# NCERT Solutions for Class-XII Maths

## Chapter-11
## Exercise- Miscellaneous

# NCERT Maths Class 12

**1.** Show that the line joining the origin to the point (2, 1, 1) is perpendicular to the line determined by the points (3, 5, −1), (4, 3, −1).

**1.** Let OA be the line joining the origin (0,0,0) and the point A(2,1,1).
Let BC be the line joining the points B(3,5,−1) and C(4,3,−1)
Direction ratios of OA = $(a_1,b_1,c_1) \equiv [(2 - 0),(1 - 0),(1 - 0)] \equiv (2,1,1)$
Direction ratios of BC = $(a_2,b_2,c_2) \equiv [(4 - 3),(3 - 5),( - 1 + 1)]$
$\equiv (1, - 2,0)$
Given -
OA is ⊥ to BC
∴ we have to prove that -
$a_1a_2 + b_1b_2 + c_1c_2 = 0$
L.H.S = $a_1a_2 + b_1b_2 + c_1c_2 = 2 \times 1 + 1 \times (-2) + 1 \times 0 = 2 - 2 = 0$
R.H.S = 0
Thus, L.H.S = R.H.S ….PROVED
Hence OA is ⊥ to BC.

**2.** If $l_1, m_1, n_1$ and $l_2, m_2, n_2$ are the direction cosines of two mutually perpendicular lines, show that the direction cosines of the line perpendicular to both of these are $m_1n_2 - m_2n_1$, $n_1l_2 - n_2l_1$, $l_1m_2 - l_2m_1$.

**2.** It is given that $l_1, m_1, n_1$ and $l_2, m_2, n_2$ are the direction cosines of two mutually perpendicular lines. Therefore,
$l_1l_2 + m_1m_2 + n_1n_2 = 0$ …(1)
$l_1^2 + m_1^2 + n_1^2 = 1$ ……(2)
$l_2^2 + m_2^2 + n_2^2 = 1$ …..(3)
Let l, m, n be the direction cosines of the line which is perpendicular to the line with direction cosines $l_1, m_1, n_1$ and $l_2, m_2, n_2$.
$\therefore ll_1 + mm_1 + nn_1 = 0$
$ll_2 + mm_2 + nn_2 = 0$

$$\therefore \frac{l}{m_1n_2 - m_2n_1} = \frac{m}{n_1l_2 - n_2l_1} = \frac{n}{l_1m_2 - l_2m_1}$$

$$\Rightarrow \frac{l^2}{(m_1n_2 - m_2n_1)^2} = \frac{m^2}{(n_1l_2 - n_2l_1)} = \frac{n^2}{(l_1m_2 - l_2m_1)^2}$$

$$\Rightarrow \frac{l^2}{(m_1n_2 - m_2n_1)^2} = \frac{m^2}{(n_1l_2 - n_2l_1)} = \frac{n^2}{(l_1m_2 - l_2m_2)^2}$$

$$= \frac{l^2 + m^2 + n^2}{(m_1n_2 - m_2n_1)^2 + (n_1l_2 - n_2l_1)^2 + (l_1m_2 - l_2m_1)^2}$$

l, m, n are the direction cosines of the line. $\therefore$ $l^2 + m^2 + n^2 = 1$ … (5)

It is known that,

$$\left(l_1^2 + m_1^2 + n_1^2\right)\left(l_2^2 + m_2^2 + n_2^2\right) - (l_1l_2 + m_1m_2 + n_1n_2)^2$$

$= (m_1n_2 - m_2n_1)^2 + (n_1l_2 - n_2l_1)^2 + (l_1m_2 - l_2m_1)^2$

From (1), (2) and (3) we obtain

$\Rightarrow 1.1 - 0 = (m_1n_2 - m_2n_1)^2 + (n_1l_2 - n_2l_1)^2 + (11m2 - l_2m_1)^2$ ….(6)

Substituting the values from equations (5) and (6) in equation (4), we obtain

$$\frac{l^2}{(m_1n_2 - m_2n_1)^2} = \frac{m^2}{(n_2l_2 - n_2l_1)^2} = \frac{n^2}{(l_1m_2 - l_2m_1)^2} = 1$$

$$\Rightarrow l = m_1n_2 - m_2n_1, m = n_1l_2 - n_2l_1, n = l_1m_2 - l_2m_1$$

Thus, the direction cosines of the required line are $m_1n_2 - m_2n_1$, $n_1l_2 - n_2l_1$, and $l_1m_2 - l_2m_1$.

**3.** Find the angle between the lines whose direction ratios are a, b, c and b − c, c − a, a − b.

**3.** Angle between the lines with direction ratios $a_1$, $b_1$, $c_1$ and $a_2$, $b_2$, $c_2$ is given by

$$cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1{}^2 + b_1{}^2 + c_1{}^2}\sqrt{a_2{}^2 + b_2{}^2 + c_2{}^2}}\right|$$

Given -

$a_1 = a$, $b_1 = b$, $c_1 = c$

$a_2 = b - c$, $b_2 = c - a$, $c_2 = a - b$

So,

$$cos\theta = \left|\frac{a(b-c) + b(c-a) + c(a-b)}{\sqrt{a^2 + b^2 + c^2}\sqrt{(b-c)^2 + (c-a)^2 + (a-b)^2}}\right|$$

$= 0$

$\therefore \cos\theta = 0$

So, $\theta = 90°$

Hence, Angle between the given pair of Lines is 90°.

