## Exercise 7(B)

### Solution 1:

(i)

$2^{2x+1} = 8$

$\Rightarrow 2^{2x+1} = 2^3$

We know that if bases are equal, the powers are equal

$\Rightarrow 2x+1=3$

$\Rightarrow 2x=3-1$

$\Rightarrow 2x = 2$

$\Rightarrow x = \frac{2}{2}$

$\Rightarrow x = 1$

(ii)

$2^{5x-1} = 4 \times 2^{3x+1}$

$\Rightarrow 2^{5x-1} = 2^2 \times 2^{3x+1}$

$\Rightarrow 2^{5x-1} = 2^{3x+1+2}$

$\Rightarrow 2^{5x-1} = 2^{3x+3}$

We know that if bases are equal, the powers are equal

$\Rightarrow 5x-1=3x+3$

$\Rightarrow 5x-3x=3+1$

$\Rightarrow 2x = 4$

$\Rightarrow x = \frac{4}{2}$

$\Rightarrow x = 2$

(iii)

$3^{4x+1} = 27^{(x+1)}$

$\Rightarrow 3^{4x+1} = \left(3^3\right)^{x+1}$

$\Rightarrow 3^{4x+1} = 3^{3x+3}$

We know that if bases are equal, the powers are equal

$\Rightarrow 4x+1=3x+3$

$\Rightarrow 4x-3x=3-1$

$\Rightarrow x = 2$

(iv)

$49^{x+4} = 7^2(343)^{(x+1)}$

$\Rightarrow (7 \times 7)^{x+4} = 7^2(7 \times 7 \times 7)^{(x+1)}$

$\Rightarrow \left(7^2\right)^{x+4} = 7^2\left(7^3\right)^{(x+1)}$

$\Rightarrow 7^{2x+8} = 7^2 \times 7^{3x+3}$

$\Rightarrow 7^{2x+8} = 7^{3x+3+2}$

$\Rightarrow 7^{2x+8} = 7^{3x+5}$

We know that if bases are equal, the powers are equal

$\Rightarrow 2x+8=3x+5$

$\Rightarrow 3x-2x=8-5$

$\Rightarrow x = 3$



**Solution 2:**

(i)

$4^{2x} = \frac{1}{32}$

$\Rightarrow (2 \times 2)^{2x} = \frac{1}{2 \times 2 \times 2 \times 2 \times 2}$

$\Rightarrow \left(2^2\right)^{2x} = \frac{1}{2^5}$

$\Rightarrow 2^{2 \times 2x} = 2^{-5}$

$\Rightarrow 2^{4x} = 2^{-5}$

We know that if bases are equal, the powers are equal

$\Rightarrow 4x=-5$

$\Rightarrow x = \frac{-5}{4}$

(ii)

$\sqrt{2^{x+3}} = 16$

$\left(2^{x+3}\right)^{\frac{1}{2}} = 2 \times 2 \times 2 \times 2$

$\Rightarrow 2^{\frac{x+3}{2}} = 2^4$

We know that if bases are equal, the powers are equal

(iii)

$$\left(\sqrt{\frac{3}{5}}\right)^{x+1} = \frac{125}{27}$$

$$\Rightarrow \left[\left(\frac{3}{5}\right)^{\frac{1}{2}}\right]^{x+1} = \frac{5 \times 5 \times 5}{3 \times 3 \times 3}$$

$$\Rightarrow \left(\frac{3}{5}\right)^{\frac{x+1}{2}} = \left(\frac{5}{3}\right)^{3}$$

$$\Rightarrow \left(\frac{3}{5}\right)^{\frac{x+1}{2}} = \left(\frac{3}{5}\right)^{-3}$$

We know that if bases are equal, the powers are equal

$$\Rightarrow \frac{x+1}{2} = -3$$

$$\Rightarrow x + 1 = -6$$

$$\Rightarrow x = -6 - 1$$

$$\Rightarrow x = -7$$

(iv)

$$\left(\sqrt[3]{\frac{2}{3}}\right)^{x-1} = \frac{27}{8}$$

$$\left[\left(\frac{2}{3}\right)^{\frac{1}{3}}\right]^{x-1} = \frac{3^3}{2^3}$$

$$\Rightarrow \left(\frac{2}{3}\right)^{\frac{x-1}{3}} = \left(\frac{3}{2}\right)^{3}$$

$$\Rightarrow \left(\frac{2}{3}\right)^{\frac{x-1}{3}} = \left(\frac{2}{3}\right)^{-3}$$

