

ML Aggarwal Solutions for Class 10 Maths

Chapter 12: Equation of Straight Line

Exercise 12.1

1. Find the slope of a line whose inclination is

(i) 45°

(ii) 30°

Solution:

The slope of a line having inclination:

(i) 45°

$$\text{Slope} = \tan 45^\circ = 1$$

(ii) 30°

$$\text{Slope} = \tan 30^\circ = 1/\sqrt{3}$$

2. Find the inclination of a line whose gradient is

(i) 1

(ii) $\sqrt{3}$

(iii) $1/\sqrt{3}$

Solution:

Given,

(i) $\tan \theta = 1$

$$\Rightarrow \theta = 45^\circ$$

(ii) $\tan \theta = \sqrt{3}$

$$\Rightarrow \theta = 60^\circ$$

(i) $\tan \theta = 1/\sqrt{3}$

$$\Rightarrow \theta = 30^\circ$$



3. Find the equation of a straight line parallel to x-axis which is at a distance

(i) 2 units above it

(ii) 3 units below it.

Solution:

(i) A line which is parallel to x-axis is $y = a$

$$\Rightarrow y = 2$$

Hence, the equation of line parallel to x-axis which is at a distance of 2 units above it is $y - 2 = 0$.

(ii) A line which is parallel to x-axis is $y = a$

$$\Rightarrow y = -3$$

Hence, the equation of line parallel to x-axis which is at a distance of 3 units below it is $y + 3 = 0$.

4. Find the equation of a straight line parallel to y-axis which is at a distance of:

(i) 3 units to the right

(ii) 2 units to the left.

Solution:

A line which is parallel to y-axis is $x = a$

(i) Here, $x = 3$

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Hence, the equation of line parallel to y-axis is at a distance of 3 units to the right is $x - 3 = 0$.

(ii) Here, $x = -2$

Hence, the equation of line parallel to y-axis at a distance of 2 units to the left is $x + 2 = 0$.

5. Find the equation of a straight line parallel to y-axis and passing through the point $(-3, 5)$.

Solution:

The equation of the line parallel to y-axis passing through $(-3, 5)$ to $x = -3$

$$\Rightarrow x + 3 = 0$$

6. Find the equation of a line whose

(i) slope = 3, y-intercept = -5

(ii) slope = $-2/7$, y-intercept = 3

(iii) gradient = $\sqrt{3}$, y-intercept = $-4/3$

(iv) inclination = 30° , y-intercept = 2

Solution:

Equation of a line whose slope and y-intercept is given by:

$y = mx + c$, where m is the slope and c is the y-intercept

(i) Given: slope = 3, y-intercept = -5

$$\Rightarrow y = 3x + (-5)$$

Hence, the equation of line is $y = 3x - 5$.

(ii) Given: slope = $-2/7$, y-intercept = 3

$$\Rightarrow y = (-2/7)x + 3$$

$$y = (-2x + 21)/7$$

$$7y = -2x + 21$$

Hence, the equation of line is $2x + 7y - 21 = 0$.

(iii) Given: gradient = $\sqrt{3}$, y-intercept = $-4/3$

$$\Rightarrow y = \sqrt{3}x + (-4/3)$$

$$y = (3\sqrt{3}x - 4)/3$$

$$3y = 3\sqrt{3}x - 4$$

Hence, the equation of line is $3\sqrt{3}x - 3y - 4 = 0$.

(iv) Given: inclination = 30° , y-intercept = 2

$$\text{Slope} = \tan 30^\circ = 1/\sqrt{3}$$

$$\Rightarrow y = (1/\sqrt{3})x + 2$$

$$y = (x + 2\sqrt{3})/\sqrt{3}$$

$$\sqrt{3}y = x + 2\sqrt{3}$$

Hence, the equation of line is $x - \sqrt{3}y + 2\sqrt{3} = 0$.

7. Find the slope and y-intercept of the following lines:

(i) $x - 2y - 1 = 0$

(ii) $4x - 5y - 9 = 0$

(iii) $3x + 5y + 7 = 0$

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(iv) $x/3 + y/4 = 1$

(v) $y - 3 = 0$

(vi) $x - 3 = 0$

Solution:

We know that, equation of line whose slope and y-intercept is given by:

$y = mx + c$, where m is the slope and c is the y-intercept

Using the above and converting to this, we find

(i) $x - 2y - 1 = 0$

$2y = x - 1$

$\Rightarrow y = (\frac{1}{2})x + (-\frac{1}{2})$

Hence, slope = $\frac{1}{2}$ and y-intercept = $-\frac{1}{2}$

(ii) $4x - 5y - 9 = 0$

$5y = 4x - 9$

$\Rightarrow y = (\frac{4}{5})x + (-\frac{9}{5})$

Hence, slope = $\frac{4}{5}$ and y-intercept = $-\frac{9}{5}$

(iii) $3x + 5y + 7 = 0$

$5y = -3x - 7$

$\Rightarrow y = (-\frac{3}{5})x + (-\frac{7}{5})$

Hence, slope = $-\frac{3}{5}$ and y-intercept = $-\frac{7}{5}$

(iv) $x/3 + y/4 = 1$

$(4x + 3y)/12 = 1$

$4x + 3y = 12$

$3y = -4x + 12$

$\Rightarrow y = (-\frac{4}{3})x + 4$

Hence, slope = $-\frac{4}{3}$ and y-intercept = 4

(v) $y - 3 = 0$

$y = 3$

$\Rightarrow y = (0)x + 3$

Hence, slope = 0 and y-intercept = 3

(vi) $x - 3 = 0$

Here, the slope cannot be defined as the line does not meet y-axis.

8. The equation of the line PQ is $3y - 3x + 7 = 0$

(i) Write down the slope of the line PQ.

(ii) Calculate the angle that the line PQ makes with the positive direction of x-axis.

Solution:

Given, equation of line PQ is $3y - 3x + 7 = 0$

Re-writing in form of $y = mx + c$, we have



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$$3y = 3x - 7$$

$$\Rightarrow y = x + (-7/3)$$

Here,

(i) Slope = 1

(ii) As $\tan \theta = 1$

$$\theta = 45^\circ$$

Hence, the angle which PQ makes with the x-axis is 45° .

9. The given figure represents the line $y = x + 1$ and $y = \sqrt{3}x - 1$. Write down the angles which the lines make with the positive direction of the x-axis. Hence determine θ .

Solution:

Given line equations, $y = x + 1$ and $y = \sqrt{3}x - 1$

On comparing with $y = mx + c$,

The slope of the line: $y = x + 1$ is 1 as $m = 1$

$$\text{So, } \tan \theta = 1 \Rightarrow \theta = 45^\circ$$

And,

The slope of the line: $y = \sqrt{3}x - 1$ is $\sqrt{3}$ as $m = \sqrt{3}$

$$\text{So, } \tan \theta = \sqrt{3} \Rightarrow \theta = 60^\circ$$

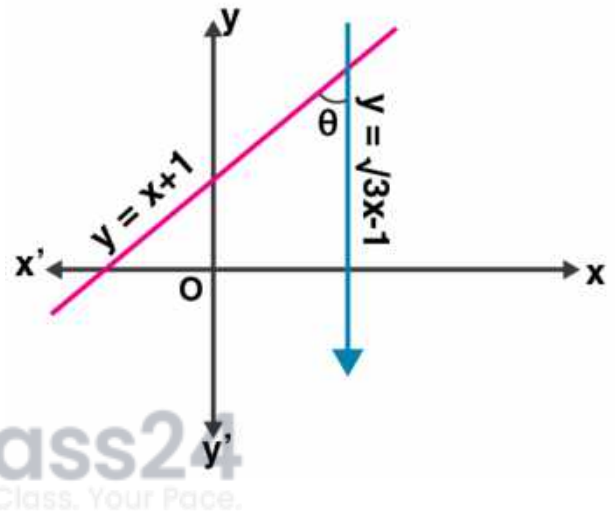
Now, in triangle formed by the given two lines and x-axis

Ext. angle = Sum of interior opposite angle

$$60^\circ = \theta + 45^\circ$$

$$\theta = 60^\circ - 45^\circ$$

$$\text{Thus, } \theta = 15^\circ$$



10. Find the value of p, given that the line $y/2 = x - p$ passes through the point $(-4, 4)$

Solution:

Given, equation of line: $y/2 = x - p$

And, it passes through the point $(-4, 4)$

Hence, it satisfies the line equation

So,

$$4/2 = (-4) - p$$

$$2 = -4 - p$$

$$p = -4 - 2$$

$$\text{Thus, } p = -6$$

11. Given that $(a, 2a)$ lies on the line $y/2 = 3x - 6$. Find the value of a.

Solution:

Given, equation of line: $y/2 = 3x - 6$

And, it passes through the point $(a, 2a)$

Hence, it satisfies the line equation

So,

$$2a/2 = 3(a) - 6$$

$$a = 3a - 6$$

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$$2a = 6$$

Thus, $a = 3$

12. The graph of the equation $y = mx + c$ passes through the points (1, 4) and (-2, -5). Determine the values of m and c .

Solution:

Given, equation of the line is $y = mx + c$

And, it passes through the points (1, 4)

So, the point will satisfy the line equation

$$\Rightarrow 4 = m \times 1 + c$$

$$4 = m + c$$

$$m + c = 4 \dots (i)$$

Also, the line passes through another point (-2, -5)

So,

$$-5 = m(-2) + c$$

$$-5 = -2m + c$$

$$\Rightarrow 2m - c = 5 \dots (ii)$$

Now, on adding (i) and (ii) we get

$$3m = 9$$

$$\Rightarrow m = 3$$

Substituting the value of m in (i), we get

$$3 + c = 4$$

$$\Rightarrow c = 4 - 3 = 1$$

Therefore, $m = 3$, $c = 1$



13. Find the equation of the line passing through the point (2, -5) and making an intercept of -3 on the y-axis.

Solution:

Given, a line equation passes through point (2, -5) and makes a y-intercept of -3

We know that,

The equation of line is $y = mx + c$, where m is the slope and c is the y-intercept

So, we have

$$y = mx - 3$$

Now, this line equation will satisfy the point (2, -5)

$$-5 = m(2) - 3$$

$$-5 = 2m - 3$$

$$2m = 3 - 5 = -2$$

$$\Rightarrow m = -1$$

Hence, the equation of the line is $y = -x + (-3) \Rightarrow x + y + 3 = 0$

14. Find the equation of a straight line passing through (-1, 2) and whose slope is $2/5$.

Solution:

Given, the equation of straight line passes through (-1, 2) and having slope as $2/5$

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So, the equation of the line will be

$$y - y_1 = m(x - x_1)$$

Here, (x_1, y_1) is $(-1, 2)$

$$\Rightarrow y - 2 = (2/5)[x - (-1)]$$

$$5(y - 2) = 2(x + 1)$$

$$5y - 10 = 2x + 2$$

Thus, the line equation is $2x - 5y + 12 = 0$.