**4.** Find the equation of a line parallel to x-axis and passing through the origin.

**4.** The line parallel to x-axis and passing through the origin is x-axis itself.

Let $A$ be a point on $x$-axis. Therefore, the coordinates of $A$ are given by $(a, 0,0)$, where $a \in R$.

Direction ratios of OA are $(a - 0) = a, 0,0$

The equation of $OA$ is given by,

$$\frac{x-0}{a} = \frac{y-0}{0} = \frac{z-0}{0}$$

$$\Rightarrow \frac{x}{1} = \frac{y}{0} = \frac{z}{0} = a$$

Thus, the equation of line parallel to $x$-axis and passing through origin is $\frac{x}{1} = \frac{y}{0} = \frac{z}{0}$

**5.** If the coordinates of the points A, B, C, D be (1, 2, 3), (4, 5, 7), (−4, 3, −6) and (2, 9, 2) respectively, then find the angle between the lines AB and CD.

**5.** Angle between the lines with direction ratios $a_1$, $b_1$, $c_1$ and $a_2$, $b_2$, $c_2$ is given by

$$cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1{}^2 + b_1{}^2 + c_1{}^2}\sqrt{a_2{}^2 + b_2{}^2 + c_2{}^2}}\right|$$

A line passing through $A(x_1,y_1,z_1)$ and $B(x_2,y_2,z_2)$ has direction ratios $(x_1 - x_2)$, $(y_1 - y_2)$, $(z_1 - z_2)$

Direction ratios of line joining the points A(1,2,3) and B(4,5,7)

= (4 - 1), (5 - 2), (7 - 3)

= (3,3,4)

$\therefore a_1 = 3, b_1 = 3, c_1 = 4$

Direction ratios of line joining the points C( - 4,3, - 6) and B(2,9,2)

= (2 - ( - 4)), (9 - 3), (2 - ( - 6))

= (6,6,8)

$\therefore a_2 = 6, b_2 = 6, c_2 = 8$

Now,

$$cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1{}^2 + b_1{}^2 + c_1{}^2}\sqrt{a_2{}^2 + b_2{}^2 + c_2{}^2}}\right|$$

$$cos\theta = \left|\frac{3 \times 6 + 3 \times 6 + 4 \times 8}{\sqrt{3^2 + 3^2 + 4^2}\sqrt{6^2 + 6^2 + 8^2}}\right|$$

$$= \left|\frac{18 + 18 + 32}{\sqrt{9 + 9 + 16}\sqrt{36 + 36 + 64}}\right|$$

$$= \left|\frac{68}{\sqrt{34}\sqrt{136}}\right|$$

$$= \left|\frac{68}{\sqrt{34}\sqrt{4 \times 34}}\right|$$

$$= \left|\frac{68}{34 \times 2}\right|$$

$\therefore \cos\theta = 1$

So, $\theta = 0°$

Hence, Angle between the lines AB and CD is 0°.

**6.** If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$ are perpendicular, find the value of k.

**6.** The direction of ratios of the lines, $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$, are $-3$, 2k, 2 and 3k, 1, $-5$ respectively.

It is known that two lines with direction ratios, $a_1, b_1, c_1$ and $a_2, b_2 c_2$, are perpendicular, if $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$

$\therefore -3(3k) + 2k \times 1 + 2(-5) = 0$

$\Rightarrow -9k + 2k - 10 = 0$

$\Rightarrow 7k = -10$

$\Rightarrow k = \frac{-10}{7}$

Therefore, for $k = -\frac{10}{7}$, the given lines are perpendicular to each other.

7. Find the vector equation of the plane passing through (1, 2, 3) and perpendicular to the plane $\vec{r}.\left(\hat{i} + 2\hat{j} - 5\hat{k}\right) + 9 = 0$.

7. The vector equation of a line passing through a point with position vector $\vec{a}$ and parallel to vector $\vec{b}$ is

$$\vec{r} = \vec{a} + \lambda\vec{b}$$

Given, the line passes through (1,2,3)

So, $\vec{a} = 1\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

Finding normal of plane

$\vec{r}.\left(\hat{\imath} + 2\hat{\jmath} - 5\hat{k}\right) + 9 = 0$

$\Rightarrow \vec{r}.\left(\hat{\imath} + 2\hat{\jmath} - 5\hat{k}\right) = -9$

$\Rightarrow -\vec{r}.\left(\hat{\imath} + 2\hat{\jmath} - 5\hat{k}\right) = 9$

$\Rightarrow \vec{r}.\left(-1\hat{\imath} - 2\hat{\jmath} + 5\hat{k}\right) + 9 = 0$

Comparing with $\vec{r}.\vec{n} = d$,

$\vec{n} = -\hat{\imath} - 2\hat{\jmath} + 5\hat{k}$

Since line is perpendicular to plane, the line will be parallel of the plane

$\therefore \vec{b} = \vec{n} = -\hat{\imath} - 2\hat{\jmath} + 5\hat{k}$

Hence,

$$\vec{r} = (1\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) + \lambda(-\hat{\imath} - 2\hat{\jmath} + 5\hat{k})$$

$\vec{r} = (1\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) - \lambda(\hat{\imath} + 2\hat{\jmath} - 5\hat{k})$

This is the required vector equation of line.

