Chapter 9. Triangles [Congruency in Triangles]

Exercise 9(A)

Solution 1:

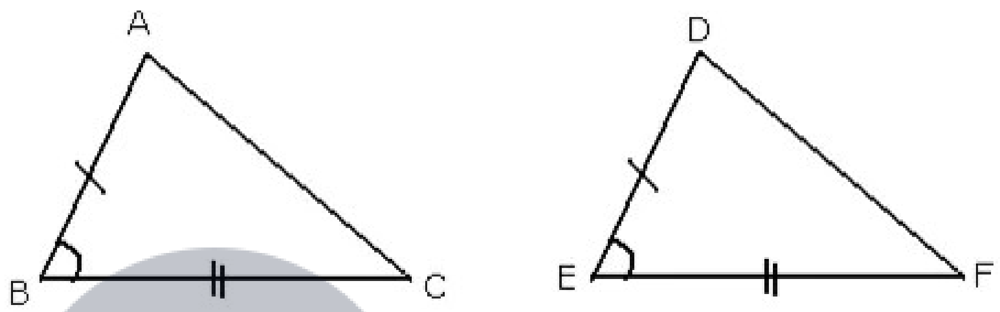
(*a*)

In ΔABC and ΔDEF

AB=DE [Given]

$\angle B = \angle E$ [Given]

BC=EF [Given]



By Side-Angle-Side criterion of congruency, the triangles ΔABC and ΔDEF are congruent to each other.

$\therefore \Delta ABC \cong \Delta DEF$

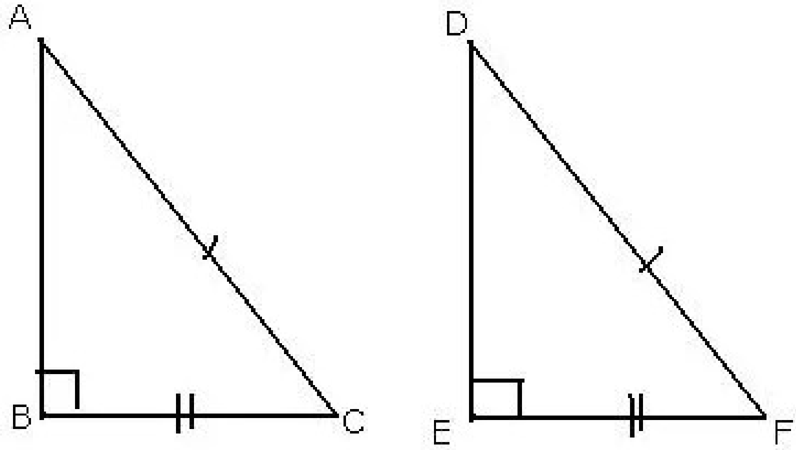
(*b*)

In ΔABC and ΔDEF

$\angle B = \angle E = 90^\circ$

Hyp. AC=Hyp.DF

BC=EF



By Right Angle-Hypotenuse-Side criterion of congruency, the triangles

ΔABC and ΔDEF are congruent to each other.

$\therefore \Delta ABC \cong \Delta DEF$

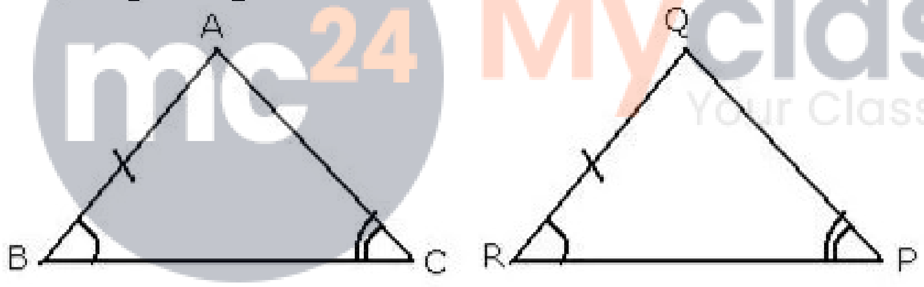
(c)

In ΔABC and ΔQRP

$\angle B = \angle R$ [Given]

$\angle C = \angle P$ [Given]

AB=QR [Given]



By Angle-Angle-Side criterion of congruency, the triangles

ΔABC and ΔQRP are congruent to each other.

