + 7I = 0$

Multiply both sides by A^2, we get

$A^2(A^2 - 5A + 7I) = A^2(0)$

$\Rightarrow A^4 - 5A^2.A + 7I.A^2$

$\Rightarrow A^4 = 5A^2.A - 7I.A^2$

$\Rightarrow A^4 = 5A^2.A - 7A^2$

As multiplying by the identity matrix, I don't change anything. Now will substitute the corresponding values we get

$\Rightarrow A^4 = 5\begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 7\begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

$\Rightarrow A^4 = 5\begin{bmatrix} 24-5 & 8+10 \\ -15-3 & -5+6 \end{bmatrix} - 7\begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

$\Rightarrow A^4 = 5\begin{bmatrix} 19 & 18 \\ -18 & 1 \end{bmatrix} - 7\begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

$\Rightarrow A^4 = \begin{bmatrix} 5\times 19 & 5\times 18 \\ 5\times(-18) & 5\times 1 \end{bmatrix} - \begin{bmatrix} 7\times 8 & 7\times 5 \\ 7\times(-5) & 7\times 3 \end{bmatrix}$

$\Rightarrow A^4 = \begin{bmatrix} 95 & 90 \\ -90 & 5 \end{bmatrix} - \begin{bmatrix} 56 & 35 \\ -35 & 21 \end{bmatrix}$

$\Rightarrow A^4 = \begin{bmatrix} 95-56 & 90-35 \\ -90+35 & 5-21 \end{bmatrix}$

$$\Rightarrow A^4 = \begin{bmatrix} 39 & 55 \\ -55 & -16 \end{bmatrix}$$

35. If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ find k such that $A^2 = kA - 2I_2$.

Solution:

Given

$$A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

I_2 is an identity matrix of size 2, so

$$2I_2 = 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

Also given,

$$A^2 = kA - 2I_2$$

Now, we will find the matrix for A^2, we get

$$A^2 = A \times A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 3 \times 3 + (-2 \times 4) & 3 \times (-2) + (-2 \times -2) \\ (4 \times 3) + (-2 \times 4) & (4 \times -2) + (-2 \times -2) \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 9 - 8 & -6 + 4 \\ 12 - 8 & -8 + 4 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} \ldots\ldots\ldots(I)$$

Now, we will find the matrix for kA, we get

$$kA = k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$\Rightarrow kA = \begin{bmatrix} k \times 3 & k \times (-2) \\ k \times 4 & k \times (-2) \end{bmatrix}$$

So,

$A^2 = kA - 2I_2$

Substitute corresponding values from eqn (i) and (ii), we get

$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k-0 \\ 4k-0 & -2k-2 \end{bmatrix}$

And to satisfy the above condition of equality, the corresponding entries of the matrices should be equal

Hence, $3k - 2 = 1 \Rightarrow k = 1$

Therefore, the value of k is 1

36. *If* $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$ *find k such that* $A^2 - 8A + kI = O$

Solution:

Given

$A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$

I is identity matrix, so

$kI = k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$

Also given, $A^2 - 8A + kI = 0$

Now, we have to find A^2, we get

$A^2 = A \times A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 1 \times 1 + 0 & 0 + 0 \\ (-1 \times 1) + 7 \times (-1) & 0 + 7 \times 7 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix}$ ---------(i)

Now, we will find the matrix for 8A, we get

$8A = 8\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$

$$\Rightarrow 8A = \begin{bmatrix} 8 \times 1 & 8 \times 0 \\ 8 \times (-1) & 8 \times 7 \end{bmatrix}$$

$$\Rightarrow 8A = \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} \text{------}(ii)$$

So,

$$A^2 - 8A + kI = 0$$

Substitute corresponding values from eqn (i) and (ii), we get

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 1 - 8 + k & 0 - 0 + 0 \\ -8 + 8 + 0 & 49 - 56 + k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

And to satisfy the above condition of equality, the corresponding entries of the matrices should be equal

Hence,

$$1 - 8 + k = 0 \Rightarrow k = 7$$

Therefore, the value of k is 7

37. *If* $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ *and* $f(x) = x^2 - 2x - 3$, *show that* $f(A) = 0$

Solution:

Given

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

To show that f (A) = 0

Substitute x = A in f(x), we get

$$f(A) = A^2 - 2A - 3I \text{------}(i)$$

I is identity matrix, so

$$3I = 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

Now, we will find the matrix for A^2, we get

$$A^2 = A \times A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 \times 1 + 2 \times 2 & 1 \times 2 + 2 \times 1 \\ 2 \times 1 + 1 \times 2 & 2 \times 2 + 1 \times 1 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 + 4 & 2 + 2 \\ 2 + 2 & 4 + 1 \end{bmatrix}$$