**8.** Find the equation of the plane passing through (a, b, c) and parallel to the plane $\vec{r}.(\hat{i}+\hat{j}+\hat{k})=2$.

**8.** Any plane parallel to the plane, $\vec{r}_1 \cdot (\hat{\imath} + \hat{\jmath} + \hat{k}) = 2$

$\vec{r} \cdot (\hat{\imath} + \hat{\jmath} + \hat{k}) = \lambda$

The plane passes through the point $(a, b, c)$. Therefore, the position vector $\vec{r}$ of this point is $\vec{r} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$

Therefore, equation (1) becomes

$(a\hat{\imath} + b\hat{\jmath} + c\hat{k}) \cdot (\hat{\imath} + \hat{\jmath} + \hat{k}) = \hat{\lambda}$

$\Rightarrow a + b + c = \lambda$

Substituting $\lambda = a + b + c$ in equation (1), we obtain $\vec{r} \cdot (\hat{\imath} + \hat{\jmath} + \hat{k}) = a + b + c$ ...(2)

This is the vector equation of the required plane. Substituting $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ in equation (2), we obtain $(x\hat{\imath} + \hat{\jmath} + z\hat{k}) \cdot (\hat{\imath} + \hat{\jmath} + \hat{k}) = a + b + c \Rightarrow x + y + z = a + b + c$

**9.** Find the shortest distance between lines $\vec{r}. = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$ and

$\vec{r} = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$

**9.** Shortest distance between lines with vector equations

$\vec{r} = \overrightarrow{a_1} + \lambda\,\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \lambda\,\overrightarrow{b_2}$ is

$$\left|\frac{(\overrightarrow{b_1} \times \overrightarrow{b_2}).(\overrightarrow{a_2} - \overrightarrow{a_1})}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}\right|$$

Given –

$\vec{r} = (6\hat{\imath} + 2\hat{\jmath} + 2\hat{k}) + \lambda(1\hat{\imath} - 2\hat{\jmath} + \hat{2})$

Comparing with $\vec{r} = \overrightarrow{a_1} + \lambda\,\overrightarrow{b_1}$, we get -

$\overrightarrow{a_1} = (6\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$ & $\overrightarrow{b_1} = (1\hat{\imath} - 2\hat{\jmath} + \hat{2})$

Similarly,

$\vec{r} = (-4\hat{\imath} - \hat{k}) + \mu(3\hat{\imath} - 2\hat{\jmath} - 2\hat{k})$

Comparing with $\vec{r} = \vec{a_2} + \lambda \vec{b_2}$, we get -

$\vec{a_2} = (-4\hat{\imath} - \hat{k})$ & $\vec{b_2} = (3\hat{\imath} - 2\hat{\jmath} - \hat{2})$

Now,

$$(\vec{a_2} - \vec{a_1}) = (-4\hat{\imath} - \hat{k}) - (6\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$$

$= ((-4-6)\hat{\imath} + (0-2)\hat{\jmath} + (-1-2)\hat{k})$

$= (-10\hat{\imath} - 2\hat{\jmath} - 3\hat{k})$

$(\vec{b_1} \times \vec{b_2}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix}$

$= \hat{\imath}[(-2 \times -2) - (-2 \times 2)] - \hat{\jmath}[(1 \times -2) - (3 \times 2)] + \hat{k}[(1 \times -2) - (3 \times -2)]$

$= \hat{\imath}[4 + 4] - \hat{\jmath}[-2 - 6] + \hat{k}[-2 + 6]$

$= 8\hat{\imath} + 8\hat{\jmath} + 4\hat{k}$

Magnitude of $\vec{b_1} \times \vec{b_2} = |\vec{b_1} \times \vec{b_2}| = \sqrt{8^2 + 8^2 + 4^2} = \sqrt{64 + 64 + 16}$

$= \sqrt{144}$

$= 12$

Also,

$(\vec{b_1} \times \vec{b_2}).(\vec{a_2} - \vec{a_1}) = (8\hat{\imath} + 8\hat{\jmath} + 4\hat{k}).(-10\hat{\imath} - 2\hat{\jmath} - 3\hat{k})$

$= -80 + (-16) + (-12)$

$= -108$

Shortest Distance $= \left|\frac{(\vec{b_1} \times \vec{b_2}).(\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|}\right| = \left|\frac{-108}{12}\right| = |-9| = 9$

Hence, the shortest distance between the given two lines is 9.

**10.** Find the coordinates of the point where the line through (5, 1, 6) and (3, 4, 1) crosses the YZ-plane

**10.** It is known that the equation of the line passing through the points, $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$, is $\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$

The line passing through the points, (5,1,6) and (3,4,1), is given by,

$$\frac{x-5}{3-5} = \frac{y-1}{4-1} = \frac{z-6}{1-6}$$

$$\Rightarrow \frac{x-5}{-2} = \frac{y-1}{3} = \frac{z-6}{-5} = k(\text{ say })$$

$$\Rightarrow x = 5 - 2k, y = 3k + 1, z = 6 - 5k$$

Any point on the line is of the form $( 5 - 2k, 3k + 1, 6 - 5k )$.