15. Find the equation of a straight line whose inclination is 60° and which passes through the point $(0, -3)$.

Solution:

Given,

Inclination of a straight line is 60°

So, the slope = $\tan 60^\circ = \sqrt{3} = m$

And, the equation of line passes through the point $(0, -3) = (x_1, y_1)$

Hence, the equation of line is given by

$$y - y_1 = m(x - x_1)$$

$$y + 3 = \sqrt{3}(x - 0)$$

$$y + 3 = \sqrt{3}x$$

$$\sqrt{3}x - y - 3 = 0$$

Thus, the line equation is $\sqrt{3}x - y - 3 = 0$.

16. Find the gradient of a line passing through the following pairs of points.

(i) $(0, -2), (3, 4)$

(ii) $(3, -7), (-1, 8)$

Solution:

Gradient of a line $(m) = y_2 - y_1 / x_2 - x_1$

(i) $(0, -2), (3, 4)$

$$m = (4 - (-2)) / (3 - 0) = 6/3 = 2$$

Hence, gradient = 2

(ii) $(3, -7), (-1, 8)$

$$m = (8 - (-7)) / (-1 - 3) = 15/-4$$

Hence, gradient = $-15/4$

17. The coordinates of two points E and F are $(0, 4)$ and $(3, 7)$ respectively. Find:

(i) The gradient of EF

(ii) The equation of EF

(iii) The coordinates of the point where the line EF intersects the x-axis.

Solution:

Given, co-ordinates of points E and F are $(0, 4)$ and $(3, 7)$ respectively

(i) The gradient of EF

$$m = y_2 - y_1 / x_2 - x_1 = (7 - 4) / (3 - 0) = 3/3$$

$$\Rightarrow m = 1$$

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(ii) Equation of line EF is given by,

$$y - y_1 = m(x - x_1)$$

$$y - 7 = 1(x - 3)$$

$$y - 7 = x - 3$$

$$x - y + 7 - 3 = 0$$

Hence, the equation of line EF is $x - y + 4 = 0$

(iii) It's seen that the co-ordinates of point of intersection of EF and the x-axis will be $y = 0$

So, substituting the value $y = 0$ in the above equation

$$x - y + 4 = 0$$

$$x - 0 + 4 = 0$$

$$x = -4$$

Hence, the co-ordinates are $(-4, 0)$.

18. Find the intercepts made by the line $2x - 3y + 12 = 0$ on the co-ordinate axis.

Solution:

Given line equation is $2x - 3y + 12 = 0$

On putting $y = 0$, we will get the intercept made on x-axis

$$2x - 3y + 12 = 0$$

$$2x - 3 \times 0 + 12 = 0,$$

$$2x - 0 + 12 = 0,$$

$$2x = -12$$

$$\Rightarrow x = -6$$

Now, on putting $x = 0$, we get the intercepts made on y-axis

$$2x - 3y + 12 = 0$$

$$2 \times 0 - 3y + 12 = 0$$

$$-3y = -12$$

$$\Rightarrow y = 4$$

Hence, the x-intercept and y-intercept of the given line is -6 and 4 respectively.

19. Find the equation of the line passing through the points P (5, 1) and Q (1, -1). Hence, show that the points P, Q and R (11, 4) are collinear.

Solution:

Given, two points P (5, 1) and Q (1, -1)

$$\text{Slope of the line (m)} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-1 - 1}{1 - 5}$$

$$= \frac{-2}{-4} = \frac{1}{2}$$

So, the equation of the line is

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{1}{2}(x - 5)$$

$$2y - 2 = x - 5$$

$$x - 2y - 3 = 0$$

Now, if point R (11, 4) is collinear to points P and Q then, R (11, 4) should satisfy the line equation

On substituting, we have

$$11 - 2(4) - 3 = 11 - 8 - 3 = 0$$

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As point R satisfies the line equation,
Hence, P, Q and R are collinear.

20. Find the value of 'a' for which the following points A (a, 3), B (2,1) and C (5, a) are collinear. Hence find the equation of the line.

Solution:

Given,

Points A (a, 3), B (2,1) and C (5, a) are collinear.

So, slope of AB = slope of BC

$$\frac{1-3}{2-a} = \frac{a-1}{5-2}$$

$$\frac{-2}{2-a} = \frac{a-1}{3}$$

$$\Rightarrow -6 = (a-1)(2-a) \quad [\text{On cross multiplying}]$$

$$-6 = 2a - 2 - a^2 + a$$

$$-6 = 3a - a^2 - 2$$

$$a^2 - 3a - 4 = 0$$

$$a^2 - 4a + a - 4 = 0$$

$$a(a-4) + (a-4) = 0$$

$$(a+1)(a-4) = 0$$

$$a = -1 \text{ or } 4$$

As $a = -1$ doesn't satisfy the equation

$$\Rightarrow a = 4$$

Now,

$$\text{Slope of BC} = (a-1)/(5-2) = (4-1)/3 = 3/3 = 1 = m$$

So, the equation of BC is

$$(y-1) = 1(x-2)$$

$$y-1 = x-2$$

$$x-y = -1+2$$

Thus, the equation of BC is $x-y=1$.

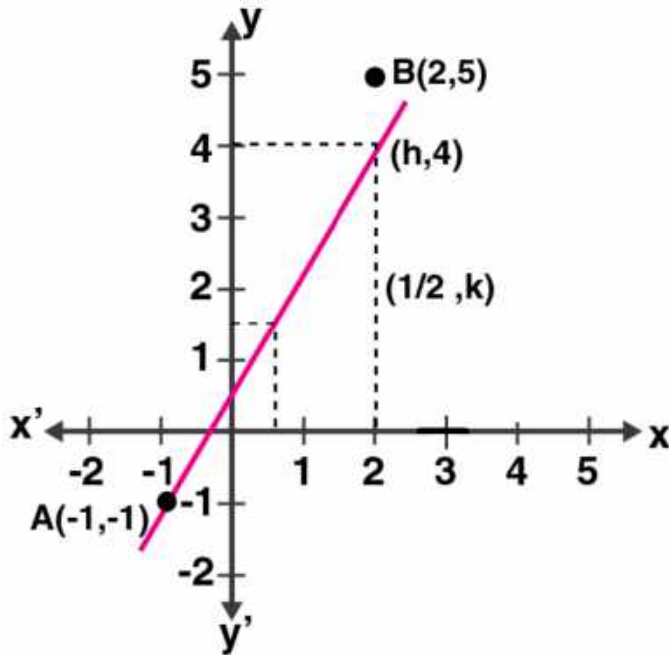
21. Use a graph paper for this question. The graph of a linear equation in x and y, passes through A (-1, -1) and B (2, 5). From your graph, find the values of h and k, if the line passes through (h, 4) and (1/2, k).

Solution:

Given,

Points (h, 4) and (1/2, k) lie on the line passing through A (-1, -1) and B (2, 5)

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From the graph, it's clearly seen that
 $h = 3/2$ and
 $k = 2$

- 22. ABCD is a parallelogram where A (x, y), B (5, 8), C (4, 7) and D (2, -4). Find**
(i) the coordinates of A
(ii) the equation of the diagonal BD.

Solution:

(i) Given,

ABCD is a parallelogram where A (x, y), B (5, 8), C (4, 7) and D (2, -4)

O is the point of intersection of the diagonals of the parallelogram.

So, the co-ordinates of O = $([5 + 2]/2, [8 - 4]/2) = (3.5, 2)$

Now, for the line AC we have

$$3.5 = \frac{(x + 4)}{2} \quad \text{and} \quad 2 = \frac{(y + 7)}{2}$$

$$7 = x + 4 \quad \text{and} \quad 4 = y + 7$$

$$x = 7 - 4 \quad \text{and} \quad y = 4 - 7$$

$$x = 3 \quad \text{and} \quad y = -3$$

Thus, the co-ordinates of A are (3, -3).

(ii) Equation of diagonal BD is given by

$$y - 8 = \frac{(-4 - 8)}{(2 - 5)} \times (x - 5)$$

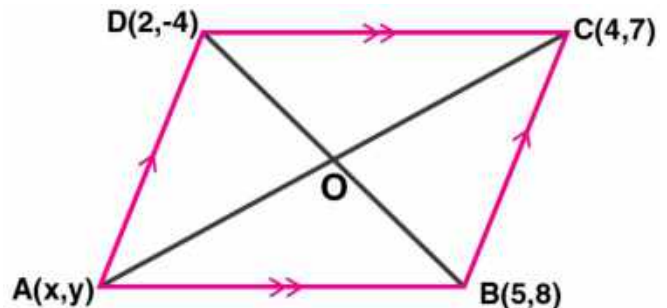
$$y - 8 = \frac{(-12/-3)}{\times} (x - 5)$$

$$y - 8 = 4(x - 5)$$

$$y - 8 = 4x - 20$$

$$4x - y - 20 + 8 = 0$$

Hence, the equation of the diagonal is $4x - y - 12 = 0$.



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23. In $\triangle ABC$, A (3, 5), B (7, 8) and C (1, -10). Find the equation of the median through A.

Solution:

Given,

$\triangle ABC$ and their vertices A (3, 5), B (7, 8) and C (1, -10).

And, AD is median

So, D is mid-point of BC

Hence, the co-ordinates of D is $(\frac{7+1}{2}, \frac{8-10}{2}) = (4, -1)$

Now,

Slope of AD, $m = \frac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{5 - (-1)}{3 - 4} = \frac{6}{-1} = -6$$

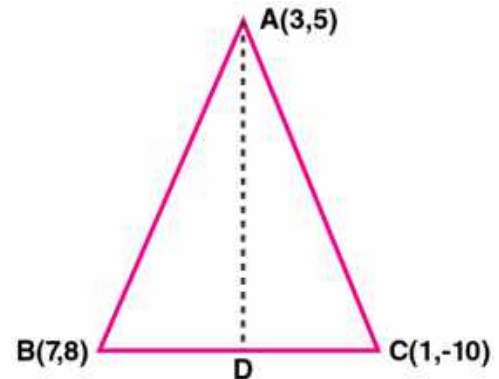
Thus, the equation of AD is given by

$$y - y_1 = m(x - x_1)$$

$$y + 1 = -6(x - 4)$$

$$y + 1 = -6x + 24$$

$$\Rightarrow 6x + y - 23 = 0$$



24. Find the equation of a line passing through the point (-2, 3) and having x-intercept 4 units.

Solution:

Given, point (-2, 3) and the x-intercept of the line passing through that point is 4 units.