$\therefore \Delta ABC \cong \Delta QRP$

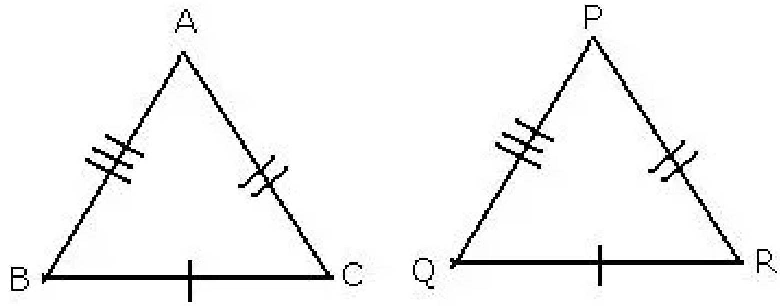
(d)

In ΔABC and ΔPQR

$AB = PQ$ [Given]

$AC = PR$ [Given]

$BC = QR$ [Given]



By Side-Side-Side criterion of congruency, the triangles ΔABC and ΔPQR are congruent to each other.

$\therefore \Delta ABC \cong \Delta PQR$

(e)

In ΔPQR

$\angle R = 40^\circ, \angle Q = 50^\circ$

$\angle P + \angle Q + \angle R = 180^\circ$ [Sum of all the angles in a triangle = 180°]

$\Rightarrow \angle P + 50^\circ + 40^\circ = 180^\circ$

$\Rightarrow \angle P + 90^\circ = 180^\circ$

$\Rightarrow \angle P = 180^\circ - 90^\circ$

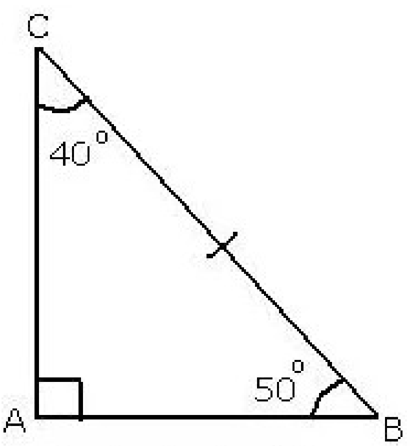
$\Rightarrow \angle P = 90^\circ$

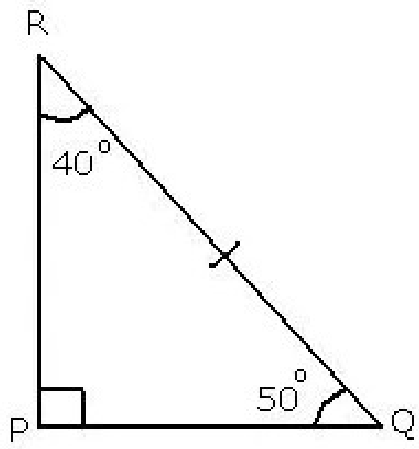
In ΔABC and ΔPQR

$\angle A = \angle P$

$\angle C = \angle R$

$BC = QR$





By Angle-Angle-Side criterion of congruency, the triangles

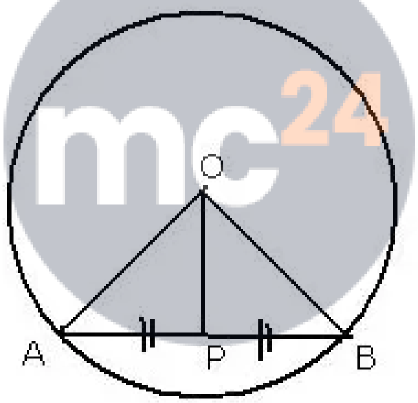
ΔABC and ΔPQR are congruent to each other.

$\therefore \Delta ABC \cong \Delta PQR$

Solution 2:

Given: In the figure, O is centre of the circle, and AB is chord. P is a point on AB such that AP =PB.

We need to prove that, $OP \perp AB$



Construction: Join OA and OB

Proof:

In ΔOAP and ΔOBP

$OA=OB$ [radii of the same circle]

$OP = OP$ [common]

$AP = PB$ [given]

$\therefore$ By Side-Side-Side criterion of congruency,

$\Delta OAP \cong \Delta OBP$

The corresponding parts of the congruent triangles are congruent.