The equation of YZ-plane is $x = 0$

Since the line passes through YZ-plane,

$5 - 2k = 0$

$\Rightarrow k = \frac{5}{2}$

$\Rightarrow 3k + 1 = 3 \times \frac{5}{2} + 1 = \frac{17}{2}$

$6 - 5k = 6 - 5 \times \frac{5}{2} = \frac{-13}{2}$

Therefore, the required point is $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$.

**11.** Find the coordinates of the point where the line through (5, 1, 6) and (3, 4, 1) crosses the ZX – plane

**11.** The vector equation of a line passing through two points with position vectors $\vec{a}$ & $\vec{b}$ is

$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$

The position vector of point A(5,1,6) is given as -

$\vec{a} = 5\hat{\imath} + \hat{\jmath} + 6\hat{k}$

The position vector of point B(3,4,1) is given as -

$\vec{b} = 3\hat{\imath} + 4\hat{\jmath} + \hat{k}$

$(\vec{b} - \vec{a}) = (3\hat{\imath} + 4\hat{\jmath} + \hat{k}) - (5\hat{\imath} + \hat{\jmath} + 6\hat{k})$

$= (3 - 5)\hat{\imath} + (4 - 1)\hat{\jmath} + (1 - 6)\hat{k}$

$= (-2\hat{\imath} + 3\hat{\jmath} - 5\hat{k})$

$\therefore \vec{r} = (5\hat{\imath} + \hat{\jmath} + 6\hat{k}) + \lambda(-2\hat{\imath} + 3\hat{\jmath} - 5\hat{k})$...(1)

Let the coordinates of the point where the line crosses the ZX plane be (0,y,z)

So, $\vec{r} = (x\hat{\imath} + 0\hat{\jmath} + z\hat{k})$…(2)

Since point lies in line, it will satisfy its equation,

Putting (2) in (1)

$(x\hat{\imath} + 0\hat{\jmath} + z\hat{k}) = (5\hat{\imath} + \hat{\jmath} + 6\hat{k}) + \lambda(-2\hat{\imath} + 3\hat{\jmath} - 5\hat{k})$

$\Rightarrow (x\hat{\imath} + 0\hat{\jmath} + z\hat{k}) = (5 - 2\lambda)\hat{\imath} + (1 + 3\lambda)\hat{\jmath} + (6 - 5\lambda)\hat{k}$

Two vectors are equal if their corresponding components are equal

So,

x = 5 - 2λ…(3)

0 = 1 + 3λ…(4)

and, z = 6 - 5λ…(5)

From equation (4), we get -

λ = - 1/3

Substitute the value of λ in equation (3) and (5), we get -

x = 5 - 2λ = 5 - 2 × ( - 1/3) = 5 + (2/3) = 17/3

and

$z = 6 - 5\lambda = 6 - 5 \times (-1/3) = 6 + (5/3) = 23/3$

Therefore, the coordinates of the required point is

(17/3, 0, 23/3).

**12.** Find the coordinates of the point where the line through (3, – 4, – 5) and (2, – 3, 1) crosses the plane 2x + y + z = 7.

**12.** It is known that the equation of the line through the points, $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$, is

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

Since the line passes through the points, $(3, -4, -5)$ and $(2, -3, 1)$, its equation is given by,

$$\frac{x - 3}{2 - 3} = \frac{y + 4}{-3 + 4} = \frac{z + 5}{1 + 5}$$

$$\Rightarrow \frac{x - 3}{-1} = \frac{y + 4}{1} = \frac{z + 5}{6} = k(\text{ say })$$

$$\Rightarrow x = 3 - k, y = k - 4, z = 6k - 5$$

Therefore, any point on the line is of the form $(3 - k, k - 4, 6k - 5)$.

This point lies on the plane, $2x + y + z = 7$

$$\therefore 2(3 - k) + (k - 4) + (6k - 5) = 7$$

$$\Rightarrow 5k - 3 = 7$$

$$\Rightarrow k = 2$$

Hence, the coordinates of the required point are $(3 - 2, 2 - 4, 6 \times 2 - 5)$ i.e., $(1, -2, 7)$.

**13.** Find the equation of the plane passing through the point (−1, 3, 2) and perpendicular to each of the planes x + 2y + 3z = 5 and 3x + 3y + z = 0.

**13.** The equation of a plane passing through $(x_1,y_1,z_1)$ is given by

$A(x - x_1) + B(y - y_1) + C(z - z_1) = 0$

where, A , B, C are the direction ratios of normal to the plane.

Now the plane passes through ( - 1,3,2)

So, equation of plane is

A(x + 1) + B(y - 3) + C(z - 2) = 0 …(1)

Since this plane is perpendicular to the given two planes.