So, the co-ordinates of the point where the line meets the x-axis is (4, 0)

Now, slope of the line passing through the points (-2, 3) and (4, 0)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{0 - 3}{4 - (-2)} = \frac{-3}{6} = -\frac{1}{2}$$

Hence, the equation of the line will be

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -\frac{1}{2}(x - 4)$$

$$2y = -x + 4$$

$$\Rightarrow x + 2y = 4$$

25. Find the equation of the line whose x-intercept is 6 and y-intercept is -4.

Solution:

Given, x-intercept of a line is 6

So,

The line will pass through the point (6, 0)

Also given, the y-intercept of the line is -4 $\Rightarrow c = -4$

So, the line will pass through the point (0, -4)

Now,

$$\text{Slope, } m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-4 - 0}{0 - 6} = \frac{-4}{-6} = \frac{2}{3}$$

Thus, the equation of the line is given by

$$y = mx + c$$

$$y = \left(\frac{2}{3}\right)x + (-4)$$

$$3y = 2x - 12$$

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$$\Rightarrow 2x - 3y - 12 = 0$$

26. Write down the equation of the line whose gradient is $\frac{3}{2}$ and which passes through P where P divides the line segment joining A (-2, 6) and B (3, -4) in the ratio 2 : 3.

Solution:

Given, P divides the line segment joining the points A (-2, 6) and B (3, -4) in the ratio 2: 3

So, the co-ordinates of P will be

$$\begin{aligned}x &= (m_1x_2 + m_2x_1)/(m_1 + m_2) \\&= (2 \times 3 + 3 \times (-2))/(2 + 3) \\&= (6 - 6)/5 \\&= 0/5 = 0\end{aligned}$$

$$\begin{aligned}y &= (m_1y_2 + m_2y_1)/(m_1 + m_2) \\&= (2 \times (-4) + 3 \times (6))/(2 + 3) \\&= (-8 + 18)/5 = 10/5 \\&= 2\end{aligned}$$

Hence, the co-ordinates of P are (0, 2)

Now, the slope (m) of the line passing through (0, 2) is $\frac{3}{2}$

Thus, the equation will be

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{3}{2}(x - 0)$$

$$2y - 4 = 3x$$

$$\Rightarrow 3x - 2y + 4 = 0$$

27. Find the equation of the line passing through the point (1, 4) and intersecting the line $x - 2y - 11 = 0$ on the y-axis.

Solution:

Given, line $x - 2y - 11 = 0$ passes through y-axis and point (1, 4)

So, putting $x = 0$ in the line equation we get the y-intercept

$$0 - 2y - 11 = 0$$

$$y = -11/2$$

The co-ordinates are (0, -11/2)

Now, the slope of the line joining the points (1, 4) and (0, -11/2) is given by

$$\begin{aligned}m &= \frac{y_2 - y_1}{x_2 - x_1} \\&= \frac{(-11/2 - 4)}{(0 - 1)} \\&= 19/2\end{aligned}$$

Thus, the line equation will be

$$y - y_1 = m(x - x_1)$$

$$y + 11/2 = 19/2(x - 0)$$

$$2y + 11 = 19x$$

$$\Rightarrow 19x - 2y - 11 = 0$$

28. Find the equation of the straight line containing the point (3, 2) and making positive equal intercepts on axes.

Solution:

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Let the line containing the point P (3, 2) pass through x-axis at A (x, 0) and y-axis at B (0, y)

Given, OA = OB

Thus, x = y

$$\begin{aligned} \text{Now, the slope of the line (m)} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{0 - y}{x - 0} \\ &= -\frac{x}{x} = -1 \end{aligned}$$

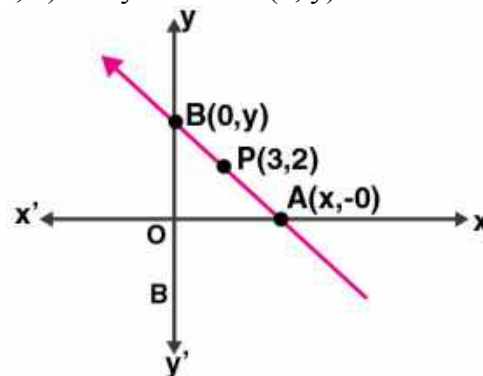
Hence, the equation of the line will be

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -1(x - 3)$$

$$y - 2 = -x + 3$$

$$\Rightarrow x + y - 5 = 0$$



29. Three vertices of a parallelogram ABCD taken in order are A (3, 6), B (5, 10) and C (3, 2) find:

(i) the coordinates of the fourth vertex D.

(ii) length of diagonal BD.

(iii) equation of side AB of the parallelogram ABCD.

Solution:

Given, the three vertices of a parallelogram ABCD taken in order are A (3, 6), B (5, 10) and C (3, 2)

(i) We know that the diagonals of a parallelogram bisect each other.

Let (x, y) be the co-ordinates of D

Hence, we have

$$\text{Mid-point of diagonal AC} = \left(\frac{3+3}{2}, \frac{6+2}{2}\right) = (3, 4)$$

$$\text{Mid-point of diagonal BD} = \left(\frac{5+x}{2}, \frac{10+y}{2}\right)$$

And, these two should be the same

On equating we get,

$$\frac{5+x}{2} = 3 \text{ and } \frac{10+y}{2} = 4$$

$$5+x = 6 \text{ and } 10+y = 8$$

$$x = 1 \text{ and } y = -2$$

Thus, the co-ordinates of D = (1, -2)

(ii) Length of diagonal BD

$$= \sqrt{(1-5)^2 + (-2-10)^2} = \sqrt{(4)^2 + (-12)^2}$$

$$= \sqrt{16+144} = \sqrt{160} \text{ units}$$

(iii) Equation of the side joining A (3, 6) and D (1, -2) is given by

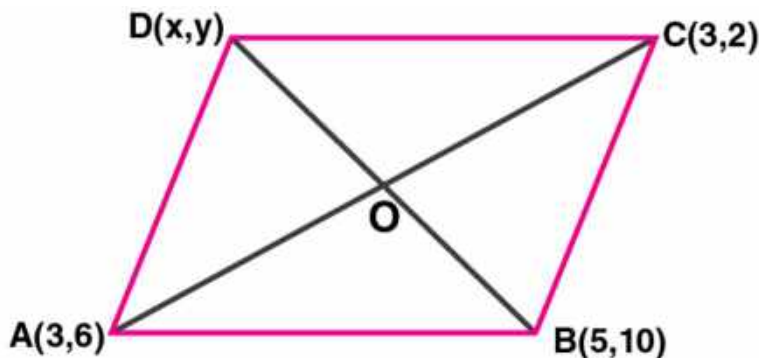
$$\frac{x-3}{3-1} = \frac{y-6}{6+2} \Rightarrow \frac{x-3}{2} = \frac{y-6}{8}$$

$$4(x-3) = y-6$$

$$4x-12 = y-6$$

$$4x-y = 6$$

Thus, the equation of the side joining A (3, 6) and D (1, -2) is $4x - y = 6$.



30. A and B are two points on the x-axis and y-axis respectively. P (2, -3) is the mid point of AB.

Find the

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- (i) the co-ordinates of A and B.
 (ii) the slope of the line AB.
 (iii) the equation of the line AB.

Solution:

Given, points A and B are on x-axis and y-axis respectively

Let co-ordinates of A be $(x, 0)$ and of B be $(0, y)$

And $P(2, -3)$ is the midpoint of AB

So, we have

$$2 = (x + 0)/2 \text{ and } -3 = (0 + y)/2$$

$$x = 4 \text{ and } y = -6$$

(i) Hence, the co-ordinates of A are $(4, 0)$ and of B are $(0, -6)$.

$$\begin{aligned} \text{(ii) Slope of AB} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-6 - 0}{0 - 4} \\ &= -6/-4 = 3/2 = m \end{aligned}$$

(iii) Equation of AB will be

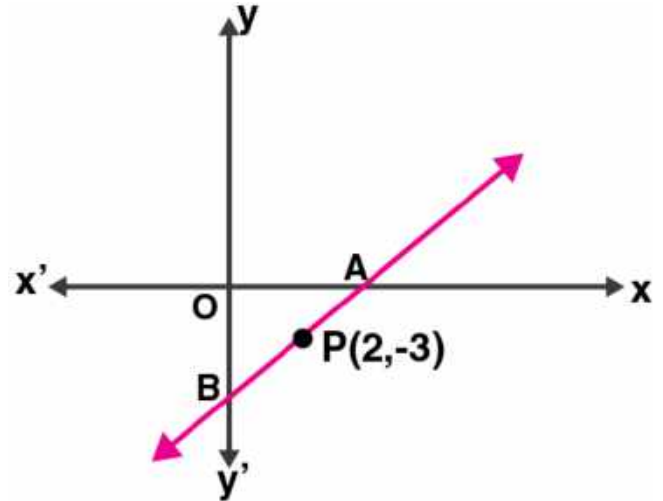
$$y - y_1 = m(x - x_1)$$

$$y - (-3) = 3/2(x - 2) \quad [\text{As } P \text{ lies on it}]$$

$$y + 3 = 3/2(x - 2)$$

$$2y + 6 = 3x - 6$$

$$3x - 2y - 12 = 0$$



31. Find the equations of the diagonals of a rectangle whose sides are $x = -1$, $x = 2$, $y = -2$ and $y = 6$.

Solution:

Given,

The equations of sides of a rectangle are

$$x_1 = -1, x_2 = 2, y_1 = -2, y_2 = 6.$$

These lines form a rectangle when they intersect at A, B, C, D respectively

Now,

The co-ordinates of A, B, C and D will be $(-1, -2)$, $(2, -2)$, $(2, 6)$ and $(-1, 6)$ respectively.

