NCERT Solutions for Class-XII Math

Chapter-2
Exercise- Miscellaneous

NCERT Math Class 12

1. Find the value of $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$.

1. Given that $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$f

We know that $\cos^{-1}(\cos x) = x$ is $x \in [0, \pi]$, which is the principal value branch of $\cos^{-1}$ x.

$\therefore \cos^{-1}\left(\cos\frac{13\pi}{6}\right) = \cos^{-1}\left[\cos\left(2\pi + \frac{\pi}{6}\right)\right]$

$= \cos^{-1}\left(\cos\frac{\pi}{6}\right) = \frac{\pi}{6} \in [0, \pi]$

Hence, $\cos^{-1}\left(\cos\frac{13\pi}{6}\right) = \frac{\pi}{6}$

2. Find the value of $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$.

2. $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$

(For $\tan^{-1}(\tan x)$ type of problem we have to always check whether the angle is in the principal range or not. This angle must be in the principal range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

So here, $\frac{7\pi}{6} \notin \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Now, $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$ can be written as,

$\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$

$= \tan^{-1}\left[\tan\left(\pi + \frac{\pi}{6}\right)\right]$

$= tan^{-1}\left(tan\frac{\pi}{6}\right)$ where, $\frac{\pi}{6}Ú\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ [since, tan(π+x) = tanx]

$= \frac{\pi}{6}$

Hence, $\tan^{-1}\left(\tan\frac{7\pi}{6}\right) = \frac{\pi}{6}$

3. Prove that $2\sin^{-1}\frac{3}{5} = \tan^{-1}\frac{24}{7}$.

3. LHS $= \sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5}$

$= \tan^{-1}\frac{8}{\sqrt{17^2-8^2}} + \tan^{-1}\frac{3}{\sqrt{5^2-3^2}}$ $\left[as \sin^{-1}\frac{a}{b} = \tan^{-1}\frac{a}{\sqrt{b^2-a^2}}\right]$

$= \tan^{-1}\frac{8}{15} + \tan^{-1}\frac{3}{4}$

$= \tan^{-1}\left[\frac{\frac{8}{15}+\frac{3}{4}}{1-\frac{8}{15}\times\frac{3}{4}}\right]$ $\left[as \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$

$= \tan^{-1}\left[\frac{\frac{32+45}{15\times4}}{\frac{15\times4-8\times3}{15\times4}}\right] = \tan^{-1}\left[\frac{\frac{77}{66}}{\frac{36}{60}}\right]$

$= \tan^{-1}\frac{77}{36} = RHS$

4. Prove that $\sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{77}{36}$

4. $\sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{77}{36}$

Let $\sin^{-1}\frac{18}{17} = x.$

Then, $\sin x = \frac{18}{17}$

$\Rightarrow \cos x = \sqrt{1-\left(\frac{18}{17}\right)^2} = \frac{15}{17}$

$\therefore \tan x = \frac{8}{15} \Rightarrow x = \tan^{-1}\frac{8}{15}$

$$\Rightarrow \sin^{-1}\frac{8}{17} = \tan^{-1}\frac{8}{15} \;\;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Let $\sin^{-1}\frac{3}{5} = y.$

Then, $\sin y = \frac{3}{5}$

$$\Rightarrow \cos y = \sqrt{1-\left(\frac{3}{5}\right)^2} = \frac{4}{5}$$

$$\therefore \tan y = \frac{3}{4} \Rightarrow y = \tan^{-1}\frac{3}{4}$$

$$\Rightarrow \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{3}{4} \;\;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Now,

$$\text{L.H.S.} = \sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5}$$

$= \tan^{-1}\frac{8}{15} + \tan^{-1}\frac{3}{4}$ Putting the value from equation (1) and (2))

$$= \tan^{-1}\frac{\frac{8}{15}+\frac{3}{4}}{1-\frac{8}{15}\times\frac{3}{4}} \quad \left[\text{Since, } \tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1-xy}\right]$$

$$= \tan^{-1}\frac{32+45}{60-24}$$

$$= \tan^{-1}\frac{77}{36}$$

=R.H.S.

Hence Proved.