$\Rightarrow A^2 = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$ ……….(ii)

Now, we will find the matrix for 2A, we get

$2A = 2\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

$\Rightarrow 2A = \begin{bmatrix} 2 \times 1 & 2 \times 2 \\ 2 \times 2 & 2 \times 1 \end{bmatrix}$

$\Rightarrow 2A = \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix}$ ……….(iii)

Substitute corresponding values from eqn (ii) and (iii) in eqn (i), we get

$f(A) = A^2 - 2A - 3I$

$\Rightarrow f(A) = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$\Rightarrow f(A) = \begin{bmatrix} 5-2-3 & 4-4-0 \\ 4-4-0 & 5-2-3 \end{bmatrix}$

$\Rightarrow f(A) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

So,

$\Rightarrow f(A) = 0$

Hence Proved

38. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then find λ, μ so that $A^2 = \lambda A + \mu I$

Solution:

Given

$A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

So

$\mu I = \mu\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \mu & 0 \\ 0 & \mu \end{bmatrix}$

Now, we will find the matrix for A^2, we get

$A^2 = A \times A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 2 \times 2 + 3 \times 1 & 2 \times 3 + 3 \times 2 \\ 1 \times 2 + 2 \times 1 & 1 \times 3 + 2 \times 2 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$ ……….(i)

Now, we will find the matrix for λA, we get

$\lambda A = \lambda\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$

$\Rightarrow \lambda A = \begin{bmatrix}\lambda\times 2 & \lambda\times 3\\ \lambda\times 1 & \lambda\times 2\end{bmatrix}$

$\Rightarrow \lambda A = \begin{bmatrix}2\lambda & 3\lambda\\ \lambda & 2\lambda\end{bmatrix}$ ………(ii)

But given, $A^2 = \lambda A + \mu I$

Substitute corresponding values from equation (i) and (ii), we get

$\Rightarrow \begin{bmatrix}7 & 12\\ 4 & 7\end{bmatrix} = \begin{bmatrix}2\lambda & 3\lambda\\ \lambda & 2\lambda\end{bmatrix} + \begin{bmatrix}\mu & 0\\ 0 & \mu\end{bmatrix}$

$\Rightarrow \begin{bmatrix}7 & 12\\ 4 & 7\end{bmatrix} = \begin{bmatrix}2\lambda+\mu & 3\lambda+0\\ \lambda+0 & 2\lambda+\mu\end{bmatrix}$

And to satisfy the above condition of equality, the corresponding entries of the matrices should be equal

Hence, $\lambda + 0 = 4 \Rightarrow \lambda = 4$

And also, $2\lambda + \mu = 7$

Substituting the obtained value of λ in the above equation, we get

$2(4) + \mu = 7 \Rightarrow 8 + \mu = 7 \Rightarrow \mu = -1$

Therefore, the value of λ and μ are 4 and − 1 respectively

39. Find the value of x for which the matrix product

$\begin{bmatrix}2 & 0 & 7\\ 0 & 1 & 0\\ 1 & -2 & 1\end{bmatrix}\begin{bmatrix}-x & 14x & 7x\\ 0 & 1 & 0\\ x & -4x & -2x\end{bmatrix}$ *equal to an identity matrix.*

Solution:

We know,

$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$

is identity matrix of size 3.

So according to the given criteria

$\begin{bmatrix}2 & 0 & 7\\ 0 & 1 & 0\\ 1 & -2 & 1\end{bmatrix}\begin{bmatrix}-x & 14x & 7x\\ 0 & 1 & 0\\ x & -4x & -2x\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$

Now we will multiply the two matrices on LHS using the formula $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{in} b_{nj}$, we get

$$\begin{bmatrix} 2\times(-x)+0+7\times x & 2\times 14x+0+7\times(-4x) & 2\times 7x+0+7\times(-2x) \\ 0+0+0 & 0+1\times 1+0 & 0+0+0 \\ 1\times(-x)+0+1\times x & 1\times 14x+(-2\times 1)+(1\times -4x) & 1\times 7x+0+1\times(-2x) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 5x & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 10x-2 & 5x \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

And to satisfy the above condition of equality, the corresponding entries of the matrices should be equal

So we get

$5x = 1 \Rightarrow x = \frac{1}{5}$

So the value of x is $\frac{1}{5}$