And, AC and BD are its diagonals

Slope of the diagonal AC

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{6 - (-2)}{2 - (-1)}$$

$$= 8/3 = m$$

So, the equation of AC will be

$$y - y_1 = m(x - x_1)$$

$$y + 2 = 8/3(x + 1)$$

$$3y + 6 = 8x + 8$$

$$\Rightarrow 8x - 3y + 2 = 0$$

32. Find the equation of a straight line passing through the origin and through the point of intersection of the lines $5x + 1y - 3$ and $2x - 3y = 7$

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Solution:

Given line equations,

$$5x + 7y = 3 \dots (i)$$

$$2x - 3y = 7 \dots (ii)$$

Now, performing multiplication of (i) by 3 and (ii) by 7, we get

$$15x + 21y = 9$$

$$14x - 21y = 49$$

On adding we get,

$$29x = 58$$

$$x = 58/29 = 2$$

Substituting the value of x in (i), we get

$$5(2) + 7y = 3$$

$$10 + 7y = 3$$

$$7y = 3 - 10$$

$$y = -7/7 = -1$$

Hence, the point of intersection of lines is (2, -1)

Now, the slope of the line joining the points (2, -1) and (0, 0) will be

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{0 - (-1)}{0 - 2}$$

$$= -1/2$$

Equation of the line is given by:

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -1/2(x - 0)$$

$$2y = -x$$

Thus, the required line equation is $x + 2y = 0$.

33. Point A (3, -2) on reflection in the x-axis is mapped as A' and point B on reflection in the y-axis is mapped onto B' (-4, 3).

(i) Write down the co-ordinates of A' and B.

(ii) Find the slope of the line A'B, hence find its inclination.

Solution:

Given,

A' is the image of A (3, -2) on reflection in the x-axis.

(i) The co-ordinates of A' will be (3, 2).

Again B' (-4, 3) is the image of A', when reflected in the y-axis

Hence, the co-ordinates of B will be (4, 3)

(ii) Slope of the line joining, the points A' (3, 2) and B (4, 3) will be

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2 - 3}{3 - 4}$$

$$= -1/-1 = 1$$

So, $\tan \theta = 45^\circ$

Thus, the angle of inclination is 45° .

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Exercise 12.2

1. State which one of the following is true: The straight lines $y = 3x - 5$ and $2y = 4x + 7$ are

(i) parallel

(ii) perpendicular

(iii) neither parallel nor perpendicular.

Solution:

Given straight lines: $y = 3x - 5$ and $2y = 4x + 7 \Rightarrow y = 2x + 7/2$

And, their slopes are 3 and 2

The product of slopes is $3 \times 2 = 6$.

Hence, as the slopes of both the lines are neither equal nor their product is -1 the given pair of straight lines are neither parallel nor perpendicular.

2. If $6x + 5y - 7 = 0$ and $2px + 5y + 1 = 0$ are parallel lines, find the value of p.

Solution:

For two lines to be parallel, their slopes must be same.

Given line equations,

$6x + 5y - 7 = 0$ and $2px + 5y + 1 = 0$

In equation $6x + 5y - 7 = 0$,

$5y = -6x + 7$

$y = (-6/5)x + 7/5$

So, the slope of the line (m_1) = $-6/5$

Again, in equation $2px + 5y + 1 = 0$

$5y = -2px - 1$

$y = (-2p/5)x - 1/5$

So, the slope of the line (m_2) = $-2p/5$

For these two lines to be parallel

$m_1 = m_2$

$-6/5 = -2p/5$

$p = (-6/5) \times (-5/2)$

Thus, $p = 3$

3. Lines $2x - by + 5 = 0$ and $ax + 3y = 2$ are parallel. Find the relation connecting a and b.

Solution:

Given lines are: $2x - by + 5 = 0$ and $ax + 3y = 2$

If two lines to be parallel then their slopes must be equal.

In equation $2x - by + 5 = 0$,

$by = 2x + 5$

$y = (2/b)x + 5/b$

So, the slope of the line (m_1) = $2/b$

And in equation $ax + 3y = 2$,

$3y = -ax + 2$

$y = (-a/3)x + 2/3$

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So, the slope of the line (m_2) = $(-a/3)$

As the lines are parallel

$$m_1 = m_2$$

$$2/b = -a/3$$

$$6 = -ab$$

Hence, the relation connecting a and b is $ab + 6 = 0$

4. Given that the line $y/2 = x - p$ and the line $ax + 5 = 3y$ are parallel, find the value of a.

Solution:

Given,

Line equation: $y/2 = x - p$

$$\Rightarrow y = 2x - 2p$$

Here, the slope of the line is 2.

And, another line equation: $ax + 5 = 3y$

$$\Rightarrow 3y = ax + 5$$

$$y = (a/3)x + 5/3$$

Hence, the slope of the line is $a/3$

As the line are parallel, their slopes must be equal

$$\Rightarrow 2 = a/3$$

$$a = 6$$

Thus, the value of a is 6.

5. If the lines $y = 3x + 7$ and $2y + px = 3$ perpendicular to each other, find the value of p.

Solution:

If two lines are perpendicular, then the product of their slopes is -1

Now, slope of the line $y = 3x + 7$ is $m_1 = 3$

And,

The slope of the line: $2y + px = 3$

$$2y = -px + 3$$

$$y = (-p/2)x + 3$$

$$m_2 = -p/2$$

As the lines as perpendicular,

$$\Rightarrow m_1 \times m_2 = -1$$

$$3 \times (-p/2) = -1$$

$$p = 2/3$$

Thus, the value of p is $2/3$.

6. If the straight lines $kx - 5y + 4 = 0$ and $4x - 2y + 5 = 0$ are perpendicular to each other. Find the value of k.

Solution:

Given,

In equation, $kx - 5y + 4 = 0$

$$\Rightarrow 5y = kx + 4$$

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$$y = (k/5)x + 4/5$$

So, the slope (m_1) = $k/5$

And, in equation $4x - 2y + 5 = 0$

$$\Rightarrow 2y = 4x + 5$$

$$y = 2x + 5/2$$

So, the slope (m_2) = 2

As the lines are perpendicular to each other

$$\Rightarrow m_1 \times m_2 = -1$$

$$k/5 \times 2 = -1$$

$$k = (-1 \times 5)/2$$

Hence, the value of $k = -5/2$

7. If the lines $3x + by + 5 = 0$ and $ax - 5y + 7 = 0$ are perpendicular to each other, find the relation connecting a and b .

Solution:

Given that the lines $3x + by + 5 = 0$ and $ax - 5y + 7 = 0$ are perpendicular to each other

Then the product of their slopes must be -1.

Slope of line $3x + by + 5 = 0$ is,

$$by = -3x - 5$$

$$y = (-3/b)x - 5/b$$

So, slope (m_1) = $-3/b$

And,

The slope of line $ax - 5y + 7 = 0$ is

$$5y = ax + 7$$

$$y = (a/5)x + 7/5$$

So, slope (m_2) = $a/5$

As the lines are perpendicular, we have

$$m_1 \times m_2 = -1$$

$$-3/b \times a/5 = -1$$

$$-3a/5b = -1$$

$$-3a = -5b$$

$$3a = 5b$$

Hence, the relation connecting a and b is $3a = 5b$.

**8. Is the line through $(-2, 3)$ and $(4, 1)$ perpendicular to the line $3x = y + 1$?
Does the line $3x = y + 1$ bisect the join of $(-2, 3)$ and $(4, 1)$.**

Solution:

Slope of the line passing through the points $(-2, 3)$ and $(4, 1)$ is given by

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{1 - 3}{4 - (-2)}$$

$$= \frac{-2}{6}$$

$$= -1/3$$

And, the slope of the line: $3x = y + 1$

$$y = 3x - 1$$

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$$\text{Slope } (m_2) = 3$$

Now,

$$m_1 \times m_2 = -1/3 \times 3 = -1$$

Thus, the lines are perpendicular to each other as the product of their slopes is -1.

Now,

Co-ordinates of the mid-point of the line joining the points (-2, 3) and (4, 1) is

$$([-2 + 4]/2, [3 + 1]/2) = (1, 2)$$

Now, if the line $3x = y + 1$ passes through the mid-point then it will satisfy the equation

$$3(1) = (2) + 1$$

$$3 = 3$$

Hence, the line $3x = y + 1$ bisects the line joining the points (-2, 3) and (4, 1).

9. The line through A (-2, 3) and B (4, b) is perpendicular to the line $2x - 4y = 5$. Find the value of b.

Solution:

The slope of the line passing through A (-2, 3) and B (4, b) will be

$$m_1 = (b - 3) / (4 + 2) = (b - 3) / 6$$

Now, the gradient of the given line $2x - 4y = 5$ is

$$4y = 2x + 5$$

$$y = (2/4)x + 5/4$$

$$y = 1/2 x + 5/4$$

$$\text{So, } m_2 = 1/2$$

As the lines are perpendicular to each other, we have

$$m_1 \times m_2 = -1$$

$$(b - 3) / 6 \times 1/2 = -1$$

$$(b - 3) / 12 = -1$$

$$b - 3 = -12$$

$$b = -12 + 3 = -9$$

Hence, the value of b is -9.

10. If the lines $3x + y = 4$, $x - ay + 7 = 0$ and $bx + 2y + 5 = 0$ form three consecutive sides of a rectangle, find the value of a and b.

Solution:

Given lines are:

$$3x + y = 4 \dots \text{(i)}$$

$$x - ay + 7 = 0 \dots \text{(ii)}$$

$$bx + 2y + 5 = 0 \dots \text{(iii)}$$

It's said that these lines form three consecutive sides of a rectangle.

So,

Lines (i) and (ii) must be perpendicular

Also, lines (ii) and (iii) must be perpendicular

We know that, for two perpendicular lines the product of their slopes will be -1.

Now,

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Slope of line (i) is

$$3x + y = 4 \Rightarrow y = -3x + 4$$

Hence, slope (m_1) = -3

And, slope of line (ii) is

$$x - ay + 7 = 0 \Rightarrow ay = x + 7$$

$$y = (1/a)x + 7/a$$

Hence, slope (m_2) = $1/a$

Finally, the slope of line (iii) is

$$bx + 2y + 5 = 0 \Rightarrow 2y = -bx - 5$$

$$y = (-b/2)x - 5/2$$

Hence, slope (m_3) = $-b/2$

As lines (i), (ii) and (iii) are consecutive sides of rectangle, we have

$$m_1 \times m_2 = -1 \quad \text{and} \quad m_2 \times m_3 = -1$$

$$(-3) \times (1/a) = -1 \quad \text{and} \quad (1/a) \times (-b/2) = -1$$

$$-3 = -a \quad \text{and} \quad -b/2a = -1$$

$$a = 3 \quad \text{and} \quad b = 2a \Rightarrow b = 2(3) = 6$$

Thus, the value of a is 3 and the value of b is 6.

11. Find the equation of a line, which has the y-intercept 4, and is parallel to the line $2x - 3y - 7 = 0$. Find the coordinates of the point where it cuts the x-axis.