5. Prove that $\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$

5. LHS $= \cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13}$

$$= \tan^{-1}\sqrt{\frac{5^2-4^2}{4}} + \tan^{-1}\frac{13^2-12^2}{12} \qquad \left[as \cos^{-1}\frac{a}{b} = \tan^{-1}\sqrt{\frac{b^2-a^2}{a}}\right]$$

$$\Rightarrow \quad = \tan^{-1}\frac{3}{4} + \tan^{-1}\frac{5}{12}$$

$$= \tan^{-1}\left[\frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\times\frac{5}{12}}\right] \qquad \left[as \tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

$$\Rightarrow \quad = \tan^{-1}\left[\frac{\frac{36+20}{4\times 12}}{\frac{4\times 12-3\times 5}{4\times 12}}\right]=\tan^{-1}\frac{56}{33}$$

$$= \cos^{-1}\frac{33}{\sqrt{56^2+33^2}} \qquad \left[as \tan^{-1}\frac{a}{b}=\cos^{-1}\frac{b}{\sqrt{a^2+b^2}}\right]$$

$$\Rightarrow \quad = \cos^{-1}\frac{33}{\sqrt{4225}}=\cos^{-1}\frac{33}{65}=RHS$$

6. Prove that $\cos^{-1}\frac{12}{13}+\sin^{-1}\frac{3}{5}=\sin^{-1}\frac{56}{65}$

6. $\cos^{-1}\frac{12}{13}+\sin^{-1}\frac{3}{5}=\sin^{-1}\frac{56}{65}$

We can also solve this problem by using the identity Sin(A + B) = sinA cosB + cosA sinB

Let $\sin^{-1}\frac{3}{5}=A$ and $\cos^{-1}\frac{12}{13}=B$

So, $\sin A=\frac{4}{5}$ and $\cos B=\frac{12}{13}$

Hence, $\cos A=\frac{3}{5}$ and $\sin B=\frac{5}{13}$

As R.H.S. is $\sin^{-1}$ we will use sin (A+B)

$\therefore\ \sin(A+B)=\sin A\cos B+\cos A\sin B$

$=\frac{4}{5}\times\frac{12}{13}+\frac{3}{5}\times\frac{5}{13}=\frac{48}{65}\times\frac{15}{65}=\frac{56}{65}$

Thus $A+B=\sin^{-1}\frac{56}{65}$

= R.H.S.

Hence Proved.

7. Prove that $\tan^{-1}\frac{63}{16}=\sin^{-1}\frac{5}{13}+\cos^{-1}\frac{3}{5}$

7. RHS $=\sin^{-1}\frac{5}{13}+\cos^{-1}\frac{3}{5}$

$$=\tan^{-1}\frac{5}{\sqrt{13^2-5^2}}+\tan^{-1}\frac{\sqrt{5^2-3^2}}{3} \qquad \left[as \cos^{-1}\frac{a}{b}=\tan^{-1}\frac{\sqrt{a^2-a^2}}{a}\ and \sin^{-1}\frac{a}{b}=\tan^{-1}\frac{a}{\sqrt{b^2-a^2}}\right]$$

$\Rightarrow \quad = \tan^{-1}\frac{5}{12} + \tan^{-1}\frac{4}{3}$

$= \tan^{-1}\left[\dfrac{\frac{5}{12}+\frac{4}{3}}{1-\frac{5}{12}\times\frac{4}{3}}\right]$ $\left[as \tan^{-1}x \tan^{-1}y = \tan^{-1}\left(\dfrac{x+y}{1-xy}\right)\right]$

$\Rightarrow \quad = \tan^{-1}\left[\dfrac{\frac{12+48}{12\times 3}}{\frac{12\times 3-5\times 4}{12\times 3}}\right] = \tan^{-1}\frac{63}{16} = RHS$

8. Prove that $\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$

8. L.H.S. $= \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{8}$

$= \tan^{-1}\dfrac{\frac{1}{5}+\frac{1}{7}}{1-\frac{1}{5}\times\frac{1}{7}} + \tan^{-1}\dfrac{\frac{1}{3}+\frac{1}{8}}{1-\frac{1}{3}\times\frac{1}{38}}$ $\left[\text{Since, } \tan^{-1}x - \tan^{-1}y = \tan^{-1}\dfrac{x-y}{1-xy}\right]$

$= \tan^{-1}\dfrac{7+5}{35-1} + \tan^{-1}\dfrac{8+3}{24-1}$

$= \tan^{-1}\dfrac{12}{34} + \tan^{-1}\dfrac{11}{23}$

$= \tan^{-1}\dfrac{6}{17} + \tan^{-1}\dfrac{11}{23}$

$= \tan^{-1}\dfrac{\frac{6}{17}+\frac{11}{23}}{1-\frac{6}{17}\times\frac{11}{23}}$ $\left[\text{Since, } \tan^{-1}x - \tan^{-1}y = \tan^{-1}\dfrac{x-y}{1-xy}\right]$

$= \tan^{-1}\dfrac{138+187}{391-66}$

$= \tan^{-1}\dfrac{325}{325} == \tan^{-1}1 = \dfrac{\pi}{4}$

= R.H.S. Hence Proved.