$\therefore \angle OPA = \angle OPB$ [by c.p.c.t]

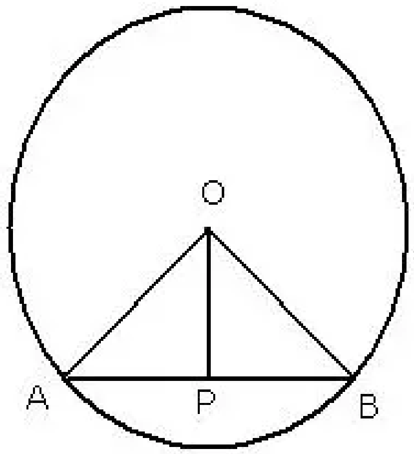
But $\angle OPA + \angle OPB = 180^\circ$ [linear pair]

$\therefore \angle OPA = \angle OPB = 90^\circ$

Hence $OP \perp AB$.

Solution 3:

Given: In the figure, O is centre of the circle,
and AB is chord. P is a point on AB such that AP =PB.
We need to prove that, $AP = BP$



Construction: Join OA and OB
Proof:
In right triangles ΔOAP and ΔOBP
Hypotenuse OA=OB [radii of the same circle]
Side $OP = OP$ [common]
∴ By Right angle-Hypotenuse-Side criterion of congruency,
ΔOAP ≅ ΔOBP
The corresponding parts of the congruent
triangles are congruent.
∴ $AP = BP$ [by c.p.c.t]
Hence proved.

Solution 4:

Given: A ΔABC in which D is the mid-point of BC.

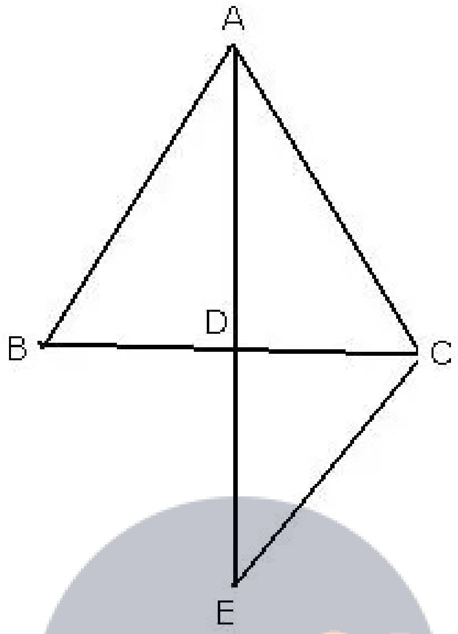
AD is produced to E so that DE=AD

We need to prove that

(i) $\Delta ABD \cong \Delta ECD$

(ii) $AB = EC$

(iii) $AB \parallel EC$



(i)In ΔABD and ΔECD

BD=DC [D is the midpoint of BC]

$\angle ADB = \angle CDE$ [vertically opposite angles]

$AD = DE$ [Given]

$\therefore$ By Side-Angle-Side criterion of congruence, we have,

$\Delta ABD \cong \Delta ECD$

(ii)The corresponding parts of the congruent triangles are congruent.

$\therefore$ AB=EC [c.p.c.t]

(iii)Also, $\angle DAB = \angle DEC$ [c.p.c.t]

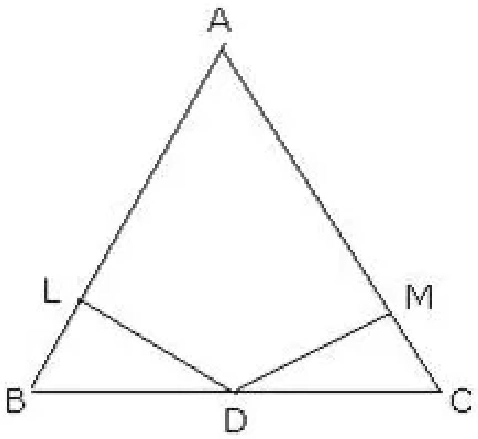
$AB \parallel EC$ [$\angle DAB$ and $\angle DEC$ are alternate angles]

Solution 5:

(i)Given: A ΔABC in which $\angle B = \angle C$.