Solution:

Given line: $2x - 3y - 7 = 0$

Its slope is,

$$3y = 2x - 7$$

$$y = (2/3)x - 7/3$$

$$\Rightarrow m = 2/3$$

So, the equation of the line parallel to the given line will be $2/3$

Also given, the y-intercept is $4 = c$

Hence, the equation of the line is given by

$$y = mx + c$$

$$y = (2/3)x + 4$$

$$3y = 2x + 12$$

$$2x - 3y + 12 = 0$$

Now, when this line intersects the x-axis the y co-ordinate becomes zero.

So, putting $y = 0$ in the line equation, we get

$$2x - 3(0) + 12 = 0$$

$$2x + 12 = 0$$

$$x = -12/2 = -6$$

Hence, the co-ordinates of the point where it cuts the x-axis is $(-6, 0)$.

12. Find the equation of a straight line perpendicular to the line $2x + 5y + 7 = 0$ and with y-intercept -3 units.

Solution:

Given line: $2x + 5y + 7 = 0$

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So, its slope is given by

$$5y = -2x - 7$$

$$y = (-2/5)x - 7/5$$

$$\Rightarrow m = -2/5$$

Now, let the slope of the line perpendicular to this line be m'

Then,

$$m \times m' = -1$$

$$(-2/5) \times m' = -1$$

$$\Rightarrow m' = 5/2$$

Also given, the y-intercept (c) = -3

Hence, the equation of the line is given by

$$y = m'x + c$$

$$y = (5/2)x + (-3)$$

$$2y = 5x - 6$$

$$5x - 2y - 6 = 0$$

13. Find the equation of a st. line perpendicular to the line $3x - 4y + 12 = 0$ and having same y-intercept as $2x - y + 5 = 0$.

Solution:

Given line: $3x - 4y + 12 = 0$

The slope of the line is given by

$$3x - 4y + 12 = 0 \Rightarrow 4y = 3x + 12$$

$$y = (3/4)x + 3$$

Thus, slope (m_1) = $3/4$

Now, let the slope of the line perpendicular to the given line be taken as m_2

So,

$$m_1 \times m_2 = -1$$

$$(3/4) \times m_2 = -1$$

$$m_2 = -4/3$$

And given, the y-intercept of the line is same as $2x - y + 5 = 0$

$$\Rightarrow y = 2x + 5$$

So, the y-intercept is $5 = c$.

Hence, the equation of line is given by

$$y = m_2x + c$$

$$y = (-4/3)x + 5$$

$$3y = -4x + 15$$

$$4x + 3y = 15$$

14. Find the equation of the line which is parallel to $3x - 2y = -4$ and passes through the point (0, 3).

Solution:

Given line: $3x - 2y = -4$

Slope (m_1) is given by

$$2y = 3x + 4$$

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$$y = \left(\frac{3}{2}\right)x + 2$$

$$\text{So, } m_1 = \frac{3}{2}$$

Now, the slope of the line parallel to the given line will have the same slope as $\frac{3}{2} = m$

And the line passes through point $(0, 3)$

Thus, the equation of the required line is given by

$$y = mx + c$$

$$y = \left(\frac{3}{2}\right)x + 3$$

$$2y = 3x + 6$$

$$3x - 2y + 6 = 0$$

15. Find the equation of the line passing through $(0, 4)$ and parallel to the line $3x + 5y + 15 = 0$.

Solution:

$$\text{Given line: } 3x + 5y + 15 = 0$$

$$5y = -3x - 15$$

$$y = \left(-\frac{3}{5}\right)x - 3$$

$$\text{So, slope (m)} = -\frac{3}{5}$$

The slope of the line parallel to the given line will be the same $-\frac{3}{5}$

And, the line passes through the point $(0, 4)$

Hence, equation of the line will be

$$y - y_1 = m(x - x_1)$$

$$y - 4 = \left(-\frac{3}{5}\right)(x - 0)$$

$$5y - 20 = -3x$$

$$3x + 5y - 20 = 0$$



16. The equation of a line is $y = 3x - 5$. Write down the slope of this line and the intercept made by it on the y-axis. Hence or otherwise, write down the equation of a line which is parallel to the line and which passes through the point $(0, 5)$.

Solution:

$$\text{Given line: } y = 3x - 5$$

$$\text{Here slope (m}_1) = 3$$

$$\text{Substituting } x = 0, \text{ we get } y = -5$$

$$\text{Hence, the y-intercept} = -5$$

Now, the slope of the line parallel to the given line will be 3 and it passes through the point $(0, 5)$.

Thus, equation of the line will be

$$y - y_1 = m(x - x_1)$$

$$y - 5 = 3(x - 0)$$

$$y = 3x + 5$$

17. Write down the equation of the line perpendicular to $3x + 8y = 12$ and passing through the point $(-1, -2)$.

Solution:

$$\text{Given line: } 3x + 8y = 12$$

$$8y = -3x + 12$$

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$$y = (-3/8)x + 12$$

So, the slope (m_1) = $-3/8$

Let's consider the slope of the line perpendicular to the given line as m_2

$$\text{Then, } m_1 \times m_2 = -1$$

$$-3/8 \times m_2 = -1$$

$$m_2 = 8/3$$

Now,

The equation of the line perpendicular to the given line and passing through the point $(-1, -2)$ will be

$$y - y_1 = m(x - x_1)$$

$$y - (-2) = (8/3)(x - (-1))$$

$$y + 2 = (8/3)(x + 1)$$

$$3y + 6 = 8x + 8$$

$$3y = 8x + 2$$

Thus, the equation of the required line is $3y = 8x + 2$.

18. (i) The line $4x - 3y + 12 = 0$ meets the x-axis at A. Write down the co-ordinates of A.

(ii) Determine the equation of the line passing through A and perpendicular to $4x - 3y + 12 = 0$.

Solution:

Given line: $4x - 3y + 12 = 0$

(i) When this line meets the x-axis, its y co-ordinate becomes 0.

So, putting $y = 0$ in the given equation, we get

$$4x - 3(0) + 12 = 0$$

$$4x + 12 = 0$$

$$x = -12/4$$

$$x = -3$$

Hence, the line meets the x-axis at A $(-3, 0)$.

(ii) Now, the slope of the line is given by

$$4x - 3y + 12 = 0$$

$$3y = 4x + 12$$

$$y = (4/3)x + 4$$

$$\Rightarrow m_1 = 4/3$$

Let's assume the slope of the line perpendicular to the given line be m_2

$$\text{Then, } m_1 \times m_2 = -1$$

$$4/3 \times m_2 = -1$$

$$m_2 = -3/4$$

Thus, the equation of the line perpendicular to the given line passing through A will be

$$y - 0 = -3/4(x + 3)$$

$$4y = -3(x + 3)$$

$$3x + 4y + 9 = 0$$

19. Find the equation of the line that is parallel to $2x + 5y - 7 = 0$ and passes through the mid-point of the line segment joining the points $(2, 7)$ and $(-4, 1)$.

Solution:

Given line: $2x + 5y - 7 = 0$

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$$5y = -2x + 7$$

$$y = (-2/5)x + 7/5$$

So, the slope is $-2/5$

Hence, the slope of the line that is parallel to the given line will be the same, $m = -2/5$

Now, the mid-point of the line segment joining points $(2, 7)$ and $(-4, 1)$ is

$$((2 - 4)/2, (7 + 1)/2) = (-1, 4)$$

Thus, the equation of the line will be

$$y - y_1 = m(x - x_1)$$

$$y - 4 = (-2/5)(x + 1)$$

$$5y - 20 = -2x - 2$$

$$2x + 5y = 18$$

20. Find the equation of the line that is perpendicular to $3x + 2y - 8 = 0$ and passes through the mid-point of the line segment joining the points $(5, -2)$ and $(2, 2)$.

Solution:

Given line: $3x + 2y - 8 = 0$

$$2y = -3x + 8$$

$$y = (-3/2)x + 4$$

Here, slope $(m_1) = -3/2$

Now, the co-ordinates of the mid-point of the line segment joining the points $(5, -2)$ and $(2, 2)$ will be

$$((5 + 2)/2, (-2 + 2)/2) = (7/2, 0)$$

Let's consider the slope of the line perpendicular to the given line be m_2

Then,

$$m_1 \times m_2 = -1$$

$$(-3/2) \times m_2 = -1$$

$$m_2 = 2/3$$

So, the equation of the line with slope m_2 and passing through $(7/2, 0)$ will be

$$y - 0 = (2/3)(x - 7/2)$$

$$3y = 2x - 7$$

$$2x - 3y - 7 = 0$$

Thus, the required line equation is $2x - 3y - 7 = 0$.

21. Find the equation of a straight line passing through the intersection of $2x + 5y - 4 = 0$ with x-axis and parallel to the line $3x - 7y + 8 = 0$.

Solution:

Let's assume the point of intersection of the line $2x + 5y - 4 = 0$ and x-axis be $(x, 0)$

Now, substituting the value $y = 0$ in the line equation, we have

$$2x + 5(0) - 4 = 0$$

$$2x - 4 = 0$$

$$x = 4/2 = 2$$

Hence, the co-ordinates of the point of intersection is $(2, 0)$

Also given, line equation: $3x - 7y + 8 = 0$

$$7y = 3x + 8$$

$$y = (3/7)x + 8/7$$

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So, the slope (m) = $3/7$

We know that the slope of any line parallel to the given line will be the same.

So, the equation of the line having slope $3/7$ and passing through the point $(2, 0)$ will be

$$y - 0 = (3/7)(x - 2)$$

$$7y = 3x - 6$$

$$3x - 7y - 6 = 0$$

Thus, the required line equation is $3x - 7y - 6 = 0$.

22. The equation of a line is $3x + 4y - 7 = 0$. Find (i) the slope of the line. (ii) the equation of a line perpendicular to the given line and passing through the intersection of the lines $x - y + 2 = 0$ and $3x + y - 10 = 0$.

Solution:

Given line equation: $3x + 4y - 7 = 0$

(i) Slope of the line is given by,

$$4y = -3x + 7$$

$$y = (-3/4)x + 7/4$$

Hence, slope (m_1) = $-3/4$

(ii) Let the slope of the perpendicular to the given line be m_2

Then, $m_1 \times m_2 = -1$

$$(-3/4) \times m_2 = -1$$

$$m_2 = 4/3$$

Now, to find the point of intersection of

$$x - y + 2 = 0 \dots (i)$$

$$3x + y - 10 = 0 \dots (ii)$$

On adding (i) and (ii), we get

$$4x - 8 = 0$$

$$4x = 8$$

$$x = 8/4 = 2$$

Putting $x = 2$ in (i), we get

$$2 - y + 2 = 0$$

$$y = 4$$

Hence, the point of intersection of the lines is $(2, 4)$

The equation of the line having slope m_2 and passing through $(2, 4)$ will be

$$y - 4 = (4/3)(x - 2)$$

$$3y - 12 = 4x - 8$$

$$4x - 3y + 4 = 0$$

Thus, the required line equation is $4x - 3y + 4 = 0$.