So, their normal to the plane would be perpendicular to normals of both planes.

we know that -

$\vec{a} \times \vec{b}$ is perpendicular to both $\vec{a}$ & $\vec{b}$

So, required normal is cross product of normals of planes

x + 2y + 3z = 5 and 3x + 3y + z = 0

Required Normal = $\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 3 \\ 3 & 3 & 1 \end{vmatrix}$

$= \hat{\imath}[2(1) - 3(3)] - \hat{\jmath}[1(1) - 3(3)] + \hat{k}[1(3) - 3(2)]$

$= \hat{\imath}[2 - 9] - \hat{\jmath}[1 - 9] + \hat{k}[3 - 6]$

$= -7\hat{\imath} + 8\hat{\jmath} - 3\hat{k}$

Hence, direction ratios = - 7, 8, - 3

∴ A = - 7, B = 8, C = - 3

Putting above values in (1), we get -

A(x + 1) + B(y - 3) + C(z - 2) = 0

⇒ - 7(x + 1) + 8(y - 3) + ( - 3)(z - 2) = 0

⇒ - 7x - 7 + 8y - 24 - 3z + 6 = 0

⇒ - 7x + 8y - 3z - 25 = 0

∴ 7x - 8y + 3z + 25 = 0

Therefore, equation of the required plane is 7x - 8y + 3z + 25 = 0.

**14.** If the points (1, 1, p) and (– 3, 0, 1) be equidistant from the plane $\vec{r}.\left(3\hat{\imath} + 4\hat{\jmath} - 12\hat{k}\right) + 13 = 0$, then find the value of p.

**14.** The position vector through the point (1,1, p) is $\vec{a}_1 = \hat{\imath} + \hat{\jmath} + p\hat{k}$ Similarly, the position vector through the point $(-3,0,1)$ is $\vec{a}_2 = -4\hat{\imath} + \hat{k}$

The equation of the given plane is $\vec{r} \cdot (3\hat{\imath} + 4\hat{\jmath} - 12\hat{k}) + 13 = 0$

It is known that the perpendicular distance between a point whose position vector is $\vec{a}$ and the plane, $\vec{r} \cdot \vec{N} = d$, is given by, $D = \frac{|\vec{a} \cdot \vec{N} - d|}{|\vec{N}|}$

Here, $\vec{N} = 3\hat{\imath} + 4\hat{\jmath} - 12\hat{k}$ and $d = -13$

Therefore, the distance between the point $(1,1,\mathrm{p})$ and the given plane is

$$D_1 = \frac{(\hat{\imath} + \hat{\jmath} + p\hat{k}) \cdot (3\hat{\imath} + 4\hat{\jmath} - 12\hat{k}) + 13}{|3\hat{\imath} + 4\hat{\jmath} - 12\hat{k}|}$$

$$\Rightarrow D_1 = \frac{|3 + 4 - 12p + 13|}{\sqrt{3^2 + 4^2 + (-12)^2}}$$

$$\Rightarrow D_1 = \frac{|20 - 12p|}{13}$$

Similarly, the distance between the point $(-3,0,1)$ and the given plane is

$$D_2 = \frac{|(-3\hat{\imath} + \hat{k}) \cdot (3\hat{\imath} + 4\hat{\jmath} - 12\hat{k}) + 13|}{|3\hat{\imath} + 4\hat{\jmath} - 12\hat{k}|}$$

$$\Rightarrow D_1 = \frac{|-9 - 12 + 13|}{\sqrt{3^2 + 4^2 + (-12)^2}}$$

$$\Rightarrow D_2 = \frac{8}{13}$$

It is given that the distance between the required plane and the points, $(1,1,p)$ and $(-3,0,1)$, is equal.

$$\therefore D_1 = D_2$$

$$\Rightarrow \frac{|20 - 12p|}{13} = \frac{8}{13}$$

$$\Rightarrow 20 - 12p = 8 \text{ or } -(20 - 12p) = 8$$

$$\Rightarrow 12p = 12 \text{ or } 12p = 28$$

$$\Rightarrow p = 1 \text{ or } p = \frac{7}{3}$$

**15.** Find the equation of the plane passing through the line of intersection of the planes $\vec{r}.(\hat{\imath} + \hat{\jmath} + \hat{k}) = 1$ and $\vec{r}.(2\hat{\imath} + 3\hat{\jmath} - \hat{k}) + 4 = 0$ and parallel to x - axis.

**15.** The equation of any plane through the line of intersection of the planes $\vec{r}.\overrightarrow{n_1} = d_1$ and $\vec{r}.\overrightarrow{n_2} = d_2$ is given by -

$(\vec{r}.\overrightarrow{n_1} - d_1) + \lambda(\vec{r}.\overrightarrow{n_2} - d_2) = 0$

So, the equation of any plane through the line of intersection of the given planes is

$$[\vec{r}.(\hat{\imath} + \hat{\jmath} + \hat{k}) - 1] + \lambda[\vec{r}.(-2\hat{\imath} - 3\hat{\jmath} + \hat{k}) - 4] = 0$$

$$\vec{r}.\left((1 - 2\lambda)\hat{\imath} + (1 - 3\lambda)\hat{\jmath} + (1 + \lambda)\hat{k}\right) - 1 - 4\lambda = 0$$

$$\therefore \vec{r}.\left((1 - 2\lambda)\hat{\imath} + (1 - 3\lambda)\hat{\jmath} + (1 + \lambda)\hat{k}\right) = 1 + 4\lambda \ldots(1)$$

Since this plane is parallel to x - axis.