We know that if bases are equal, the powers are equal

$$\Rightarrow \frac{x-1}{3} = -3$$

$$\Rightarrow x - 1 = -9$$

$$\Rightarrow x = -9 + 1$$

$$\Rightarrow x = -8$$

**Solution 3:**

(i)

$4^{x-2} - 2^{x+1} = 0$

$\Rightarrow 4^{x-2} = 2^{x+1}$

$\Rightarrow \left(2^2\right)^{x-2} = 2^{x+1}$

$\Rightarrow 2^{2x-4} = 2^{x+1}$

We know that if bases are equal, the powers are equal

$\Rightarrow 2x - 4 = x + 1$

$\Rightarrow 2x - x = 4 + 1$

$\Rightarrow x = 5$

(ii)

$3^{x^2} : 3^x = 9 : 1$

$\frac{3^{x^2}}{3^x} = \frac{9}{1}$

$\Rightarrow 3^{x^2} = 9 \times 3^x$

$\Rightarrow 3^{x^2} = 3^2 \times 3^x$

$\Rightarrow 3^{x^2} = 3^{x+2}$

We know that if bases are equal, the powers are equal

$\Rightarrow x^2 = x + 2$

$\Rightarrow x^2 - x - 2 = 0$

$\Rightarrow x^2 - 2x + x - 2 = 0$

$\Rightarrow x(x - 2) + 1(x - 2) = 0$

$\Rightarrow (x + 1)(x - 2) = 0$

$\Rightarrow x + 1 = 0$ or $x - 2 = 0$

$\Rightarrow x = -1$ or $x = 2$

**Solution 4:**

(i)

$8\times 2^{2x}+4\times 2^{x+1}=1+2^{x}$

$\Rightarrow 8\times\left(2^{x}\right)^{2}+4\times 2^{x}\times 2^{1}=1+2^{x}$

$\Rightarrow 8\times\left(2^{x}\right)^{2}+4\times\left(2^{x}\right)\times 2^{1}-1-2^{x}=0$

$\Rightarrow 8\times\left(2^{x}\right)^{2}+\left(2^{x}\right)\times(8-1)-1=0$

$\Rightarrow 8\times\left(2^{x}\right)^{2}+7\left(2^{x}\right)-1=0$

$\Rightarrow 8y^{2}+7y-1=0 \quad [y=2^{x}]$

$\Rightarrow 8y^{2}+8y-y-1=0$

$\Rightarrow 8y(y+1)-1(y+1)=0$

$\Rightarrow (8y-1)(y+1)=0$

$\Rightarrow 8y=1$ or $y=-1$

$\Rightarrow y=\frac{1}{8}$ or $y=-1$

$\Rightarrow 2^{x}=\frac{1}{8}$ or $2^{x}=-1$

$\Rightarrow 2^{x}=\frac{1}{2^{3}}$ or $2^{x}=-1$

$\Rightarrow 2^{x}=2^{-3}$ or $2^{x}=-1$

$\Rightarrow x=-3$

$[\because 2^{x}=-1$ is not possible$]$

(ii)

$2^{2x}+2^{x+2}-4\times 2^3=0$

$\Rightarrow (2^x)^2+2^x\cdot 2^2-4\times 2\times 2\times 2=0$

$\Rightarrow (2^x)^2+2^x\cdot 2^2-32=0$

$\Rightarrow y^2+4y-32=0 \quad [y=2^x]$

$\Rightarrow y^2+8y-4y-32=0$

$\Rightarrow y(y+8)-4(y+8)=0$

$\Rightarrow (y+8)(y-4)=0$

$\Rightarrow y+8=0$ or $y-4=0$

$\Rightarrow y=-8$ or $y=4$

$\Rightarrow 2^x=-8$ or $2^x=4$

$\Rightarrow 2^x=2^2 \quad [\because 2^x=-8$ is not possible$]$

$\Rightarrow x=2$

(iii)

$(\sqrt{3})^{x-3}=(\sqrt[4]{3})^{x+1}$

$\Rightarrow \left(3^{\frac{1}{2}}\right)^{x-3}=\left(3^{\frac{1}{4}}\right)^{x+1}$

$\Rightarrow 3^{\frac{x-3}{2}}=3^{\frac{x+1}{4}}$

$\Rightarrow \frac{x-3}{2}=\frac{x+1}{4}$

$\Rightarrow 4(x-3)=2(x+1)$

$\Rightarrow 4x-12=2x+2$

$\Rightarrow 4x-2x=12+2$

$\Rightarrow 2x=14$

$\Rightarrow x=\frac{14}{2}$

$\Rightarrow x=7$

**Solution 5:**

$$4^{2m} = \left(\sqrt[3]{16}\right)^{-\frac{6}{n}} = \left(\sqrt{8}\right)^2$$

$$\Rightarrow 4^{2m} = \left(\sqrt{8}\right)^2 \ldots.(1)$$

and

$$\left(\sqrt[3]{16}\right)^{-\frac{6}{n}} = \left(\sqrt{8}\right)^2 \ldots.(2)$$