23. Find the equation of the line perpendicular from the point $(1, -2)$ on the line $4x - 3y - 5 = 0$. Also find the co-ordinates of the foot of perpendicular.

Solution:

Given line equation: $4x - 3y - 5 = 0$

$$3y = 4x - 5$$

$$y = (4/3)x - 5/3$$

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Slope of the line (m_1) = $4/3$

Let the slope of the line perpendicular to the given line be m_2

Then, $m_1 \times m_2 = -1$

$$(4/3) \times m_2 = -1$$

$$m_2 = -3/4$$

Now, the equation of the line having slope m_2 and passing through the point (1, -2) will be

$$y + 2 = (-3/4)(x - 1)$$

$$4y + 8 = -3x + 3$$

$$3x + 4y + 5 = 0$$

Next, for finding the co-ordinates of the foot of the perpendicular which is the point of intersection of the lines

$$4x - 3y - 5 = 0 \dots (1) \text{ and}$$

$$3x + 4y + 5 = 0 \dots (2)$$

On multiplying (1) by 4 and (2) by 3, we get

$$16x - 12y - 20 = 0$$

$$9x + 12y + 15 = 0$$

Adding we get,

$$25x - 5 = 0$$

$$x = 5/25$$

$$x = 1/5$$

Putting the value of x in (1), we have

$$4(1/5) - 3y - 5 = 0$$

$$4/5 - 3y - 5 = 0$$

$$3y = 4/5 - 5 = (4 - 25)/5$$

$$3y = -21/5$$

$$y = -7/5$$

Thus, the co-ordinates are $(1/5, -7/5)$

24. Prove that the line through (0, 0) and (2, 3) is parallel to the line through (2, -2) and (6, 4).

Solution:

Let the slope of the line through (0, 0) and (2, 3) be m_1

$$\text{So, } m_1 = (y_2 - y_1) / (x_2 - x_1)$$

$$= (3 - 0) / (2 - 0)$$

$$= 3/2$$

And, let the slope of the line through (2, -2) and (6, 4) be m_2

$$\text{So, } m_2 = (y_2 - y_1) / (x_2 - x_1)$$

$$= (4 + 2) / (6 - 2)$$

$$= 6/4 = 3/2$$

It's clearly seen that the slopes $m_1 = m_2$

Thus, the lines are parallel to each other.

25. Prove that the line through (-2, 6) and (4, 8) is perpendicular to the line through (8, 12) and (4, 24).

Solution:

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Let the slope of the line through points $(-2, 6)$ and $(4, 8)$ be m_1

$$\begin{aligned}\text{So, } m_1 &= (y_2 - y_1) / (x_2 - x_1) \\ &= (8 - 6) / (4 + 2) \\ &= 2/6 \\ &= 1/3\end{aligned}$$

And, let the slope of the line through $(8, 12)$ and $(4, 24)$ be m_2

$$\begin{aligned}\text{So, } m_2 &= (y_2 - y_1) / (x_2 - x_1) \\ &= (24 - 12) / (4 - 8) \\ &= 12 / (-4) \\ &= -3\end{aligned}$$

Now, product of slopes is

$$m_1 \times m_2 = 1/3 \times (-3) = -1$$

Thus, the lines are perpendicular to each other.

26. Show that the triangle formed by the points A (1, 3), B (3, -1) and C (-5, -5) is a right-angled triangle by using slopes.

Solution:

Given, points A (1, 3), B (3, -1) and C (-5, -5) form a triangle

Now,

$$\text{Slope of the line AB} = m_1 = (-1 - 3) / (3 - 1) = -4/2 = -2$$

And,

$$\text{Slope of the line BC} = m_2 = (-5 + 1) / (-5 - 3) = -4/-8 = 1/2$$

Hence,

$$m_1 \times m_2 = (-2) \times (1/2) = -1$$

So, the lines AB and BC are perpendicular to each other.

Therefore, ΔABC is a right-angled triangle.

27. Find the equation of the line through the point $(-1, 3)$ and parallel to the line joining the points $(0, -2)$ and $(4, 5)$.

Solution:

Slope of the line joining the points $(0, -2)$ and $(4, 5)$ is

$$\begin{aligned}m &= (5 + 2) / (4 - 0) \\ &= 7/4\end{aligned}$$

Now, the slope of the line parallel to it and passing through $(-1, 3)$ will be also be $7/4$

Hence, the equation of the line is

$$y - y_1 = m(x - x_1) \Rightarrow y - 3 = 7/4(x + 1)$$

$$4y - 12 = 7x + 7$$

$$7x - 4y + 19 = 0$$

28. are the vertices of a triangle.

(i) Find the coordinates of the centroid G of the triangle.

(ii) Find the equation of the line through G and parallel to AC.

Solution:

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Given, A (-1, 3), B (4, 2), C (3, -2)

(i) Co-ordinates of centroid G is

$$\begin{aligned} G(x, y) &= ((x_1 + x_2 + x_3)/3, (y_1 + y_2 + y_3)/3) \\ &= ((-1 + 4 + 3)/3, (3 + 2 - 2)/3) \\ &= (6/3, 3/3) = (2, 1) \end{aligned}$$

Hence, the co-ordinates of the centroid G of the triangle is (2, 1)

(ii) Slope of AC = $(y_2 - y_1)/(x_2 - x_1) = (-2 - 3)/(3 - (-1)) = -5/4$

So, the slope of the line parallel to AC is also -5/4

Now, the equation of line through G is

$$y - 1 = (-5/4)(x - 2)$$

$$4y - 4 = -5x + 10$$

$$5x + 4y = 14$$

Thus, the required line equation is $5x + 4y = 14$.

29. The line through P (5, 3) intersects y-axis at Q. (i) Write the slope of the line. (ii) Write the equation of the line. (iii) Find the coordinates of Q.

Solution:

(i) Here, $\theta = 45^\circ$

So, the slope of the line = $\tan \theta = \tan 45^\circ = 1$

(ii) Equation of the line through P and Q is

$$y - 3 = 1(x - 5)$$

$$x - y - 2 = 0$$

(iii) Let the co-ordinates of Q be (0, y)

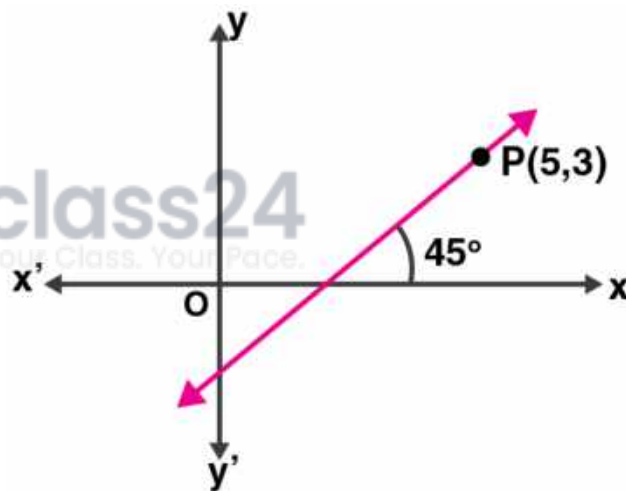
Then, $m = (y_2 - y_1)/(x_2 - x_1)$

$$1 = (3 - y)/(5 - 0)$$

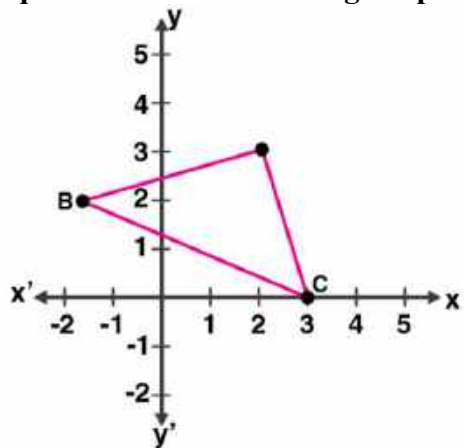
$$5 = 3 - y$$

$$y = 3 - 5 = -2$$

Thus, co-ordinates of Q are (0, -2).



30. In the adjoining diagram, write down (i) the co-ordinates of the points A, B and C. (ii) the equation of the line through A parallel to BC.



Solution:

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From the given figure, it's clearly seen that

Co-ordinates of A are (2, 3) and of B are (-1, 2) and of C are (3, 0).

Now,

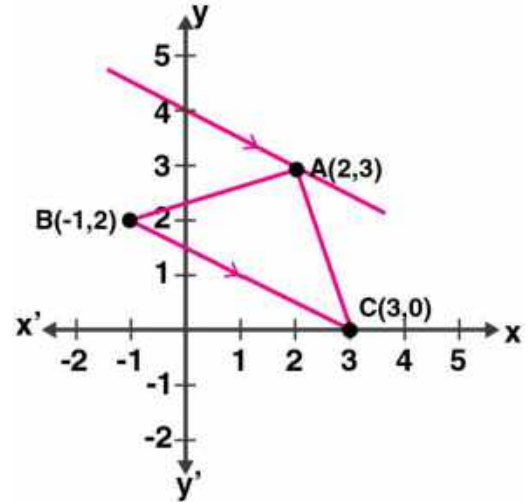
$$\begin{aligned}\text{Slope of BC} &= (0 - 2) / (3 - (-1)) \\ &= -2/4 \\ &= -1/2\end{aligned}$$

So, the slope of the line parallel to BC is also $-1/2$

And, the line passes through A (2, 3)

Hence, the equation will be

$$\begin{aligned}y - 3 &= (-1/2)(x - 2) \\ 2y - 6 &= -x + 2 \\ x + 2y &= 8\end{aligned}$$



31. Find the equation of the line through (0, -3) and perpendicular to the line joining the points (-3, 2) and (9, 1).

Solution:

The slope of the line joining the points (-3, 2) and (9, 1) is

$$m_1 = (1 - 2) / (9 + 3) = -1/12$$

Now, let the slope of the line perpendicular to the above line be m_2

$$\text{Then, } m_1 \times m_2 = -1$$

$$(-1/12) \times m_2 = -1$$

$$m_2 = 12$$

So, the equation of the line passing through (0, -3) and having slope of m_2 will be

$$y - (-3) = 12(x - 0)$$

$$y + 3 = 12x$$

$$12x - y = 3$$

Thus, the required line equation is $12x - y = 3$.