So, the normal vector of the plane (1) will be perpendicular to x - axis.

Direction ratios of Normal$(a_1, b_1, c_1) \equiv [(1 - 2\lambda), (1 - 3\lambda), (1 + \lambda)]$

Direction ratios of x - axis$(a_2, b_2, c_2) \equiv (1,0,0)$

Since the two lines are perpendicular,

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\Rightarrow (1 - 2\lambda) \times 1 + (1 - 3\lambda) \times 0 + (1 + \lambda) \times 0 = 0$

$\Rightarrow (1 - 2\lambda) = 0$

$\therefore \lambda = 1/2$

Putting the value of λ in (1), we get -

$$\vec{r}.\left((1 - 2\lambda)\hat{\imath} + (1 - 3\lambda)\hat{\jmath} + (1 + \lambda)\hat{k}\right) = 1 + 4\lambda$$

$\vec{r}.\left(\left(1-2\left(\frac{1}{2}\right)\right)\hat{\imath} + \left(1-3\left(\frac{1}{2}\right)\right)\hat{\jmath} + \left(1+\frac{1}{2}\right)\hat{k}\right) = 1 + 4\left(\frac{1}{2}\right)$

$\vec{r}.\left(0\hat{\imath} + \left(\frac{-1}{2}\right)\hat{\jmath} + \left(\frac{3}{2}\right)\hat{k}\right) = 1 + 2$

$\vec{r}.\left(0\hat{\imath} - \hat{\jmath} + 3\hat{k}\right) = 6$

Hence, the equation of the required plane is

$\vec{r}.\left(0\hat{\imath} - \hat{\jmath} + 3\hat{k}\right) = 6$

**16.** If O be the origin and the coordinates of P be (1, 2, – 3), then find the equation of the plane passing through P and perpendicular to OP.

**16.** The coordinates of the points, $O$ and $P_f$ are $(0,0,0)$ and $(1,2,-3)$ respectively.

Therefore, the direction ratios of OP are $(1-0)=1,(2-0)=2$, and $(-3-0)=-3$

It is known that the equation of the plane passing through the point $(x_1, y_1 z_1)$ is

$a(x-x_1)+b(y-y_1)+c(z-z_1)=0$ where, $a_1$, b, and c are the direction ratios of normal.

Here, the direction ratios of normal are 1,2 , and $-3$ and the point $P$ is $(1,2,-3)$. Thus, the equation of the required plane is

$1(x-1)+2(y-2)-3(z+3)=0$

$\Rightarrow x+2y-3z-14=0$



**17.** Find the equation of the plane which contains the line of intersection of the planes $\vec{r}.\left(\hat{\imath} + 2\hat{\jmath} + 3\hat{k}\right) - 4 = 0$ and $\vec{r}.\left(2\hat{\imath} + \hat{\jmath} - \hat{k}\right) + 5 = 0$. And which is perpendicular to the plane $\vec{r}.\left(5\hat{\imath} + 3\hat{\jmath} - 6\hat{k}\right) + 8 = 0$.

**17.** The equation of any plane through the line of intersection of the planes $\vec{r}.\overrightarrow{n_1} = d_1$ and $\vec{r}.\overrightarrow{n_2} = d_2$ is given by -

$(\vec{r}.\overrightarrow{n_1} - d_1) + \lambda(\vec{r}.\overrightarrow{n_2} - d_2) = 0$

So, the equation of any plane through the line of intersection of the given planes is

$\left[\vec{r}.\left(\hat{\imath} + 2\hat{\jmath} + 3\hat{k}\right) - 4\right] + \lambda\left[\vec{r}.\left(-2\hat{\imath} - \hat{\jmath} + \hat{k}\right) - 5\right] = 0$

$\vec{r}.\left((1-2\lambda)\hat{\imath} + (2-\lambda)\hat{\jmath} + (3+\lambda)\hat{k}\right) - 4 - 5\lambda = 0$

$\therefore \vec{r}.\left((1-2\lambda)\hat{\imath} + (2-\lambda)\hat{\jmath} + (3+\lambda)\hat{k}\right) = 4 + 5\lambda \ldots(1)$

Since this plane is perpendicular to the plane

$\vec{r}.\left(5\hat{\imath} + 3\hat{\jmath} - 6\hat{k}\right) + 8 = 0$

$\vec{r}.\left(5\hat{\imath} + 3\hat{\jmath} - 6\hat{k}\right) = -8$

$-\vec{r}.\left(5\hat{\imath} + 3\hat{\jmath} - 6\hat{k}\right) = 8$

$\boldsymbol{\vec{r}.\left(-5\hat{\imath} - 3\hat{\jmath} + 6\hat{k}\right) = 8\ldots(2)}$

So, the normal vector of the plane (1) will be perpendicular to the normal vector of plane (2).