From (1)

$$4^{2m} = \left(\sqrt{8}\right)^2$$

$$\Rightarrow \left(2^2\right)^{2m} = \left(\sqrt{2^3}\right)^2$$

$$\Rightarrow 2^{4m} = \left[\left(2^3\right)^{\frac{1}{2}}\right]^2$$

$$\Rightarrow 2^{4m} = \left[2^{3\times\frac{1}{2}}\right]^2$$

$$\Rightarrow 2^{4m} = 2^{3\times\frac{1}{2}\times 2}$$

$$\Rightarrow 2^{4m} = 2^3$$

$$\Rightarrow 4m = 3$$

$$\Rightarrow m = \frac{3}{4}$$

From (2), we have

$$\left(\sqrt[3]{16}\right)^{-\frac{6}{n}} = \left(\sqrt{8}\right)^2$$

$$\Rightarrow \left(\sqrt[3]{2\times2\times2\times2}\right)^{-\frac{6}{n}} = \left(\sqrt{2\times2\times2}\right)^2$$

$$\Rightarrow \left(\sqrt[3]{2^4}\right)^{-\frac{6}{n}} = \left(\sqrt{2^3}\right)^2$$

$$\Rightarrow \left[\left(2^4\right)^{\frac{1}{3}}\right]^{-\frac{6}{n}} = \left[\left(2^3\right)^{\frac{1}{2}}\right]^2$$

$$\Rightarrow \left[2^{\frac{4}{3}}\right]^{-\frac{6}{n}} = \left[2^{\frac{3}{2}}\right]^2$$

$$\Rightarrow 2^{\frac{4}{3}\times\left(-\frac{6}{n}\right)} = 2^{\frac{3}{2}\times 2}$$

$$\Rightarrow 2^{\left(-\frac{8}{n}\right)} = 2^3$$

$$\Rightarrow -\frac{8}{n} = 3$$

$$\Rightarrow n = \frac{-8}{3} \qquad \text{Thus } m = \frac{3}{4} \; n = \frac{-8}{3}$$

**Solution 6:**

Consider the equation

$$\left(\sqrt{32}\right)^{x} \div 2^{y+1} = 1$$

$$\Rightarrow \left(\sqrt{2 \times 2 \times 2 \times 2 \times 2}\right)^{x} \div 2^{y+1} = 1$$

$$\Rightarrow \left(\sqrt{2^5}\right)^{x} \div 2^{y+1} = 1$$

$$\Rightarrow \left[\left(2^5\right)^{\frac{1}{2}}\right]^{x} \div 2^{y+1} = x^0$$

$$\Rightarrow 2^{\frac{5x}{2}} \div 2^{y+1} = x^0$$

$$\Rightarrow \frac{5x}{2} - (y + 1) = 0$$

$$\Rightarrow 5x - 2(y + 1) = 0$$

$$\Rightarrow 5x - 2y - 2 = 0.....(1)$$

Now consider the other equation

$$8^y - 16^{4-\frac{x}{2}} = 0$$

$$\Rightarrow \left(2^3\right)^{y} - \left(2^4\right)^{4-\frac{x}{2}} = 0$$

$$\Rightarrow 2^{3y} - 2^{4\left(4-\frac{x}{2}\right)} = 0$$

$$\Rightarrow 2^{3y} = 2^{4\left(4-\frac{x}{2}\right)}$$

$$\Rightarrow 3y = 4\left(4 - \frac{x}{2}\right)$$

$$\Rightarrow 3y = 16 - 2x$$

$$\Rightarrow 2x + 3y = 16.....(2)$$

Thus we have two equations,

$5x - 2y = 2$ .....(1)

$2x + 3y = 16$.....(2)

Multiplying (1) by 3 and (2) by 2, we have

$15x - 6y = 6$....(3)

$4x + 6y = 32$....(4)

Adding (3) and (4), we have

$19x=38$

$\Rightarrow x=2$

Substituting the value of x in equation (1), we have,

$5(2) - 2y = 2$

$\Rightarrow 10 - 2y = 2$

$\Rightarrow 2y = 10 - 2$

$\Rightarrow 2y = 8$

$\Rightarrow y = \frac{8}{2}$

$\Rightarrow y = 4$

Thus the values of x and y are:

$x=2$ and $y=4$

## Solution 7:

(i)

$$\text{L.H.S.} = \left(\frac{x^a}{x^b}\right)^{a+b-c} \times \left(\frac{x^b}{x^c}\right)^{b+c-a} \times \left(\frac{x^c}{x^a}\right)^{c+a-b}$$

$$= \left(x^{a-b}\right)^{(a+b-c)} \times \left(x^{b-c}\right)^{(b+c-a)} \times \left(x^{c-a}\right)^{(c+a-b)}$$

$$= x^{(a-b)(a+b-c)} \times x^{(b-c)(b+c-a)} \times x^{(c-a)(c+a-b)}$$

$$= x^{a^2+ab-ac-ab-b^2+bc} \times x^{b^2+bc-ab-cb-c^2+ac} \times x^{c^2+ac-bc-ac-a^2+ab}$$

$$= x^{a^2-ac-b^2+bc+b^2-ab-c^2+ac+c^2-bc-a^2+ab}$$

$$= x^0$$

$$= 1$$

$= \text{R.H.S}$

(ii)

We need to prove that

$$\frac{x^{a(b-c)}}{x^{b(a-c)}} \div \left(\frac{x^b}{x^a}\right)^c = 1$$

$$\text{L.H.S.} = x^{a(b-c)-b(a-c)} \div \frac{x^{bc}}{x^{ac}}$$

$$\Rightarrow \quad = x^{ab-ac-ab+bc} \div x^{bc-ac}$$

$$\Rightarrow \quad = x^{ab-ac-ab+bc-(bc-ac)}$$

$$\Rightarrow \quad = x^{ab-ac-ab+bc-bc+ac}$$

$$\Rightarrow \quad = x^0$$

$$\Rightarrow \quad = 1$$

$\Rightarrow \quad = \text{R.H.S}$

**Solution 8:**

We are given that

$a^x = b, b^y = c$ and $c^z = a$

Consider the equation

$a^x = b$

$\Rightarrow a^{xyz} = b^{yz}$ [raising to the power yz on both sides]

$\Rightarrow a^{xyz} = (b^y)^z$

$\Rightarrow a^{xyz} = (c)^z$ $[\because b^y = c]$

$\Rightarrow a^{xyz} = c^z$

$\Rightarrow a^{xyz} = a$ $[\because c^z = a]$

$\Rightarrow a^{xyz} = a^1$

$\Rightarrow xyz = 1$

**Solution 9:**

Let $a^x = b^y = c^z = k$

$\therefore a = k^{\frac{1}{x}}; b = k^{\frac{1}{y}}; c = k^{\frac{1}{z}}$

Also, we have $b^2 = ac$

$\therefore \left(k^{\frac{1}{y}}\right)^2 = \left(k^{\frac{1}{x}}\right) \times \left(k^{\frac{1}{z}}\right)$

$\Rightarrow k^{\frac{2}{y}} = k^{\frac{1}{x}+\frac{1}{z}}$

$\Rightarrow k^{\frac{2}{y}} = k^{\frac{z+x}{xz}}$

Comparing the powers we have

$\frac{2}{y} = \frac{z+x}{xz}$

$\Rightarrow y = \frac{2xz}{z+x}$

**Solution 10:**

Let $5^{-p} = 4^{-q} = 20^{r} = k$

$5^{-p} = k \Rightarrow 5 = k^{-\frac{1}{p}} [\because a^p = b^q \Rightarrow a = b^{\frac{q}{p}}]$

$4^{-q} = k \Rightarrow 4 = k^{-\frac{1}{q}} [\because a^p = b^q \Rightarrow a = b^{\frac{q}{p}}]$

$20^{r} = k \Rightarrow 20 = k^{\frac{1}{r}} [\because a^p = b^q \Rightarrow a = b^{\frac{q}{p}}]$

$5 \times 4 = 20$

$\Rightarrow k^{-\frac{1}{p}} \times k^{-\frac{1}{q}} = k^{\frac{1}{r}}$

$\Rightarrow k^{-\frac{1}{p}-\frac{1}{q}} = k^{\frac{1}{r}}$

$\Rightarrow k^{0} = k^{\frac{1}{p}+\frac{1}{q}+\frac{1}{r}}$

$\Rightarrow \frac{1}{p} + \frac{1}{q} + \frac{1}{r} = 0$ [If bases are equal, powers are also equal]

**Solution 11:**

$(m+n)^{-1}(m^{-1}+n^{-1}) = m^x n^y$

$\Rightarrow \frac{1}{(m+n)} \times \left(\frac{1}{m} + \frac{1}{n}\right) = m^x n^y$