32. The vertices of a triangle are A (10, 4), B (4, -9) and C (-2, -1). Find the equation of the altitude through A. The perpendicular drawn from a vertex of a triangle to the opposite side is called altitude.

Solution:

Given, vertices of a triangle are A (10, 4), B (4, -9) and C (-2, -1)

Now,

$$\text{Slope of line BC } (m_1) = (-1 + 9) / (-2 - 4) = 8 / (-6) = -4/3$$

Let the slope of the altitude from A (10, 4) to BC be m_2

$$\text{Then, } m_1 \times m_2 = -1$$

$$(-4/3) \times m_2 = -1$$

$$m_2 = 3/4$$

So, the equation of the line will be

$$y - 4 = 3/4(x - 10)$$

$$4y - 16 = 3x - 30$$

$$3x - 4y - 14 = 0$$

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33. A (2, -4), B (3, 3) and C (-1, 5) are the vertices of triangle ABC. Find the equation of:

- (i) the median of the triangle through A
- (ii) the altitude of the triangle through B.

Solution:

Given, A (2, -4), B (3, 3) and C (-1, 5) are the vertices of triangle ABC

(i) D is the mid-point of BC

So, the co-ordinates of D will be

$$((3 - 1)/2, (3 + 5)/2) = (2/2, 8/2) = (1, 4)$$

Now,

The slope of AC (m_1) = $(5 + 4)/(-1 - 2) = 9/-3 = -3$

Let the slope of BE be m_2

Then, $m_1 \times m_2 = -1$

$$-3 \times m_2 = -1$$

$$m_2 = 1/3$$

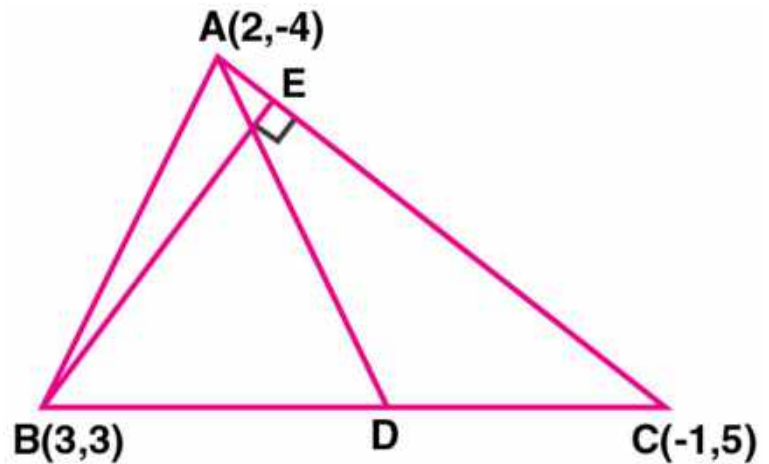
so, the equation of BE will be

$$y - 3 = 1/3 (x - 3)$$

$$3y - 9 = x - 3$$

$$x - 3y + 6 = 0$$

Thus, the required line equation is $x - 3y + 6 = 0$.



34. Find the equation of the right bisector of the line segment joining the points (1, 2) and (5, -6).

Solution:

The slope of the line joining the points (1, 2) and (5, -6) is

$$m_1 = (-6 - 2)/(5 - 1) = -8/4 = -2$$

Now, if m_2 is the slope of the right bisector of the above line

Then,

$$m_1 \times m_2 = -1$$

$$-2 \times m_2 = -1$$

$$m_2 = 1/2$$

The mid-point of the line segment joining (1, 2) and (5, -6) will be

$$((1 + 5)/2, (2 - 6)/2) = (6/2, -4/2) = (3, -2)$$

So, equation of the line is

$$y + 2 = 1/2 (x - 3)$$

$$2y + 4 = x - 3$$

$$x - 2y - 7 = 0$$

Thus, the equation of the required right bisector is $x - 2y - 7 = 0$.

35. Points A and B have coordinates (7, -3) and (1, 9) respectively. Find

(i) the slope of AB.

(ii) the equation of the perpendicular bisector of the line segment AB.

(iii) the value of 'p' if (-2, p) lies on it.

Solution:

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Given, co-ordinates of points A are (7, -3) and of B are (1, 9)

(i) The slope of AB (m) = $(9 + 3) / (1 - 7) = 12 / (-6) = -2$

(ii) Let PQ be the perpendicular bisector of AB intersecting it at M

Now, the co-ordinates of M will be the mid-point of AB

Co-ordinates of M will be

$$= (7 + 1)/2, (-3 + 9)/2 = 8/2, 6/2$$

$$= (4, 3)$$

The slope of line PQ will be = $-1/m = -1/(-2) = 1/2$

Thus, the equation of PQ is

$$y - 3 = 1/2 (x - 4)$$

$$2y - 6 = x - 4$$

$$x - 2y + 2 = 0$$

(iii) As point (-2, p) lies on the above line

The point will satisfy the line equation

$$-2 - 2p + 2 = 0$$

$$-2p = 0$$

$$p = 0$$

Thus, the value of p is 0.

36. The points B (1, 3) and D (6, 8) are two opposite vertices of a square ABCD. Find the equation of the diagonal AC.

Solution:

Given, points B (1, 3) and D (6, 8) are two opposite vertices of a square ABCD

Slope of BD is given by

$$m_1 = (8 - 3) / (6 - 1) = 5/5 = 1$$

We know that, the diagonal AC is a perpendicular bisector of diagonal BD

So, the slope of AC (m_2) will be

$$m_1 \times m_2 = -1$$

$$1 \times m_2 = -1$$

$$m_2 = -1$$

And, the co-ordinates of mid-point of BD and AC will be

$$((1 + 6)/2, (3 + 8)/2) = (7/2, 11/2)$$

So, the equation of AC is

$$y - 11/2 = -1 (x - 7/2)$$

$$2y - 11 = -2x - 7$$

$$2x + 2y - 7 - 11 = 0 \Rightarrow 2x + 2y - 18 = 0$$

Thus, the equation of diagonal AC is $x + y - 9 = 0$.

37. ABCD is a rhombus. The co-ordinates of A and C are (3, 6) and (-1, 2) respectively. Write down the equation of BD.

Solution:

Given, ABCD is a rhombus and co-ordinates of A are (3, 6) and of C are (-1, 2)

$$\text{Slope of AC } (m_1) = (2 - 6) / (-1 - 3) = -4/-4 = 1$$

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We know that, the diagonals of a rhombus bisect each other at right angles.

So, the diagonal BD is perpendicular to diagonal AC

Let the slope of BD be m_2

Then, $m_1 \times m_2 = -1$

$$m_2 = -1/(m_1)$$

$$= -1/(1) = -1$$

Now, the co-ordinates of the mid-point of AC is given by

$$((3 - 1)/2, (6 + 2)/2) = (2/2, 8/2) = (1, 4)$$

So, the equation of BD will be

$$y - 4 = -1 (x - 1)$$

$$y - 4 = -x + 1$$

$$x + y = 5$$

Thus, the equation of BD is $x + y = 5$.

38. Find the equation of the line passing through the intersection of the lines $4x + 3y = 1$ and $5x + 4y = 2$ and

(i) parallel to the line $x + 2y - 5 = 0$

(ii) perpendicular to the x-axis.

Solution:

Given, line equations:

$$4x + 3y = 1 \dots (1)$$

$$5x + 4y = 2 \dots (2)$$

On solving the above equation to find the point of intersection, we have

Multiplying (1) by 4 and (2) by 3

$$16x + 12y = 4$$

$$15x + 12y = 6$$

On subtracting, we get

$$x = -2$$

Putting the value of x in (1), we have

$$4(-2) + 3y = 1$$

$$-8 + 3y = 1$$

$$3y = 1 + 8 = 9$$

$$y = 9/3 = 3$$

Hence, the point of intersection is $(-2, 3)$.

(i) Given line, $x + 2y - 5 = 0$

$$2y = -x + 5$$

$$y = -(1/2)x + 5/2$$

$$\text{Slope (m)} = -1/2$$

A line parallel to this line will have the same slope $m = -1/2$

So, the equation of line having slope m and passing through $(-2, 3)$ will be

$$y - 3 = (-1/2)(x + 2)$$

$$2y - 6 = -x - 2$$

$$x + 2y = 4$$

(ii) As any line perpendicular to x-axis will be parallel to y-axis.

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So, the equation of line will be
 $x = -2 \Rightarrow x + 2 = 0$

39. (i) Write down the co-ordinates of the point P that divides the line joining A (– 4, 1) and B (17, 10) in the ratio 1 : 2.
(ii) Calculate the distance OP where O is the origin
(iii) In what ratio does the y-axis divide the line AB?

Solution:

(i) Given, co-ordinates of the line joining A (– 4, 1) and B (17, 10) and point P divides the line segment in the ratio 1 : 2

Let the co-ordinates of P be (x, y)

Then,

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{1 \times 17 + 2 \times (-4)}{1 + 2}$$
$$= \frac{17 - 8}{3} = \frac{9}{3} = 3$$

$$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} = \frac{1 \times 10 + 2 \times 1}{1 + 2}$$
$$= \frac{10 + 2}{3} = \frac{12}{3} = 4$$

Thus, Co-ordinates of P will be (3, 4)

(ii) O is the origin

So, Distance between O and P

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(0 - 3)^2 + (0 - 4)^2} = \sqrt{(-3)^2 + (-4)^2}$$
$$= \sqrt{9 + 16} = \sqrt{25} = 5 \text{ units.}$$

(iii) Let y-axis divides AB in the ratio of $m_1 : m_2$

Then, $x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$

$$\Rightarrow 0 = \frac{m_1 \times 17 + m_2 \times (-4)}{m_1 + m_2}$$

$$17 m_1 - 4 m_2 = 0 \Rightarrow 17 m_1 = 4 m_2$$

$$\frac{m_1}{m_2} = \frac{4}{17}$$

$$\Rightarrow m_1 : m_2 = 4:17$$

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40. Find the image of the point (1, 2) in the line $x - 2y - 7 = 0$.