DL is the perpendicular from D to AB

DM is the perpendicular from D to AC



We need to prove that

$DL = DM$

Proof:

In ΔDLB and ΔDMC

$\angle DLB = \angle DMC = 90°$ $[DL \perp AB$ and $DM \perp AC]$

$\angle B = \angle C$ [Given]

$BD = DC$ [D is the midpoint of BC]

$\therefore$ By Angle-Angle-Side criterion of congruence,

$\Delta DLB \cong \Delta DMC$

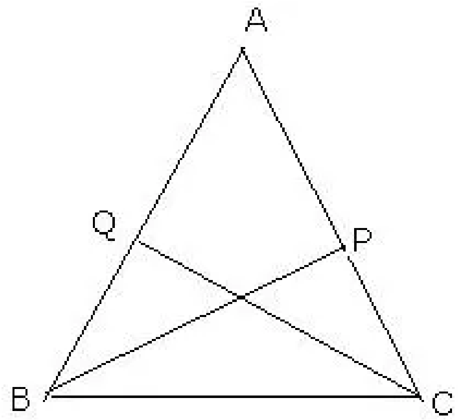
The corresponding parts of the congruent triangles are congruent.

$\therefore DL = DM$ [c.p.c.t]

(ii)Given: A ΔABC in which $\angle B = \angle C$.

BP is the perpendicular from D to AC

CQ is the perpendicular from C to AB



We need to prove that

$BP = CQ$

Proof:

In ΔBPC and ΔCQB

$\angle B = \angle C$ [Given]

$\angle BPC = \angle CQB = 90^\circ$ [$BP \perp AC$ and $CQ \perp AB$]

$BC = BC$ [Common]

$\therefore$ By Angle-Angle-Side criterion of congruence,

$\Delta BPC \cong \Delta CQB$

The corresponding parts of the congruent triangles are congruent.

$\therefore BP = CQ$ [c.p.c.t]

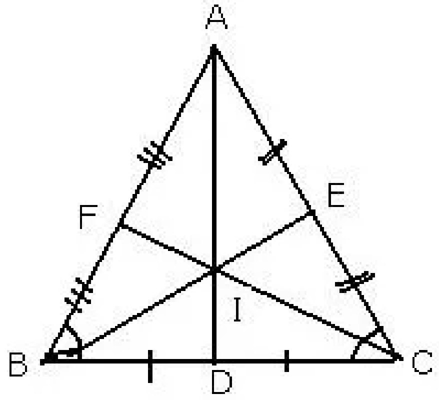
Solution 6:

Given: A ΔABC in which AD is the perpendicular bisector of BC

BE is the perpendicular bisector of CA

CF is the perpendicular bisector of AB

AD, BE and CF meet at I



We need to prove that

$IA = IB = IC$

Proof:

In ΔBID and ΔCID

$BD = DC$ [Given]

$\angle BDI = \angle CDI = 90°$ [AD is the perpendicular bisector of BC]

$BC = BC$ [Common]

$\therefore$ By Side-Angle-Side criterion of congruence,

$\Delta BID \cong \Delta CID$

The corresponding parts of the congruent triangles are congruent.

$\therefore IB = IC$ [c.p.c.t]

Similarly, in ΔCIE and ΔAIE

$CE = AE$ [Given]

$\angle CEI = \angle AEI = 90°$ [AD is the perpendicular bisector of BC]

$IE = IE$ [Common]

$\therefore$ By Side-Angle-Side criterion of congruence,

$\Delta CIE \cong \Delta AIE$

The corresponding parts of the congruent triangles are congruent.

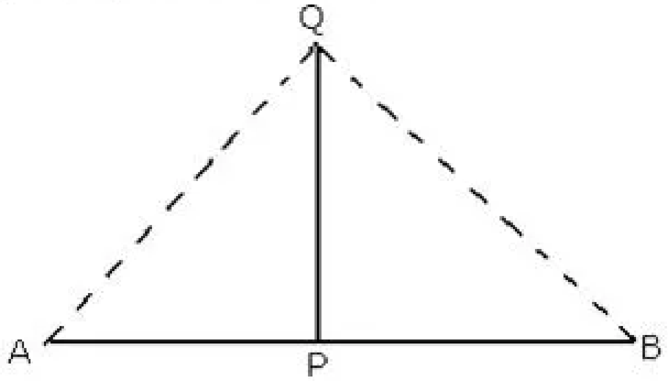
$\therefore IC = IA$ [c.p.c.t]

Thus, $IA = IB = IC$

Solution 7:

Given: A ΔABC in which AB is bisected at P

PQ is perpendicular to AB



We need to prove that

$QA = QB$

Proof:

In ΔAPQ and ΔBPQ

$AP = PB$	[P is the mid-point of AB]
$\angle APQ = \angle BPQ = 90°$	[PQ is perpendicular to AB]
$PQ = PQ$	[Common]

$\therefore$ By Side-Angle-Side criterion of congruence,

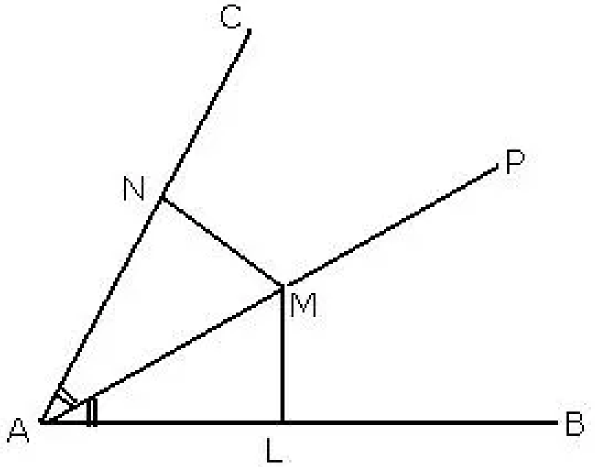
$\Delta APQ \cong \Delta BPQ$

The corresponding parts of the congruent triangles are congruent.

$\therefore$ $QA = QB$ [c.p.c.t]

Solution 8:

From M, draw ML such that ML is perpendicular to AB and MN is perpendicular to AC



In ΔALM and ΔANM

$\angle LAM = \angle MAN$ [$\because$ AP is the bisector of $\angle BAC$]

$\angle ALM = \angle ANM = 90°$ [$\because ML \perp AB, MN \perp AC$]

$AM = AM$ [Common]

$\therefore$ By Angle-Angle-Side criterion of congruence,

$\Delta ALM \cong \Delta ANM$

The corresponding parts of the congruent triangles are congruent.

$\therefore ML = MN$ [c.p.c.t]

Hence proved.

Solution 9:

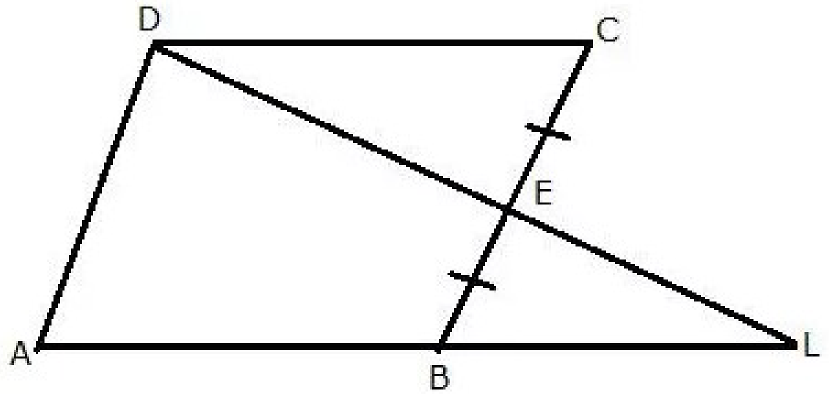
Given: ABCD is a parallelogram in which E is the mid-point of BC.

We need to prove that

(i) $\Delta DCE \cong \Delta LBE$

(ii) $AB = BL$

(iii) $AL = 2DC$



(i) In ΔDCE and ΔLBE

$\angle DCE = \angle EBL$ [DC ∥ AB, alternate angles]

CE=EB [E is the midpoint of BC]

$\angle DEC = \angle LEB$ [vertically opposite angles]

∴ By Angle-Side-Angle criterion of congruence, we have,

$\Delta DCE \cong \Delta LBE$

The corresponding parts of the congruent triangles are congruent.

∴ DC=LB [c.p.c.t] ...(1)

(ii) DC=AB [opposite sides of a parallelogram]...(2)

From (1) and (2), AB=BL ...(3)

(iii) AL=AB+BL ...(4)

From (3) and (4), AL=AB+AB

⇒ AL=2AB

⇒ AL=2DC [from (2)]

Solution 10:

Given: In the figure AB=DB, AC=DC, $\angle ABD=58°$,

$\angle DBC=(2x-4)°$, $\angle ACB=(y+15)°$ and $\angle DCB=63°$

We need to find the values of x and y.

