

EXERCISE 19.10

1. $\int x^2 \sqrt{x+2} dx$

Solution:

Let $I = \int x^2 \sqrt{x+2} dx$

Substituting, $x + 2 = t \Rightarrow dx = dt$,

$I = \int (t-2)^2 \sqrt{t} dt$

$\Rightarrow I = \int (t^2 - 4t + 4) \sqrt{t} dt$

$\Rightarrow I = \int \left(t^{\frac{5}{2}} - 4t^{\frac{3}{2}} + 4t^{\frac{1}{2}} \right) dt$

$\Rightarrow I = \frac{2}{7} t^{\frac{7}{2}} - \frac{8}{5} t^{\frac{5}{2}} + \frac{8}{3} t^{\frac{3}{2}} + c$

$\Rightarrow I = \frac{2}{7} (x+2)^{\frac{7}{2}} - \frac{8}{5} (x+2)^{\frac{5}{2}} + \frac{8}{3} (x+2)^{\frac{3}{2}} + c$

Therefore, $\int x^2 \sqrt{x+2} dx = \frac{2}{7} (x+2)^{\frac{7}{2}} - \frac{8}{5} (x+2)^{\frac{5}{2}} + \frac{8}{3} (x+2)^{\frac{3}{2}} + c$

2. $\int \frac{x^2}{\sqrt{x-1}} dx$

Solution:

Let $I = \int \frac{x^2}{\sqrt{x-1}} dx$

Substituting $x - 1 = t \Rightarrow dx = dt$,

Now substituting the values we get

$$\Rightarrow I = \int \frac{(t+1)^2}{\sqrt{t}} dt$$

Expanding using $(a+b)^2$ formula

$$\Rightarrow I = \int \frac{t^2 + 2t + 1}{\sqrt{t}} dt$$

On simplification

$$\Rightarrow I = \int \left(t^{\frac{3}{2}} + 2t^{\frac{1}{2}} + t^{-\frac{1}{2}} \right) dt$$

On integrating we get

$$\Rightarrow I = \frac{2}{5} t^{\frac{5}{2}} + 2t^{\frac{1}{2}} + \frac{4}{3} t^{\frac{3}{2}} + c$$

Again taking LCM

$$\Rightarrow I = \frac{\left(6t^{\frac{5}{2}} + 30t^{\frac{1}{2}} + 20t^{\frac{3}{2}} \right)}{15} + c$$

$$\Rightarrow I = \frac{2}{15} t^{\frac{1}{2}} (3t^2 + 15 + 10t) + c$$

Substituting the value of t we get

$$\Rightarrow I = \frac{2}{15} (x-1)^{\frac{1}{2}} (3(x-1)^2 + 15 + 10(x-1)) + c$$

$$\Rightarrow I = \frac{2}{15} (x-1)^{\frac{1}{2}} (3(x^2 - 2x + 1) + 15 + 10x - 10) + c$$

By simplifying we get

$$\Rightarrow I = \frac{2}{15} (x-1)^{\frac{1}{2}} (3x^2 + 4x + 8) + c$$

Therefore, $\int \frac{x^2}{\sqrt{x-1}} dx = \frac{2}{15} (x-1)^{\frac{1}{2}} (3x^2 + 4x + 8) + c$

$$3. \int \frac{x^2}{\sqrt{3x+4}} dx$$

$$\text{Let } I = \int \frac{x^2}{\sqrt{3x+4}} dx$$

Substituting $3x+4=t \Rightarrow 3dx=dt$,

Substituting the values of x

$$\Rightarrow I = \int \frac{\left(\frac{t-4}{3}\right)^2}{3\sqrt{t}} dt$$

Expanding the above given function using $(a-b)^2$ formula

$$\Rightarrow I = \frac{1}{27} \int \frac{t^2 + 16 - 8t}{\sqrt{t}} dt$$

On simplifying, we get

$$\Rightarrow I = \frac{1}{27} \int \left(t^{\frac{3}{2}} - 8t^{\frac{1}{2}} + 16t^{-\frac{1}{2}} \right) dt$$

On integrating, we get

$$\Rightarrow I = \frac{1}{27} \left[\frac{2}{5} t^{\frac{5}{2}} - \frac{16}{3} t^{\frac{3}{2}} + 32t^{\frac{1}{2}} \right] + c$$

$$\Rightarrow I = \frac{1}{27} \left[\frac{2}{5} (3x+4)^{\frac{5}{2}} - \frac{16}{3} (3x+4)^{\frac{3}{2}} + 32(3x+4)^{\frac{1}{2}} \right] + c$$

$$\Rightarrow I = \frac{2}{135} (3x+4)^{\frac{5}{2}} - \frac{16}{81} (3x+4)^{\frac{3}{2}} + \frac{32}{27} (3x+4)^{\frac{1}{2}} + c$$

$$\text{Therefore, } \int \frac{x^2}{\sqrt{3x+4}} dx$$

$$= \frac{2}{135} (3x+4)^{\frac{5}{2}} - \frac{16}{81} (3x+4)^{\frac{3}{2}} + \frac{32}{27} (3x+4)^{\frac{1}{2}} + c \quad \text{Solution:}$$

$$4. \int \frac{2x-1}{(x-1)^2} dx$$

Solution:

$$\text{Let } I = \int \frac{2x - 1}{(x - 1)^2} dx$$

Substituting $x - 1 = t \Rightarrow dx = dt$

Substituting the values of x

$$\Rightarrow I = \int \frac{2(t + 1) - 1}{t^2} dt$$

Multiplying and simplifying we get

$$\Rightarrow I = \int \frac{2t + 1}{t^2} dt$$

$$\Rightarrow I = \int \left(\frac{2}{t} + \frac{1}{t^2} \right) dt$$

On integration

$$\Rightarrow I = 2 \log|t| - \frac{1}{t} + c$$

$$\Rightarrow I = 2 \log|x - 1| - \frac{1}{x - 1} + c$$

$$\text{Therefore, } \int \frac{2x - 1}{(x - 1)^2} dx = 2 \log|x - 1| - \frac{1}{x - 1} + c$$

$$5. \int (2x^2 + 3)\sqrt{x + 2} dx$$

Solution:

$$\text{Let } I = \int (2x^2 + 3)\sqrt{x + 2} dx$$

Substituting $x + 2 = t \Rightarrow dx = dt$

Substituting the values of x in given equation, we get

$$\Rightarrow I = \int [2(t-2)^2 + 3]\sqrt{t} dt$$

Expanding above equation using $(a-b)^2$ formula

$$\Rightarrow I = \int [2t^2 - 8t + 8 + 3]\sqrt{t} dt$$

On simplification

$$\Rightarrow I = \int \left[2t^{\frac{5}{2}} - 8t^{\frac{3}{2}} + 11t^{\frac{1}{2}} \right] dt$$

On integrating we get

$$\Rightarrow I = \frac{4}{7} t^{\frac{7}{2}} - \frac{16}{5} t^{\frac{5}{2}} + \frac{22}{3} t^{\frac{3}{2}} + c$$

$$\Rightarrow I = \frac{4}{7} (x+2)^{\frac{7}{2}} - \frac{16}{5} (x+2)^{\frac{5}{2}} + \frac{22}{3} (x+2)^{\frac{3}{2}} + c$$

$$\therefore \int (2x^2 + 3)\sqrt{x+2} dx = \frac{4}{7} (x+2)^{\frac{7}{2}} - \frac{16}{5} (x+2)^{\frac{5}{2}} + \frac{22}{3} (x+2)^{\frac{3}{2}} + c$$

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