Solution:

Given line equation: $x - 2y - 7 = 0 \dots (i)$

Draw a perpendicular from point P (1, 2) on the line

Let P' be the image of P and let its co-ordinates be (x, y)

The slope of the given line is given as,

$$2y = x - 7$$

$$y = (1/2)x - 7$$

$$\text{Slope } (m_1) = 1/2$$

Let the slope of line segment PP' be m_2

As PP' is perpendicular to the given line, product of slopes: $m_1 \times m_2 = -1$

$$\text{So, } 1/2 \times m_2 = -1$$

$$m_2 = -2$$

So, the equation of the line perpendicular to the given line and passing through P (1, 2) is

$$y - 2 = (-2)(x - 1)$$

$$y - 2 = -2x + 2$$

$$2x + y - 4 = 0 \dots (ii)$$

Let the intersection point of lines (i) and (ii) be taken as M.

Solving both the line equations, we have

Multiplying (ii) by 2 and adding with (i)

$$x - 2y - 7 = 0$$

$$4x + 2y - 8 = 0$$

$$\text{-----}$$

$$5x - 15 = 0$$

$$x = 15/5 = 3$$

Putting value of x in (i), we get

$$3 - 2y - 7 = 0$$

$$2y = -4$$

$$y = -4/2 = -2$$

So, the co-ordinates of M are (3, -2)

Hence, it is seen that M should be the mid-point of the line segment PP'

$$(3, -2) = ((x + 1)/2, (y + 2)/2)$$

$$(x + 1)/2 = 3$$

$$x + 1 = 6$$

$$x = 6 - 1 = 5$$

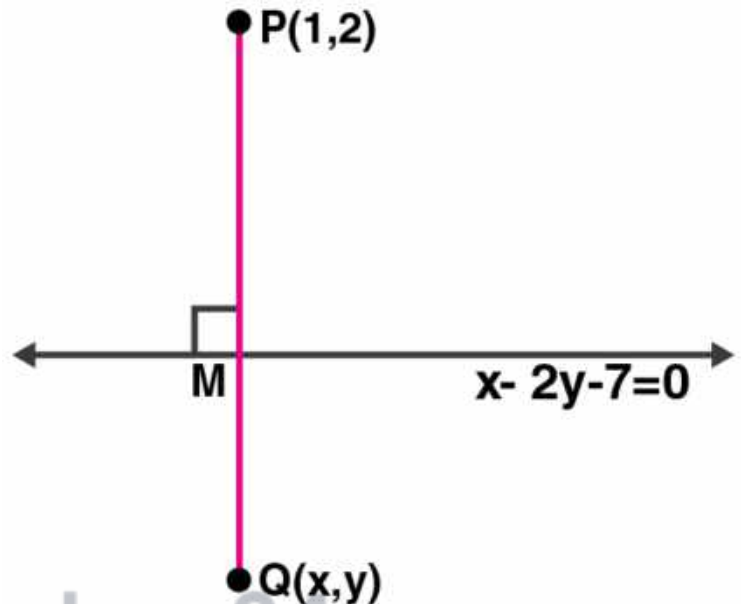
And,

$$(y + 2)/2 = -2$$

$$y + 2 = -4$$

$$y = -4 - 2 = -6$$

Therefore, the co-ordinates of P' are (5, -6).



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41. If the line $x - 4y - 6 = 0$ is the perpendicular bisector of the line segment PQ and the co-ordinates of P are (1, 3), find the co-ordinates of Q.

Solution:

Given, line equation: $x - 4y - 6 = 0 \dots$ (i)

Co-ordinates of P are (1, 3)

Let the co-ordinates of Q be (x, y)

Now, the slope of the given line is

$$4y = x - 6$$

$$y = (1/4)x - 6/4$$

$$\text{slope (m)} = 1/4$$

So, the slope of PQ will be $(-1/m)$ [As the product of slopes of perpendicular lines is -1]

$$\text{Slope of PQ} = -1 / (1/4) = -4$$

Now, the equation of line PQ will be

$$y - 3 = (-4)(x - 1)$$

$$y - 3 = -4x + 4$$

$$4x + y = 7 \dots$$
 (ii)

On solving equations (i) and (ii), we get the coordinates of M

Multiplying (ii) by 4 and adding with (i), we

get

$$x - 4y - 6 = 0$$

$$16x + 4y = 28$$

$$17x = 34$$

$$x = 34/17 = 2$$

Putting the value of x in (i)

$$2 - 4y - 6 = 0$$

$$-4 - 4y = 0$$

$$4y = -4$$

$$y = -1$$

So, the co-ordinates of M are (2, -1)

But, M is the mid-point of line segment PQ

$$(2, -1) = (x + 1)/2, (y + 3)/2$$

$$(x + 1)/2 = 2$$

$$x + 1 = 4$$

$$x = 3$$

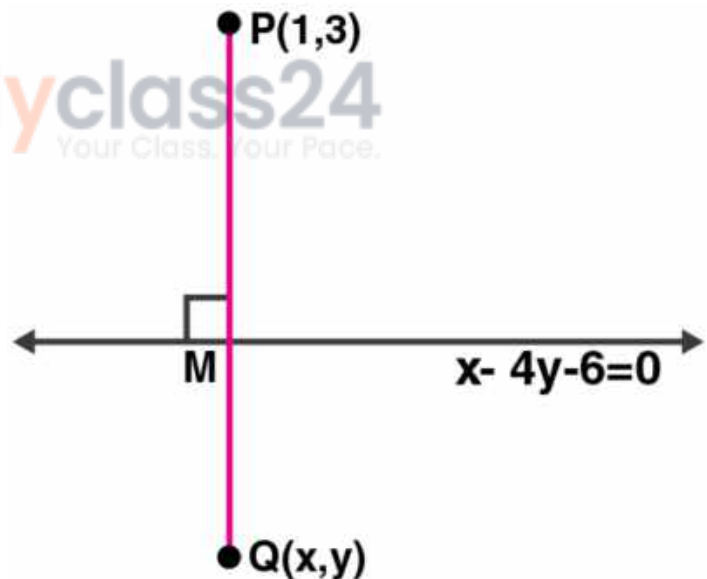
And,

$$(y + 3)/2 = -1$$

$$y + 3 = -2$$

$$y = -5$$

Thus, the co-ordinates of Q are (3, -5).



42. OABC is a square, O is the origin and the points A and B are (3, 0) and (p, q). If OABC lies in the first quadrant, find the values of p and q. Also write down the equations of AB and BC.

Solution:

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Given, OABC is a square

Co-ordinates of A and B are (3, 0) and (p, q) respectively

By distance formula, we have

$$\begin{aligned} OA &= \sqrt{(3-0)^2 + (0-0)^2} \\ &= \sqrt{(3)^2 + (0)^2} \\ &= \sqrt{9+0} = \sqrt{9} = 3 \end{aligned}$$

$$\begin{aligned} AB &= \sqrt{(3-p)^2 + (0-q)^2} \\ &= \sqrt{(3-p)^2 + q^2} \end{aligned}$$

As $OA = AB$ (sides of a square)

$$\Rightarrow \sqrt{(3-p)^2 + q^2} = 3$$

$$(3-p)^2 + q^2 = 9 \quad (\text{Squaring both sides})$$

$$9 + p^2 - 6p + q^2 = 9$$

$$p^2 + q^2 - 6p = 0 \quad \dots(i)$$

$$\text{Now, } OB = \sqrt{(p-0)^2 + (q-0)^2} = \sqrt{p^2 + q^2}$$

$$\text{But } OB^2 = OA^2 + AB^2$$

$$\Rightarrow (\sqrt{p^2 + q^2})^2 = 3^2 + (\sqrt{(3-p)^2 + q^2})^2$$

$$p^2 + q^2 = 9 + (3-p)^2 + q^2$$

$$p^2 + q^2 = 9 + 9 + p^2 - 6p + q^2$$

$$(p^2 + q^2) = 18 + (p^2 - 6p + q^2) \quad [\text{Using (i)}]$$

$$6p = 18 \Rightarrow p = \frac{18}{6} = 3$$

Substituting the value of p in (i)

$$(3)^2 + q^2 - 6(3) = 0 \Rightarrow 9 + q^2 - 18 = 0$$

$$q^2 - 9 = 0 \Rightarrow q^2 = 9 \Rightarrow q = 3$$

$$\therefore p = 3, q = 3$$

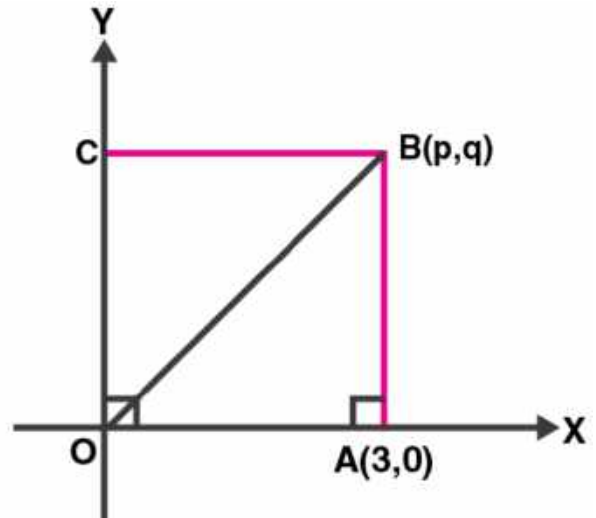
As, AB parallel to y-axis

\therefore Equation AB will be $x = 3$

$$\Rightarrow x - 3 = 0$$

and equation of BC will be $y = 3$

$$\Rightarrow y - 3 = 0 \quad (\because BC \parallel x\text{-axis})$$



ML Aggarwal Solutions for Class 10 Maths

Chapter 12: Equation of Straight Line

Chapter Test

1. Find the equation of a line whose inclination is 60° and y-intercept is -4 .

Solution:

Given, inclination = 60° and y-intercept (c) = -4

So, slope (m) = $\tan 60^\circ = \sqrt{3}$

Hence, the equation of the line is given by

$$y = mx + c$$

$$y = \sqrt{3}x - 4$$

2. Write down the gradient and the intercept on the y-axis of the line $3y + 2x = 12$.

Solution:

Given line equation: $3y + 2x = 12$

$$3y = -2x + 12$$

$$y = (-2/3)x + 12/3$$

$$y = (-2/3)x + 4$$

Hence, gradient = $-2/3$ and the intercept on the y-axis is 4 .

3. If the equation of a line is $y = \sqrt{3}x + 1$, find its inclination.

Solution:

Given line equation: $y = \sqrt{3}x + 1$

$$y = \sqrt{3}x + 1$$

Here, slope = $\sqrt{3}$

$$\Rightarrow \tan \theta = \sqrt{3}$$

$$\theta = 60^\circ$$

Hence, the inclination of the line is 60° .

