NCERT Solutions for Class-XI Maths

Chapter-8 Exercise-8.2

NCERT Math Class 11

1. Find the coefficient of x^5 in $(x+3)^8$

1. It is known that $(r+1)^{th}$ term, $\left(T_{r+1}\right)$, in the binomial expansion of $(a+b)^n$ is given by $T_{r+1} = {}^nC_r a^{n-r} b^r$.

Assuming that x^5 occurs in the $(r+1)^{th}$ term of the expansion $(x+3)^8$, we obtain

$T_{r+1} = {}^8C_r (x)^{8-r} (3)^r$

Comparing the indices of x in x^5 and in T_{r+1},

we obtain $r = 3$

Thus, the coefficient of x^5 is ${}^8C_3 (3)^3 = \frac{8!}{3!5!} \times 3^3 = \frac{8 \cdot 7 \cdot 6 \cdot 5!}{3 \cdot 2.5!} \cdot 3^3 = 1512$

2. a^5b^7 in $(a - 2b)^{12}$.

2. The general term T_{r+1} in the binomial expansion is given by $T_{r+1} = {}^nC_r a^{n-r} b^r$

Here a =a , b = -2b & n =12

Putting values

$T_{r+1} = {}^{12}C_r a^{12-r} (-2b)^r$..........1

To find a^5

We equate $a^{12-r} = a^5$

r=7

Putting r = 7 in 1

$T_8 = {}^{12}C_7 a^5 (-2b)^7$

$T_8 = \frac{12!}{7!5!} \times a^5 \times (-2)^7 b^7$

$= -101376 a^5 b^7$

Hence the coefficient of a^5b^7= -101376

3. Write the general term in the expansion of $\left(x^2 - y\right)^6$

3. It is known that the general term T_{r+1} {which is the $(r+1)^{th}$ term } in the binomial expansion of $(a+b)^n$ is given by $T_{r+1} = {}^nC_r a^{n-r} b^r$.

Thus, the general term in the expansion of $\left(x^2 - y^6\right)$ is

$$T_{r+1} = {}^6C_r \left(x^2\right)^{6-r} (-y)^r = (-1)^r \, {}^6C_r \cdot x^{12-2r} \cdot y^r$$

4. $(x^2 - yx)^{12}$, $x \neq 0$.

4. The general term T_{r+1} in the binomial expansion is given by $T_{r+1} = {}^nC_r \, a^{n-r} \, b^r$

Here n = 12, a= x^2 and b = -yx

Putting the values

$T_{n+1} = {}^{12}C_r \times x^{2(12-r)} \, (-1)^r \, y^r \, x^r$

$= \dfrac{12!}{r!(12-r)!} \times x^{24-2r} \; -1^r \; y^r \; x^r$

$= -1^r \; \dfrac{12!}{r!(12-r)!} \; x^{24-r} \; y^r$

5. Find the 4^{th} term in the expansion of $(x-2y)^{12}$.

5. It is known that $(r+1)^{th}$ term, $\left(T_{r+1}\right)$, in the binomial expansion of $(a+b)^n$ is given by $T_{r+1} = {}^nC_r a^{n-r} b^r$.

Thus, the 4^{th} term in the expansion of $(x-2y)^{12}$ is

$T_4 = T_{3+1} = {}^{12}C_3 (x)^{12-3} (-2y)^3 = (-1)^3 \cdot \dfrac{12!}{3!9!} \cdot x^9 \cdot (2)^3 \cdot y^3$

