EXERCISE 1.8

If A and B are two sets such that n (A ∪ B) = 50, n (A) = 28 and n (B) = 32, find n (A ∩ B).

Solution:

We have,

n (A ∪ B) = 50

n (A) = 28

n (B) = 32

We know, n (A ∪ B) = n (A) + n (B) – n (A ∩ B)

Substituting the values we get

50 = 28 + 32 – n (A ∩ B)

50 = 60 – n (A ∩ B)

–10 = – n (A ∩ B)

∴ n (A ∩ B) = 10

1. If P and Q are two sets such that P has 40 elements, P ∪ Q has 60 elements and P ∩ Q has 10 elements, how many elements does Q have?

Solution:

We have,

n (P) = 40

n (P ∪ Q) = 60

n (P ∩ Q) =10

We know, n (P ∪ Q) = n (P) + n (Q) – n (P ∩ Q)

Substituting the values we get

60 = 40 + n (Q)–10

60 = 30 + n (Q)

N (Q) = 30

∴ Q has 30 elements.

2. In a school, there are 20 teachers who teach mathematics or physics. Of these, 12 teach mathematics, and 4 teach physics and mathematics. How many teach physics?

Solution:

We have,

Teachers teaching physics or math = 20

Teachers teaching physics and math = 4

Teachers teaching maths = 12

Let teachers who teach physics be ‘n (P)’ and for Maths be ‘n (M)’

Now,

20 teachers who teach physics or math = n (P ∪ M) = 20
4 teachers who teach physics and math = n (P ∩ M) = 4
12 teachers who teach maths = n (M) = 12
We know,
n (P ∪ M) = n (M) + n (P) – n (P ∩ M)
Substituting the values we get,
20 = 12 + n (P) – 4
20 = 8 + n (P)
n (P) =12
∴ There are 12 physics teachers.

3. In a group of 70 people, 37 like coffee, 52 like tea and each person likes at least one of the two drinks. How many like both coffee and tea?
Solution:
We have,
A total number of people = 70
Number of people who like Coffee = n (C) = 37
Number of people who like Tea = n (T) = 52
Total number = n (C ∪ T) = 70
Person who likes both would be n (C ∩ T)
We know,
n (C ∪ T) = n (C) + n (T) – n (C ∩ T)
Substituting the values we get
70 = 37 + 52 – n (C ∩ T)
70 = 89 – n (C ∩ T)
n (C ∩ T) =19
∴ There are 19 persons who like both coffee and tea.

4. Let A and B be two sets such that: n (A) = 20, n (A ∪ B) = 42 and n (A ∩ B) = 4. Find
(i) n (B)
(ii) n (A – B)
(iii) n (B – A)
Solution:
(i) n (B)
We know,
n (A ∪ B) = n (A) + n (B) – n (A ∩ B)
Substituting the values we get
42 = 20 + n (B) – 4

42 = 16 + n (B)
n (B) = 26
∴ n (B) = 26

(ii) n (A – B)
We know,
n (A – B) = n (A ∪ B) – n (B)
Substituting the values we get
n (A – B) = 42 – 26
= 16
∴ n (A – B) = 16

(iii) n (B – A)
We know,
n (B – A) = n (B) – n (A ∩ B)
Substituting the values we get
n (B – A) = 26 – 4
= 22
∴ n (B – A) = 22

5. A survey shows that 76% of the Indians like oranges, whereas 62% like bananas. What percentage of the Indians like both oranges and bananas?
Solution:
We have,
People who like oranges = 76%
People who like bananas = 62%

Let people who like oranges be n (O)
Let people who like bananas be n (B)
Total number of people who like oranges or bananas = n (O ∪ B) = 100
People who like both oranges and bananas = n (O ∩ B)
We know,
n (O ∪ B) = n (O) + n (B) – n (O ∩ B)
Substituting the values we get
100 = 76 + 62 – n (O ∩ B)
100 = 138 – n (O ∩ B)
n (O ∩ B) = 38
∴ People who like both oranges and banana is 38%.

6. In a group of 950 persons, 750 can speak Hindi and 460 can speak English. Find:
(i) How many can speak both Hindi and English.
(ii) How many can speak Hindi only.
(iii) how many can speak English only.
Solution:
We have,
Let, total number of people be n (P) = 950
People who can speak English n (E) = 460
People who can speak Hindi n (H) = 750

(i) How many can speak both Hindi and English.
People who can speak both Hindi and English = $n (H \cap E)$
We know,
$n (P) = n (E) + n (H) – n (H \cap E)$
Substituting the values we get
$950 = 460 + 750 – n (H \cap E)$
$950 = 1210 – n (H \cap E)$
$n (H \cap E) = 260$
∴ Number of people who can speak both English and Hindi are 260.

(ii) How many can speak Hindi only.
We can see that H is disjoint union of $n (H–E)$ and $n (H \cap E)$.
(If A and B are disjoint then $n (A \cup B) = n (A) + n (B)$)
∴ $H = n (H–E) \cup n (H \cap E)$
$n (H) = n (H–E) + n (H \cap E)$
$750 = n (H – E) + 260$
$n (H–E) = 490$
∴ 490 people can speak only Hindi.

(iii) How many can speak English only.
We can see that E is disjoint union of $n (E–H)$ and $n (H \cap E)$
(If A and B are disjoint then $n (A \cup B) = n (A) + n (B)$)
∴ $E = n (E–H) \cup n (H \cap E)$.
$n (E) = n (E–H) + n (H \cap E)$.
$460 = n (H – E) + 260$
$n (H–E) = 460 – 260 = 200$
∴ 200 people can speak only English.