EXERCISE 4.2

Find the domain of definition of $f(x) = \cos^{-1}(x^2 - 4)$ Solution:

Given $f(x) = \cos^{-1}(x^2 - 4)$

We know that domain of $\cos^{-1}(x^2 - 4)$ lies in the interval $[-1, 1]$

Therefore, we can write as

$-1 \le x^2 - 4 \le 1$

$4 - 1 \le x^2 \le 1 + 4$

$3 \le x^2 \le 5$

$\pm\sqrt{3} \le x \le \pm\sqrt{5}$

$-\sqrt{5} \le x \le -\sqrt{3}$ and $\sqrt{3} \le x \le \sqrt{5}$

Therefore domain of $\cos^{-1}(x^2 - 4)$ is $[-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$

1. Find the domain of $f(x) = \cos^{-1} 2x + \sin^{-1} x$.

Solution:

Given that $f(x) = \cos^{-1} 2x + \sin^{-1} x$.

Now we have to find the domain of f(x),

We know that domain of $\cos^{-1} x$ lies in the interval $[-1, 1]$

Also know that domain of $\sin^{-1} x$ lies in the interval $[-1, 1]$

Therefore, the domain of $\cos^{-1}(2x)$ lies in the interval $[-1, 1]$

Hence we can write as,

$-1 \le 2x \le 1$

$-\frac{1}{2} \le x \le \frac{1}{2}$

Hence, domain of $\cos^{-1}(2x) + \sin^{-1} x$ lies in the interval $[-\frac{1}{2}, \frac{1}{2}]$