

EXERCISE 19.26

Evaluate the following integrals:

1. $\int e^x(\cos x - \sin x) dx$

Solution:

Let $I = \int e^x(\cos x - \sin x) dx$

Using integration by parts,

$$= \int e^x \cos x dx - \int e^x \sin x dx$$

We know that, $\frac{d}{dx} \cos x = -\sin x$

$$= \cos x \int e^x - \int \frac{d}{dx} \cos x \int e^x dx - \int e^x \sin x dx$$

$$= e^x \cos x + \int e^x \sin x dx - \int e^x \sin x dx$$

$$= e^x \cos x + c$$

2. $\int e^x \left(\frac{1}{x^2} - \frac{2}{x^3} \right) dx$

Solution:

Let $I = \int e^x \left(\frac{1}{x^2} - \frac{2}{x^3} \right) dx$

$$= \int e^x x^{-2} dx - 2 \int e^x x^{-3} dx$$

Integrating by parts

$$= x^{-2} \int e^x dx - \int \frac{d}{dx} x^{-2} \int e^x dx - 2 \int e^x x^{-3} dx$$

We know that,

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

$$= e^x x^{-2} + 2 \int e^x x^{-3} dx - 2 \int e^x x^{-3} dx$$

$$= \frac{e^x}{x^2} + c$$

3. $\int e^x \left(\frac{1 + \sin x}{1 + \cos x} \right) dx$

Solution:

Let $I = \int e^x \left(\frac{1 + \sin x}{1 + \cos x} \right) dx$

We know that, $\sin^2 x + \cos^2 x = 1$ and $\sin x = 2 \sin \frac{x}{2} \cos \frac{x}{2}$

$$= e^x \left(\frac{\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} + 2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \cos^2 \frac{x}{2}} \right)$$

$$= \frac{e^x \left(\sin \frac{x}{2} + \cos \frac{x}{2} \right)^2}{2 \cos^2 \frac{x}{2}}$$

$$= \frac{1}{2} e^x \left(\frac{\sin \frac{x}{2} + \cos \frac{x}{2}}{\cos \frac{x}{2}} \right)^2$$

$$= \frac{1}{2} e^x \left[\tan \frac{x}{2} + 1 \right]^2$$

$$= \frac{1}{2} e^x \left[1 + \tan \frac{x}{2} \right]^2$$

$$= \frac{1}{2} e^x \left[1 + \tan^2 \frac{x}{2} + 2 \tan \frac{x}{2} \right]$$

$$= \frac{1}{2} e^x \left[\sec^2 \frac{x}{2} + 2 \tan \frac{x}{2} \right]$$

$$= e^x \left[\frac{1}{2} \sec^2 \frac{x}{2} + \tan \frac{x}{2} \right] \dots \dots (1)$$

Let $\tan \frac{x}{2} = f(x)$

$$f'(x) = \frac{1}{2} \sec^2 \frac{x}{2}$$

We know that,

$$\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + c$$

From equation (1), we obtain

$$\int e^x \left(\frac{1 + \sin x}{1 + \cos x} \right) dx = e^x \tan \frac{x}{2} + c$$

4. $\int e^x (\cot x - \operatorname{cosec}^2 x) dx$

Solution:

$$\text{Let } I = \int e^x (\cot x - \operatorname{cosec}^2 x) dx$$

$$= \int e^x \cot x dx - \int e^x \operatorname{cosec}^2 x dx$$

Integrating by parts,

$$= \cot x \int e^x dx - \int \frac{d}{dx} \cot x \int e^x dx - \int e^x \operatorname{cosec}^2 x dx$$

$$= \cot x e^x + \int e^x \operatorname{cosec}^2 x dx - \int e^x \operatorname{cosec}^2 x dx$$

$$= e^x \cot x + c$$

5. $\int e^x \left(\frac{x-1}{2x^2} \right) dx$

Solution:

Given

$$\int e^x \left(\frac{x-1}{2x^2} \right) dx$$

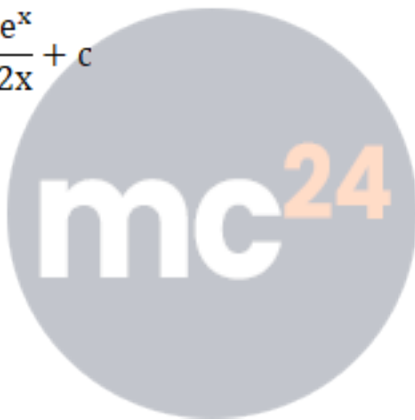
$$\text{Let } I = \int e^x \frac{1}{2x} dx - \int e^x \frac{1}{2x^2} dx$$

Integrating by parts,

$$= \frac{e^x}{2x} - \int e^x \left(\frac{d}{dx} \left(\frac{1}{2x} \right) \right) dx - \int \frac{e^x}{2x^2} dx$$

$$= \frac{e^x}{2x} + \int \frac{e^x}{2x^2} dx - \int \frac{e^x}{2x^2} dx$$

$$= \frac{e^x}{2x} + c$$



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