EXERCISE 19.26

Evaluate the following integrals:

1. $\int e^x(\cos x - \sin x)\, dx$

Solution:

Let $I = \int e^x(\cos x - \sin x)dx$

Using integration by parts,

$$= \int e^x \cos x\, dx - \int e^x \sin x\, dx$$

We know that, $\frac{d}{dx}\cos x = -\sin x$

$$= \cos x \int e^x - \int \frac{d}{dx}\cos x \int e^x dx - \int e^x \sin x\, dx$$

$$= e^x \cos x + \int e^x \sin x\, dx - \int e^x \sin x\, dx$$

$$= e^x \cos x + c$$

2. $\int e^x\left(\frac{1}{x^2} - \frac{2}{x^3}\right) dx$

Solution:

Let $I = \int e^x\left(\frac{1}{x^2} - \frac{2}{x^3}\right) dx$

$$= \int e^x x^{-2} dx - 2\int e^x x^{-3} dx$$

Integrating by parts

$$= x^{-2}\int e^x dx - \int \frac{d}{dx}x^{-2}\int e^x dx - 2\int e^x x^{-3} dx$$

We know that,

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

$$= e^x x^{-2} + 2\int e^x x^{-3} dx - 2\int e^x x^{-3} dx$$

$$= \frac{e^x}{x^2} + c$$

3. $\int e^x \left(\frac{1+\sin x}{1+\cos x}\right) dx$

Solution:

Let $I = \int e^x \left(\frac{1+\sin x}{1+\cos x}\right) dx$

We know that, $\sin^2 x + \cos^2 x = 1$ and $\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$

$$= e^x \left(\frac{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)$$

$$= \frac{e^x\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2}{2\cos^2\frac{x}{2}}$$

$$= \frac{1}{2}e^x \left(\frac{\sin\frac{x}{2} + \cos\frac{x}{2}}{\cos\frac{x}{2}}\right)^2$$

$$= \frac{1}{2}e^x \left[\tan\frac{x}{2} + 1\right]^2$$

$$= \frac{1}{2}e^x \left[1 + \tan\frac{x}{2}\right]^2$$

$$= \frac{1}{2}e^x \left[1 + \tan^2\frac{x}{2} + 2\tan\frac{x}{2}\right]$$

$$= \frac{1}{2}e^x\left[\sec^2\frac{x}{2} + 2\tan\frac{x}{2}\right]$$

$$= e^x\left[\frac{1}{2}\sec^2\frac{x}{2} + \tan\frac{x}{2}\right] \dots \dots (1)$$

Let $\tan\frac{x}{2} = f(x)$

$$f'(x) = \frac{1}{2}\sec^2\frac{x}{2}$$

We know that,

$$\int e^x\{f(x) + f'(x)\}dx = e^x f(x) + c$$

From equation (1), we obtain

$$\int e^x\left(\frac{1 + \sin x}{1 + \cos x}\right)dx = e^x\tan\frac{x}{2} + c$$

4. $\int e^x(\cot x - \boldsymbol{cosec^2 x})\, dx$

Solution:

Let $I = \int e^x(\cot x - \text{cosec}^2 x)dx$

$$= \int e^x \cot x dx - \int e^x \text{cosec}^2 x dx$$

Integrating by parts,

$$= \cot x\int e^x dx - \int \frac{d}{dx}\cot x\int e^x dx - \int e^x \text{cosec}^2 x dx$$

$$= \cot x\, e^x + \int e^x \text{cosec}^2 x dx - \int e^x \text{cosec}^2 x dx$$

$$= e^x \cot x + c$$

5. $\int e^x\left(\frac{x-1}{2x^2}\right)dx$

Solution:

Given

$$\int e^x\left(\frac{x-1}{2x^2}\right)dx$$

Let $I=\int e^x\frac{1}{2x}dx-\int e^x\frac{1}{2x^2}dx$

Integrating by parts,

$$=\frac{e^x}{2x}-\int e^x\left(\frac{d}{dx}\left(\frac{1}{2x}\right)\right)dx-\int\frac{e^x}{2x^2}dx$$

$$=\frac{e^x}{2x}+\int\frac{e^x}{2x^2}dx-\int\frac{e^x}{2x^2}dx$$

$$=\frac{e^x}{2x}+c$$