Exercise 7(A)

Given, a: b = 5: 3
So, a/b = 5/3
Now,

$$\frac{5a-3b}{5a+3b}=\frac{5\left(\frac{a}{b}\right)-3}{5\left(\frac{a}{b}\right)+3}$$ (Dividing each term by b)

$$=\frac{5\left(\frac{5}{3}\right)-3}{5\left(\frac{5}{3}\right)+3}=\frac{\frac{25}{3}-3}{\frac{25}{3}+3}=\frac{25-9}{25+9}$$

$$=\frac{16}{34}=\frac{8}{17}$$

1. Solution:

Given, x: y = 4: 7
So, x/y = 4/7

$$\frac{3x+2y}{5x+y}=\frac{3\left(\frac{x}{y}\right)+2}{5\left(\frac{x}{y}\right)+1}$$ (Dividing each term by y)

$$=\frac{3\left(\frac{4}{7}\right)+2}{5\left(\frac{4}{7}\right)+1}=\frac{\frac{12}{7}+2}{\frac{20}{7}+1}=\frac{12+14}{20+7}$$

$$=\frac{26}{27}$$

Solution:

Given, a: b = 3: 8
So, a/b = 3/8

$$\frac{4a+3b}{6a-b} = \frac{4\left(\frac{a}{b}\right)+3}{6\left(\frac{a}{b}\right)-1}$$ (Dividing each term by b)

$$= \frac{4\left(\frac{3}{8}\right)+3}{6\left(\frac{3}{8}\right)-1} = \frac{\frac{3}{2}+3}{\frac{9}{4}-1} = \frac{\frac{9}{2}}{\frac{5}{4}}$$

$$= \frac{18}{5}$$

Solution:

Given,
(a - b)/ (a + b) = 1/ 11
11a – 11b = a + b
10a = 12b
a/b = 12/10 = 6/5
Now, lets take a = 6k and b = 5k
So,

$$\frac{5a+4b+15}{5a-4b+3} = \frac{5(6k)+4(5k)+15}{5(6k)-4(5k)+3}$$

$$= \frac{30k+20k+15}{30k-20k+3}$$

$$= \frac{50k+15}{10k+3}$$

$$= \frac{5(10k+3)}{10k+3}$$

$$= 5$$

Therefore, (5a + 4b + 15): (5a – 4b + 3 = 5: 1

Solution:

Let consider the required number to be x/y
Now, given that
Ratio of 8/21 to 4/9 = (8/21)/ (4/9) = (8/21) x (9/4) = 6/7
Hence, we have
(x/y)/ (7/33) = 6/7
x/y = (6/7)/ (7/33)
= (6/7) x (7/33)
= 2/11

Therefore, the required number is 2/11.

2. $If \frac{m+n}{m+3n} = \frac{2}{3}, find: \frac{2n^2}{3m^2+mn}.$

Solution:

Given,

$$\frac{m+n}{m+3n} = \frac{2}{3}$$

3(m + n) = 2(m + 3n)

3m + 3n = 2m + 6n

m = 3n

m/n = 3/1

Now,

$$\frac{2n^2}{3m^2+mn} = \frac{2}{3\left(\frac{m}{n}\right)^2+\left(\frac{m}{n}\right)} \text{ (Dividing each term by } n^2\text{)}$$

$$= \frac{2}{3\left(\frac{3}{1}\right)^2+\left(\frac{3}{1}\right)}$$

$$= \frac{2}{27+3} = \frac{1}{15}$$

3. Find x/y; when $x^2 + 6y^2 = 5xy$

Solution:

Given,

$x^2 + 6y^2 = 5xy$

Dividing by y^2 both side, we have

$$\frac{x^2}{y^2} + \frac{6y^2}{y^2} = \frac{5xy}{y^2}$$

$$\left(\frac{x}{y}\right)^2 + 6 = 5\left(\frac{x}{y}\right)$$

$$\left(\frac{x}{y}\right)^2 - 5\left(\frac{x}{y}\right) + 6 = 0$$

Let x/y = a

So,

$a^2 – 5a + 6 = 0$

(a - 2) (a - 3) = 0

a = 2 or a = 3

Therefore, x/y = 2 or 3

4. If the ratio between 8 and 11 is the same as the ratio of 2x - y to x + 2y, find the value of 7x/ 9y.
Solution:

Given,
(2x - y)/ (x + 2y) = 8/11
On cross multiplying, we get
11(2x - y) = 8(x + 2y)
22x – 11y = 8x + 16y
14x = 27y
x/y = 27/14
So,
7x / 9y = (7 x 27)/ (9 x 14) = 3/2

5. Divide Rs 1290 into A, B and C such that A is 2/5 of B and B: C = 4: 3.
Solution:

Given,
B: C = 4: 3 so, B/C = 4/3 ⇒ C = (3/4) B
And, A = (2/5) B
We know that,
A + B + C = Rs 1290
(2/5) B + B + (3/4) B = 1290
Taking L.C.M,
(8B + 20B + 15B)/ 20 = 1290
43B = 1290 x 20
B = 1290 x 20/ 43 = 600
So,
A = (2/5) x 600 = 240
And,
C = (3/4) x 600 = 450

Therefore,
A gets Rs 600, B gets Rs 240 and C gets Rs 450

6. A school has 630 students. The ratio of the number of boys to the number of girls is 3: 2. This ratio changes to 7: 5 after the admission of 90 new students. Find the number of newly admitted boys.
Solution:

Let's consider the number of boys be 3x.
Then, the number of girls = 2x
⇒ 3x + 2x = 630
5x = 630
x = 126
So, the number of boys = 3x = 3 x 126 = 378
And, number of girls = 2x = 2 x 126 = 252

After admission of 90 new students,
Total number of students = 630 + 90 = 720
Here, let take the number of boys to be 7x
And, the number of girls = 5x
⇒ 7x + 5x = 720
12x = 720
x = 720/12
x = 60
So, the number of boys = 7x = 7 x 60 = 420
And, the number of girls = 5x = 5 x 60 = 300
Therefore, the number of newly admitted boys = 420 – 378 = 42

7. What quantity must be subtracted from each term of the ratio 9: 17 to make it equal to 1: 3?
Solution:

Let x be subtracted from each term of the ratio 9: 17.

$$\frac{9-x}{17-x} = \frac{1}{3}$$

27 – 3x = 17 – x
10 = 2x
x = 5
Therefore, the required number which should be subtracted is 5.

8. The monthly pocket money of Ravi and Sanjeev are in the ratio 5: 7. Their expenditures are in the ratio 3: 5. If each saves Rs. 80 every month, find their monthly pocket money.
Solution:

Given,
The pocket money of Ravi and Sanjeev are in the ratio 5: 7
So, we can assume the pocket money of Ravi as 5k and that of Sanjeev as 7k.
Also, give that
The expenditure of Ravi and Snajeev are in the ratio 3: 5
So, it can be taken as the expenditure of Ravi as 3m and that of Sanjeev as 5m.
And, each of them saves Rs 80
This can be expressed as below:
5k – 3m = 80 ….. (a)
7k – 5m = 80 .....(b)
Solving equations (a) and (b), we have
k = 40 and m = 40
Therefore, the monthly pocket money of Ravi is Rs 5k = Rs 5 x 40 = Rs 200 and that of Sanjeev is Rs 7k = Rs 7 x 40 = Rs 280.

9. The work done by (x - 2) men in (4x + 1) days and the work done by (4x + 1) men in (2x - 3) days are in the ratio 3: 8. Find the value of x.
Solution:

On assuming that the same amount of work is done one day by all the men and one day work of each man = 1 units, we have
Amount of work done by (x - 2) men in (4x + 1) days
= Amount of work done by (x - 2)*(4x + 1) men in one day
= (x - 2)(4x + 1) units of work

Similarly, we have
Amount of work done by (4x + 1) men in (2x - 3) days
= (4x + 1)*(2x - 3) units of work
Then according to the question, we have

$$\frac{(x-2)(4x+1)}{(4x+1)(2x-3)} = \frac{3}{8}$$

$$\frac{x-2}{2x-3} = \frac{3}{8}$$

8x – 16 = 6x – 9
2x = 7
x = 7/2

10. The bus fare between two cities is increased in the ratio 7: 9. Find the increase in the fare, if:
(i) the original fare is Rs 245;
(ii) the increased fare is Rs 207.
Solution:

From the question we have,
Increased (new) bus fare = (9/7) x original bus fare
(i) We have,
Increased (new) bus fare = 9/7 x Rs 245 = Rs 315
Thus, the increase in fare = Rs 315 - Rs 245 = Rs 70
(ii) Here we have,
Rs 207 = (9/7) x original bus fare
Original bus fare = Rs 207 x 7/9 = Rs 161
Thus, the increase in fare = Rs 207 - Rs 161 = Rs 46

11. By increasing the cost of entry ticket to a fair in the ratio 10: 13, the number of visitors to the fair has decreased in the ratio 6: 5. In what ratio has the total collection increased or decreased?
Solution:

Let's take the cost of the entry ticket initially and at present to be 10x and 13x respectively.
And let the number of visitors initially and at present be 6y and 5y respectively.
So,
Initially, the total collection = 10x × 6y = 60 xy
And at present, the total collection = 13x × 5y = 65 xy
Hence,
The ratio of total collection = 60 xy: 65 xy = 12: 13
Therefore, it's seen that the total collection has been increased in the ratio 12: 13.

Exercise 7(B)

Solution:

(i) Let's assume the fourth proportional to 1.5, 4.5 and 3.5 be x.
1.5: 4.5 = 3.5: x
1.5 × x = 3.5 × 4.5
x = (3.5 x 4.5)/ 1.5
x = 10.5

(ii) Let's assume the fourth proportional to 3a, $6a^2$ and $2ab^2$ be x.
$3a: 6a^2 = 2ab^2: x$
$3a \times x = 2ab^2 \times 6a^2$
$3a \times x = 12a^3b^2$
$x = 4a^2b^2$

(i) Solution:

(i) Let's take the third proportional to $2\frac{2}{3}$ and 4 be x.

So, $2\frac{2}{3}$, 4, x are in continued proportion.
8/3: 4 = 4: x
(8/3)/ 4 = 4/x
x = 16 x 3/8 = 6

(ii) Let's take the third proportional to a - b and $a^2 - b^2$ be x.
So, a - b, $a^2 - b^2$, x are in continued proportion.
$a - b: a^2 - b^2 = a^2 - b^2: x$

$$\frac{a-b}{a^2-b^2} = \frac{a^2-b^2}{x}$$

$$x = \frac{(a^2-b^2)^2}{a-b}$$

$$x = \frac{(a+b)(a-b)(a^2-b^2)}{a-b}$$

$$x = (a+b)(a^2-b^2)$$

2. Find the mean proportional between:
(i) $6 + 3\sqrt{3}$ and $8 - 4\sqrt{3}$