Direction ratios of Normal of plane(1) =($a_1, b_1, c_1$)

$\equiv [(1 - 2\lambda), (2 - \lambda), (3 + \lambda)]$

Direction ratios of Normal of plane (2) =($a_2, b_2, c_2$)

$\equiv (-5, -3, 6)$

Since the two lines are perpendicular,

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\Rightarrow (1 - 2\lambda) \times (-5) + (2 - \lambda) \times (-3) + (3 + \lambda) \times 6 = 0$

$\Rightarrow -5 + 10\lambda - 6 + 3\lambda + 18 + 6\lambda = 0$

$\Rightarrow 19\lambda + 7 = 0$

$\therefore \lambda = -7/19$

Putting the value of λ in (1), we get -

$$\vec{r}.\left((1-2\lambda)\hat{\imath} + (2-\lambda)\hat{\jmath} + (3+\lambda)\hat{k}\right) = 4 + 5\lambda$$

$$\vec{r}.\left(\left(1-2(\tfrac{-7}{19})\right)\hat{\imath} + \left(2-(\tfrac{-7}{19})\right)\hat{\jmath} + (3 + (\tfrac{-7}{19}))\hat{k}\right) = 4 + 5(\tfrac{-7}{19})$$

$$\vec{r}.\left(\frac{33}{19}\hat{\imath} + \frac{45}{19}\hat{\jmath} + \frac{50}{19}\hat{k}\right) = \frac{41}{19}$$

$$\vec{r}.(33\hat{\imath} + 45\hat{\jmath} + 50\hat{k}) = 41$$

Hence, the equation of the required plane is

$$\vec{r}.(33\hat{\imath} + 45\hat{\jmath} + 50\hat{k}) = 41$$



**18.** Find the distance of the point (– 1, – 5, – 10) from the point of intersection of the line $\vec{r} = (2\hat{\imath} - \hat{\jmath} + 2\hat{k}) + \lambda(3\hat{\imath} + 4\hat{\jmath} + 2\hat{k})$ and the plane $\vec{r}.(\hat{\imath} - \hat{\jmath} + \hat{k}) = 5$.

**18.** The equation of the given line is

$\vec{r} + 2\hat{\imath} - \hat{\jmath} + 2\hat{k} + \lambda(3\hat{\imath} + 4\hat{\jmath} + 2\hat{k})$

The equation of the given plane is

$$\vec{r} \cdot (\hat{\imath} - \hat{\jmath} + \hat{k}) = 5$$

Substituting the value of $\vec{}_{\text{rom}}$ equation (1) in equation (2), we obtain

$$[2\hat{\imath} - \hat{\jmath} + 2\hat{k} + \lambda(3\hat{\imath} + 4\hat{\jmath} + 2\hat{k})] \cdot (\hat{\imath} - \hat{\jmath} + \hat{k}) = 5$$

$$\Rightarrow [(3\lambda + 2)\hat{\imath} + (4\lambda - 1)\hat{\jmath} + (2\lambda + 2)\hat{k}] \cdot (\hat{\imath} - \hat{\jmath} + \hat{k}) = 5.$$

$$\Rightarrow (3\lambda + 2) - (4\lambda - 1) + (2\lambda + 2) = 5$$

$$\Rightarrow \lambda = 0$$

Substituting this value in equation (1), we obtain the equation of the line as $\vec{r} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$

This means that the position vector of the point of intersection of the line and the plane is $\vec{r} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$

This shows that the point of intersection of the given line and plane is given by the coordinates, $(2, -1, 2)$. The point is $(-1, -5, -10)$.

The distance $d$ between the points, $(2, -1, 2)$ and $(-1, -5, -10)$, is

$d = \sqrt{(-1-2)^2 + (-5+1)^2 + (-10-2)^2} = \sqrt{9+16+144} = \sqrt{169} = 13$

**19.** Find the vector equation of the line passing through (1, 2, 3) and parallel to the planes $\vec{r}.(\hat{\imath} - \hat{\jmath} + 2\hat{k}) = 5$ and $\vec{r}.(3\hat{\imath} + \hat{\jmath} + \hat{k}) = 6$ .

**19.** The vector equation of a line passing through a point with position vector $\vec{a}$ and parallel to a vector $\vec{b}$ is

$\vec{r} = \vec{a} + \lambda\vec{b}$

Given, the line passes through (1,2,3)

So, $\vec{a} = 1\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

Given, line is parallel to both planes

∴ Line is perpendicular to normal of both planes.

i.e $\vec{b}$ is perpendicular to normal of both planes.

we know that -

$\vec{a} \times \vec{b}$ is perpendicular to both $\vec{a}$ & $\vec{b}$

So, $\vec{b}$ is cross product of normals of planes

$\vec{r}.(\hat{\imath} - \hat{\jmath} + 2\hat{k}) = 5$ and $\vec{r}.(3\hat{\imath} + \hat{\jmath} + \hat{k}) = 6$

Required Normal $= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -1 & 2 \\ 3 & 1 & 1 \end{vmatrix}$

$= \hat{\imath}[(-1)(1) - 1(2)] - \hat{\jmath}[1(1) - 3(2)] + \hat{k}[1(1) - 3(-1)]$

$= \hat{\imath}[-1-2] - \hat{\jmath}[1-6] + \hat{k}[1+3]$

$= -3\hat{\imath} + 5\hat{\jmath} + 4\hat{k}$

Thus, $\vec{b} = -3\hat{\imath} + 5\hat{\jmath} + 4\hat{k}$

Now, putting the value of $\vec{a}$ & $\vec{b}$ in formula

$\vec{r} = \vec{a} + \lambda\vec{b}$

$= (1\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) + \lambda(-3\hat{\imath} + 5\hat{\jmath} + 4\hat{k})$

Therefore, the equation of the line is

$\vec{r} = (1\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) + \lambda(-3\hat{\imath} + 5\hat{\jmath} + 4\hat{k})$

**20.** Find the vector equation of the line passing through the point (1, 2, – 4) and perpendicular to the two lines:

$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.