$= -\dfrac{12 \cdot 11 \cdot 10}{3 \cdot 2} \cdot (2)^3 x^9 y^3$

$= -1760 x^9 y^3$

6. Find the 13^{th} term in the expansion of $\left(9x - \dfrac{1}{3\sqrt{x}}\right)^{18}, x \neq 0$

6. The general term T_{r+1} in the binomial expansion is given by $T_{r+1} = {}^nC_r \, a^{n-r} \, b^r$

Here a=9x , $b = -\dfrac{1}{3\sqrt{x}}$, n =18 and r = 12

Putting values

$T_{13} = \dfrac{18!}{12!6!} 9x^{18-12} \left(-\dfrac{1}{3\sqrt{x}}\right)^{12}$

$= \dfrac{(18 \times 17 \times 16 \times 15 \times 14 \times 13 \times 12!)}{12! \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} \times 3^{12} \times x^6 \times \dfrac{1}{x^6} \times \dfrac{1}{3^{12}}$ $\quad (9 = (3^2)^6 = 3^{12})$

= 18564

7. Find the middle terms in the expansions of $\left(3-\frac{x^3}{6}\right)^7$

7. It is known that in the expansion of $(a+b)^n$, if n is odd, then there are two middle terms, namely $\left(\frac{n+1}{2}\right)^{th}$ term and $\left(\frac{n+1}{2}+1\right)^{th}$ term.

Therefore, the middle terms in the expansion $\left(3-\frac{x^3}{6}\right)^7$ are $\left(\frac{7+1}{2}\right)^{th}=4^{th}$ term and $\left(\frac{7+1}{2}+1\right)^{th}=5^{th}$ term

$$T_4=T_{3+1}={}^7C_3(3)^{7-3}\left(-\frac{x^3}{6}\right)^3=(-1)^3\frac{7!}{3!4!}\cdot 3^4\cdot\frac{x^9}{6^3}$$

$$=-\frac{7\cdot6\cdot5\cdot4!}{3\cdot2\cdot4!}\cdot3^4\cdot\frac{1}{2^3\cdot3^3}\cdot x^9=-\frac{105}{8}x^9$$

$$T_5=T_{4+1}={}^7C_4(3)^{7-4}\left(-\frac{x^3}{6}\right)^4=(-1)^4\frac{7!}{4!3!}(3)^3\cdot\frac{x^{12}}{6^4}$$

$$=\frac{7\cdot6\cdot5\cdot4!}{4!\cdot3\cdot2}\cdot\frac{3^3}{2^4\cdot3^4}\cdot x^{12}=\frac{35}{48}x^{12}$$

8. $\left(\frac{x}{3}+9y\right)^{10}$

8. Here n is even so the middle term will be given by $\left(\frac{n+1}{2}\right)^{th}$ term $=6^{th}$ term

The general term T_{r+1} in the binomial expansion is given by $T_{r+1}={}^nC_r\,a^{n-r}\,b^r$

Now $a=\frac{x}{3}$, $b=9y$, $n=10$ and $r=5$

Putting the values

$$T_6=\frac{10!}{5!5!}\times\left(\frac{x}{3}\right)^5\times(9y)^5$$

$$=\frac{10\times9\times8\times7\times6\times5!}{5!\times5\times4\times3\times2\times1}\times\frac{x^5}{3^5}\times3^{10}\times y^5$$

$= 61236\, x^5y^5$

9. In the expansion of $(1+a)^{m+n}$, prove that coefficients of a^m and a^n are equal.

9. It is known that $(r+1)^{th}$ term, $\left(T_{r+1}\right)$, in the binomial expansion of $(a+b)^n$ is given by $T_{r+1} = {}^nC_r a^{n-r} b^r$.

Assuming that a^m occurs in the $(r+1)^{th}$ term of the expansion $(1+a)^{m+n}$, we obtain

$T_{r+1} = {}^{m+n}C_r (1)^{m+n-r}(a)^r = {}^{m+n}C_r a^r$

Comparing the indices of a in a^m and in T_{r+1},

we obtain $r = m$

Therefore, the coefficient of a^m is

$${}^{m+n}C_m = \frac{(m+n)!}{m!(m+n-m)!} = \frac{(m+n)!}{m!n!}$$

Assuming that a^n occurs in the $(k+1)^{th}$ term of the expansion $(1+a)^{m+n}$, we obtain

$T_{k+1} = {}^{m+n}C_k (1)^{m+n-k}(a)^k = {}^{m+n}C_k (a)^k$

Comparing the indices of a in a^n and in T_{k+1},

we obtain $k = n$

Therefore, the coefficient of a^n is

$${}^{m+n}C_n = \frac{(m+n)!}{n!(m+n-n)!} = \frac{(m+n)!}{n!m!}$$

Thus, from (1) and (2), it can be observed that the coefficients of a^m and a^n in the expansion of $(1+a)^{m+n}$ are equal.