$\Rightarrow \frac{1}{(m+n)} \times \left(\frac{m+n}{mn}\right) = m^x n^y$

$\Rightarrow \frac{1}{mn} = m^x n^y$

$\Rightarrow m^{-1} n^{-1} = m^x n^y$

Comparing the coefficients of $x$ and $y$, we get

$x = -1$ and $y = -1$

$LHS$,

$x + y + 2 = (-1) + (-1) + 2 = 0 = RHS$

**Solution 12:**

$5^{x+1} = 25^{x-2}$

$\Rightarrow 5^{x+1} = \left(5^2\right)^{x-2}$

$\Rightarrow 5^{x+1} = 5^{2x-4}$ [If bases are equal, powers are also equal]

$\Rightarrow x + 1 = 2x - 4$

$\Rightarrow 2x - x = 4 + 1$

$\Rightarrow x = 5$

$\therefore 3^{x-3} \times 2^{3-x} = 3^{5-3} \times 2^{3-5} = 3^2 \times 2^{-2} = 9 \times \frac{1}{4} = \frac{9}{4}$

**Solution 13:**

$4^{x+3} = 112 + 8 \times 4^x$

$\Rightarrow 4^x \times 4^3 = 112 + 8 \times 4^x$

$\Rightarrow 64 \times 4^x = 112 + 8 \times 4^x$

Let $4^x = y$

$64y = 112 + 8y$

$\Rightarrow 56y = 112$

$\Rightarrow y = 2$

Substituting we get,

$4^x = 2$

$\Rightarrow 2^{2x} = 2$

$\Rightarrow 2x = 1$

$\Rightarrow x = \frac{1}{2}$

$(18x)^{3x} = \left(\frac{18}{2}\right)^{3 \times \frac{1}{2}} = 9^{3 \times \frac{1}{2}} = \left(9^{\frac{1}{2}}\right)^3 = 3^3 = 27$

## Solution 14(i):

(i)

$$4^{x-1} \times (0.5)^{3-2x} = \left(\frac{1}{8}\right)^{-x}$$

$$\Rightarrow \left(2^2\right)^{x-1} \times \left(\frac{1}{2}\right)^{3-2x} = \left(\frac{1}{2^3}\right)^{-x}$$

$$\Rightarrow 2^{2x-2} \times 2^{-(3-2x)} = \left(2^{-3}\right)^{-x}$$

$$\Rightarrow 2^{2x-2-3+2x} = 2^{3x}$$

$$\Rightarrow 2^{4x-5} = 2^{3x}$$

$$\Rightarrow 4x - 5 = 3x$$

$$\Rightarrow 4x - 3x = 5$$

$$\Rightarrow x = 5$$

## Solution 14(ii):

$$a^{2(3x+5)} \times a^{4x} = a^{8x+12}$$

$$\Rightarrow a^{6x+10+4x} = a^{8x+12}$$

$\Rightarrow 10x + 10 = 8x + 12$ [If bases are the same, powers are also same]

$$\Rightarrow 2x = 2$$

$$\Rightarrow x = 1$$



## Solution 14(iii):

$$(81)^{\frac{3}{4}} - \left(\frac{1}{32}\right)^{-\frac{2}{5}} + x\left(\frac{1}{2}\right)^{-1} . 2^0 = 27$$

$$\Rightarrow 3^{4 \times \frac{3}{4}} - \left(2^{-5}\right)^{-\frac{2}{5}} + x(2) = 27$$

$$\Rightarrow 3^3 - 2^2 + 2x = 27$$

$$\Rightarrow 2x + 27 - 4 = 27$$

$$\Rightarrow 2x = 4$$

$$\Rightarrow x = 2$$

**Solution 14(iv):**

$2^{3x} \times 2^3 = 2^{3x} \times 2 + 48$

$\Rightarrow 8 \times 2^{3x} = 2^{3x} \times 2 + 48$

$\Rightarrow 2^{3x} (8 - 2) = 48$

$\Rightarrow 2^{3x} \times 6 = 48$

$\Rightarrow 2^{3x} = 8$

$\Rightarrow 2^{3x} = 2^3$

$\Rightarrow 3x = 3$

$\Rightarrow x = 1$

**Solution 14(v):**

$3 \times 2^x + 3 - 2^x \times 2^2 + 5 = 0$

$\Rightarrow 2^x (3 - 4) + 8 = 0$

$\Rightarrow -2^x = -8$

$\Rightarrow 2^x = 8$

$x=3$