**20.** Let the required line be parallel to the vector $\vec{b}$ given by, $\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$ The position vector of the point $(1,2,-4)$ is $\vec{a} = \hat{\imath} + 2\hat{\jmath} - 4\hat{k}$

The equation of the line passing through $(1,2,-4)$ and parallel to vector $\vec{b}$ is

$$\vec{r} = \vec{a} + \lambda\vec{b}$$
$$\Rightarrow \vec{r}(\hat{\imath} + 2\hat{\jmath} - 4\hat{k}) + \lambda\left(b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}\right)$$

The equations of the lines are

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$$
$$\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$

Line (1) and line (2) are perpendicular to each other.

$$\therefore 3b_1 - 16b_2 + 7b_3 = 0$$

Also, line (1) and line (3) are perpendicular to each other.

$$\therefore 3b_1 + 8b_2 - 5b_1 = 0$$

From equations (4) and (5), we obtain

$$\frac{b_1}{(-16)(-5) - 8 \times 7} = \frac{b_2}{7 \times 3 - 3(-5)} = \frac{b_3}{3 \times 8 - 3(-16)}$$
$$\Rightarrow \frac{b_1}{24} = \frac{b_1}{36} = \frac{b_3}{72}$$
$$\Rightarrow \frac{b_1}{2} = \frac{b_2}{3} = \frac{b_3}{6}$$

Direction ratios of $\vec{b}$ are 2,3 , and 6 .

$$\therefore \bar{b} = 2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$$

Substituting $\bar{b} = 2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$ in equation (1), we obtain

$$\vec{r} = (\hat{\imath} + 2\hat{\jmath} - 4\hat{k}) + \lambda(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$$

This is the equation of the required line.

**21.** Prove that if a plane has the intercepts a, b, c and is at a distance of p units from the origin, then $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{p^2}$

**21.** Distance of the point $(x_1,y_1,z_1)$ from the plane Ax + By + Cz = D is

$$\left|\frac{Ax_1 + By_1 + Cz_1 - D}{\sqrt{A^2 + B^2 + C^2}}\right|$$

The equation of a plane having intercepts a, b, c on the x - , y - , z - axis respectively is

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Comparing with Ax + By + Cz = D, we get -

A = 1/a, B = 1/b, C = 1/c, D = 1

Given, the plane is at a distance of 'p' units from the origin.

So, The point is O(0,0,0)

$\therefore x_1 = 0, y_1 = 0, z_1 = 0$

Now,

Distance = $\left|\frac{Ax_1 + By_1 + Cz_1 - D}{\sqrt{A^2 + B^2 + C^2}}\right|$

Substituting all values, we get -

$$\Rightarrow p = \left|\frac{\frac{1}{a} \times 0 + \frac{1}{b} \times 0 + \frac{1}{c} \times 0 - 1}{\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2}}\right|$$

$$\Rightarrow p = \left|\frac{0 + 0 + 0 - 1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}}\right|$$

$$\Rightarrow p = \left|\frac{-1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}}\right|$$

$$\Rightarrow p = \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}}$$

$$\frac{1}{p} = \sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}$$

squaring both sides, we get -

$$\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$$

Hence Proved.

**22.** Distance between the two planes: 2x + 3y + 4z = 4 and 4x + 6y + 8z = 12 is

(a) 2 units

(b) 4 units

(c) 8 units

(d) $\frac{2}{\sqrt{29}}$ units

**22.** The equations of the planes are

$$2x + 3y + 4z = 4$$

$$4x + 6y + 8z = 12$$

$$\Rightarrow 2x + 3y + 4z = 6$$

It can be seen that the given planes are parallel.

It is known that the distance between two parallel planes, $ax + by + cz = d_1$ and $ax +$ by $+cz = d_2$, is given by,

$$D = \left|\frac{d_2 - d_1}{\sqrt{a^2 + b^2 + c^2}}\right|$$

$$\Rightarrow D = \left|\frac{6 - 4}{\sqrt{(2)^2 + (3)^2 + (4)^2}}\right|$$

$$D = \frac{2}{\sqrt{29}}$$

Thus, the distance between the lines is $\frac{2}{\sqrt{29}}$ units.

Hence, the correct answer is D.

**23.** The planes: 2x – y + 4z = 5 and 5x – 2.5y + 10z = 6 are

(a) Perpendicular
(b) Parallel
(c) intersect y - axis
(d) passes through

**23.** Given -

First Plane is

2x – y + 4z = 5

Multiply both sides by 2.5, we get -

5x - 2.5y + 10z = 12.5…(1)

Second Plane is

5x – 2.5y + 10z = 6…(2)

Clearly, the direction ratios of normals of both the plane (1) and (2) are same.

Hence, Both the given planes are parallel.