10. The coefficients of the $(r-1)^{th}$, r^{th} and $(r+1)^{th}$ terms in the expansion of $(x+1)^n$ are in the ratio 1 : 3 : 5. Find n and r.

10. The general term T_{r+1} in the binomial expansion is given by $T_{r+1} = {}^nC_r a^{n-r} b^r$

Here the binomial is $(1+x)^n$ with a = 1 , b = x and n = n

The $(r+1)^{th}$ term is given by

$T_{(r+1)} = {}^nC_r 1^{n-r} x^r$

$T_{(r+1)} = {}^nC_r x^r$

The coefficient of $(r+1)^{th}$ term is ${}^nC_r$

The r^{th} term is given by $(r-1)^{th}$ term

$T_{(r+1-1)} = {}^nC_{r-1} x^{r-1}$

$T_r = {}^nC_{r-1} x^{r-1}$

∴ the coefficient of r^{th} term is ${}^nC_{r-1}$

For $(r-1)^{th}$ term we will take $(r-2)^{th}$ term

$T_{r-2+1} = {}^nC_{r-2} x^{r-2}$

$T_{r-1} = {}^nC_{r-2} x^{r-2}$

$\therefore$ the coefficient of $(r-1)^{th}$ term is ${}^nC_{r-2}$

Given that the coefficient of $(r-1)^{th}$, r^{th} and $r+1^{th}$ term are in ratio 1:3:5

$$\frac{\text{the coefficient of } r-1^{th} \text{ term}}{\text{coefficient of } r^{th} \text{ term}} = \frac{1}{3}$$

$$\Rightarrow \frac{\frac{n!}{(r-2)!(n-r+2)!}}{\frac{n!}{(r-1)!(n-r+1)!}} = \frac{1}{3}$$

$$\frac{n!}{(r-2)!(n-r+2)!} \times \frac{(r-1)!(n-r+1)!}{n!} = \frac{1}{3}$$

$$\Rightarrow \frac{(r-1)(r-2)!(n-r+1)!}{(r-2)!(n-r+2)!} = \frac{1}{3}$$

$$\Rightarrow \frac{(r-1)(n-r+1)!}{(n-r+2)(n-r+1)!} = \frac{1}{3}$$

$$\Rightarrow \frac{(r-1)}{(n-r+2)} = \frac{1}{3}$$

$\Rightarrow 3r - 3 = n - r + 2$

$\Rightarrow n - 4r + 5 = 0$……….1

Also

$$\frac{\text{the coefficient of } r^{th} \text{ term}}{\text{coefficient of } r+1^{th} \text{ term}} = \frac{3}{5}$$

$$\Rightarrow \frac{\frac{n!}{(r-1)!(n-r+1)!}}{\frac{n!}{r!(n-r)!}} = \frac{3}{5}$$

$$\Rightarrow \frac{n!}{(r-1)!(n-r+1)!} \times \frac{r!(n-r)!}{n!} = \frac{3}{5}$$