In ΔABC and ΔDBC

$AB = DB$ [given]

AC=DC [given]

$BC = BC$ [common]

$\therefore$ By Side-Side-Side criterion of congruence, we have,

$\Delta ABC \cong \Delta DBC$

The corresponding parts of the congruent triangles are congruent.

$\therefore \angle ACB=\angle DCB$ [c.p.c.t]

$\Rightarrow y° + 15°=63°$

$\Rightarrow y°=63° - 15°$

$\Rightarrow y°=48°$

and $\angle ABC=\angle DBC$[c.p.c.t]

But, $\angle DBC= (2x-4)°$

We have $\angle ABC+\angle DBC=\angle ABD$

$\Rightarrow (2x-4)° + (2x-4)° = 58°$

$\Rightarrow 4x - 8° = 58°$

$\Rightarrow 4x = 58° + 8°$

$\Rightarrow 4x = 66°$

$\Rightarrow x = \frac{66°}{4}$

$\Rightarrow x = 16.5°$

Thus the values of x and y are:

$x = 16.5°$ and $y=48°$

Solution 11:

In the given figure AB||FD,

$\Rightarrow \angle ABC = \angle FDC$

Also AC||GE,

$\Rightarrow \angle ACB = \angle GEB$

Consider the two triangles ΔGBE and ΔFDC

$\angle B = \angle D$

$\angle C = \angle E$

Also given that

BD = CE

$\Rightarrow BD + DE = CE + DE$

$\Rightarrow BE = DC$

$\therefore$ By Angle – Side – Angle criterian of congruence

$\Delta GBE \cong \Delta FDC$

$\therefore \frac{GB}{FD} = \frac{BE}{DC} = \frac{GE}{FC}$

But BE=DC

$\Rightarrow \frac{BE}{DC} = \frac{BE}{BE} = 1$

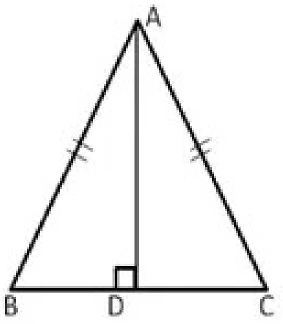
$\therefore \frac{GB}{FD} = \frac{BE}{DC} = 1$

$\Rightarrow GB = FD$

$\therefore \frac{GE}{FC} = \frac{BE}{DC} = 1$

$\Rightarrow GE = FC$

Solution 12:



In ΔADB and ΔADC,

AB = AC (Since ΔABC is an isosceles triangle)

AD = AD (common side)

$\angle ADB = \angle ADC$ (Since AD is the altitude so each is 90°)

$\Rightarrow \Delta ADB \cong \Delta ADC$ (RHS congruence criterion)

BD = DC (cpct)

$\Rightarrow$ AD is the median.

Solution 13:

In ΔDLB and ΔDMC,

$BL = CM$ (given)

$\angle DLB = \angle DMC$ (Both are $90°$)

$\angle BDL = \angle CDM$ (vertically opposite angles)

$\therefore \Delta DLB \cong \Delta DMC$ (AAS congruence criterion)

$BD = CD$ (cpct)

Hence, AD is the median of ΔABC.

Solution 14:

(i) In ΔADB and ΔADC,

$\angle ADB = \angle ADC$ (Since AD is perpendicular to BC)

$AB = AC$ (given)

$AD = AD$ (common side)

$\therefore \Delta ADB \cong \Delta ADC$ (RHS congruence criterion)

$\Rightarrow BD = CD$ (cpct)

(ii) In ΔEFB and ΔEDB,

$\angle EFB = \angle EDB$ (both are $90°$)

$EB = EB$ (common side)

$\angle FBE = \angle DBE$ (given)

$\therefore \Delta EFB \cong \Delta EDB$ (AAS congruence criterion)

$\Rightarrow EF = ED$ (cpct)

that is, $ED = EF$.

Solution 15:

In ΔABC and ΔEFD,

$AB \parallel EF \Rightarrow \angle ABC = \angle EFD$ (alternate angles)

$AC = ED$ (given)

$\angle ACB = \angle EDF$ (given)

$\therefore \Delta ABC \cong \Delta EFD$ (AAS congruence criterion)

$\Rightarrow AB = FE$ (cpct)

and $BC = DF$ (cpct)

$\Rightarrow BD + DC = CF + DC$ (B - D - C - F)

$\Rightarrow BD = CF$