$$\Rightarrow \frac{r(r-1)!(n-r)!}{(r-1)!(n-r+1)!} = \frac{3}{5}$$

$$\Rightarrow \frac{r(n-r)!}{(n-r+1)!} = \frac{3}{5}$$

$$\Rightarrow \frac{r(n-r)!}{(n-r+1)(n-r)!} = \frac{3}{5}$$

$$\Rightarrow \frac{r}{(n-r+1)} = \frac{3}{5}$$

$\Rightarrow$ 5r = 3n - 3r + 3

$\Rightarrow$8r – 3n - 3 =0………….2

We have 1 and 2 as

n - 4r ∓5 =0…………1

8r – 3n - 3 =0…………….2

Multiplying equation 1 by number 2

2n -8r +10 =0……………….3

Adding equation 2 and 3

2n -8r +10 =0

+ -3n – 8r - 3 =0

$\Rightarrow$ -n = -7

n =7

and r = 3

11. Prove that the coefficient of x^n in the expansion of $(1+x)^{2n}$ is twice the coefficient of x^n in the expansion of $(1+x)^{2n-1}$.

11. It is known that $(r+1)^{th}$ term, (T_{r+1}), in the binomial expansion of $(a+b)^n$ is given by $T_{r+1} = {}^nC_r a^{n-r} b^r$.

Assuming that x^n occurs in the $(r+1)^{th}$ term of the expansion of $(1+x)^{2n}$, we obtain

$T_{r+1} = {}^{2n}C_r (1)^{2n-r} (x)^r = {}^{2n}C_r (x)^r$

Comparing the indices of x in x^n and in T_{r+1}, we obtain $r = n$

Therefore, the coefficient of x^n in the expansion of $(1+x)^{2n}$ is

$${}^{2n}C_n = \frac{(2n)!}{n!(2n-n)!} = \frac{(2n)!}{n!n!} = \frac{(2n)!}{(n!)^2}$$

Assuming that x^n occurs in the $(k+1)^{th}$ term of the expansion $(1+x)^{2n-1}$, we obtain

$T_{k+1} = {}^{2n-1}C_k (1)^{2n-1-k} (x)^k = {}^{2n-1}C_k (x)^k$

Comparing the indices of x in x^n and T_k+1, we obtain $k = n$

Therefore, the coefficient of x^n in the expansion of $(1+x)^{2n-1}$ is

$${}^{2n-1}C_n = \frac{(2n-1)!}{n!(2n-1-n)!} = \frac{(2n-1)!}{n!(n-1)!}$$

$$= \frac{2n\cdot(2n-1)!}{2n\cdot n!(n-1)!} = \frac{(2n)!}{2\cdot n!n!} = \frac{1}{2}\left[\frac{(2n)!}{(n!)^2}\right]$$

From (1) and (2), it is observed that

$\frac{1}{2}\left({}^{2n}C_n\right) = {}^{2n-1}C_n$

$\Rightarrow {}^{2n}C_n = 2\left({}^{2n-1}C_n\right)$

Therefore, the coefficient of x^n in the expansion of $(1+x)^{2n}$ is twice the coefficient of x^n in the expansion of $(1+x)^{2n-1}$.

Hence, proved.

12. Find a positive value of m for which the coefficient of x^2 in the expansion $(1 + x)^m$ is 6.

12. The general term T_{r+1} in the binomial expansion is given by $T_{r+1} = {}^nC_r\, a^{n-r}\, b^r$

Here a = 1, b = x and n = m

Putting the value

$T_{r+1} = {}^mC_r\, 1^{m-r}\, x^r$

$= {}^mC_r\, x^r$

We need coefficient of x^2

∴ putting r = 2

$T_{2+1} = {}^mC_2\, x^2$

The coefficient of $x^2 = {}^mC_2$

Given that coefficient of $x^2 = {}^mC_2 = 6$

$\Rightarrow \frac{m!}{2!(m-2)!} = 6$

$\Rightarrow \frac{m(m-1)(m-2)!}{2\times1\times(m-2)!} = 6$

$\Rightarrow$ m(m-1) = 12

$\Rightarrow m^2 - m - 12 = 0$

$\Rightarrow m^2 - 4m + 3m - 12 = 0$

$\Rightarrow$ m(m-4) + 3(m-4) = 0

$\Rightarrow$ (m+3)(m - 4)= 0

$\Rightarrow$ m = - 3 , 4

we need positive value of m so m = 4