9. Prove that $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\dfrac{1-x}{1+x}\right), x \in [0,1]$

9. LHS $= \tan^{-1}\sqrt{x} = \frac{1}{2}\times 2\tan^{-1}\sqrt{x} = \frac{1}{2}\times 2\tan^{-1}\sqrt{x}$

$$=\frac{1}{2}\cos^{-1}\left[\frac{1-(\sqrt{x})^2}{1+(\sqrt{x})^2}\right] \qquad \left[as\,2\tan^{-1}x=\cos^{-1}\left[\frac{1-x^2}{1+x^2}\right]\right]$$

$$=\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)=RHS$$

10. Prove that $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2}, x\in\left(0,\frac{\pi}{4}\right)$

10. $\cot^{-1}\left(\frac{\sqrt{1+\text{sinx}}+\sqrt{1-\text{sinx}}}{\sqrt{1+\text{sinx}}-\sqrt{1-\text{sinx}}}\right)=\frac{\text{x}}{2}$

Consider, $\left(\frac{\sqrt{1+\text{sinx}}+\sqrt{1-\text{sinx}}}{\sqrt{1+\text{sinx}}-\sqrt{1-\text{sinx}}}\right)$

On Rationalizing, we get,

$$=\frac{\left(\sqrt{1+\text{sinx}}+\sqrt{1-\text{sinx}}\right)^2}{\left(\sqrt{1+\text{sinx}}\right)^2-\left(\sqrt{1-\text{sinx}}\right)^2}$$

$$=\frac{(1+\text{sinx})+(1-\text{sinx})+2\sqrt{(1+\text{sinx})(1-\text{sinx})}}{1+\text{sinx}-1+\text{sinx}}$$

$$=\frac{2\left(1+\sqrt{1-\sin^2}\right)}{2\text{sinx}}=\frac{1-\text{cosx}}{\text{sinx}}=\frac{2\cos^2\frac{\text{x}}{2}}{2\sin\frac{\text{x}}{2}\cos\frac{\text{x}}{2}}=\cot\frac{\text{x}}{2}$$

Now,

$$\text{L.H.S.}=\cot^{-1}\left(\frac{\sqrt{1+\text{sinx}}+\sqrt{1-\text{sinx}}}{\sqrt{1+\text{sinx}}-\sqrt{1-\text{sinx}}}\right)$$

$$=\cot^{-1}\left(\cot\frac{\text{x}}{2}\right)=\frac{\text{x}}{2}=\text{R.H.S.}$$

Hence Proved

11. Prove that $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x, -\frac{1}{\sqrt{2}}x\le x\le 1$

11. LHS $=\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$

$$=\tan^{-1}\left(\frac{\sqrt{1+\cos y}-\sqrt{1-\cos y}}{\sqrt{1+\cos y}+\sqrt{1-\cos y}}\right) \qquad [\text{Let } x=\cos y]$$

$$= \tan^{-1}\left(\frac{\sqrt{2\cos^2\frac{y}{2}}-\sqrt{2\sin^2\frac{y}{2}}}{\sqrt{2\cos^2\frac{y}{2}}+\sqrt{2\sin^2\frac{y}{2}}}\right) \quad \left[as 1+\cos y = 2\cos^2\frac{y}{2} and 1-\cos y = 2\sin^2\frac{y}{3}\right]$$

$$= \tan^{-1}\left(\frac{\sqrt{2}\cos\frac{y}{2}-\sqrt{2}\sin^2\frac{y}{2}}{\sqrt{2}\cos^2\frac{y}{2}+\sqrt{2}\sin^2\frac{y}{2}}\right)$$

$$= \tan^{-1}\left(\frac{1-\tan\frac{y}{2}}{1+\tan\frac{y}{2}}\right) \quad \left[\text{dividingeachtermby}\sqrt{2}\cos\frac{y}{2}\right]$$

$$= \tan^{-1}\left(\frac{\tan\frac{\pi}{4}-\tan\frac{y}{2}}{1+\tan\frac{\pi}{4}.\tan\frac{y}{2}}\right) = \tan^{-1}\left[\tan\left(\frac{x}{4}-\frac{y}{2}\right)\right]$$

$$= \frac{\pi}{4}-\frac{y}{2} = \frac{\pi}{4}-\frac{1}{2}\cos^{-1}x = RHS$$

12. Prove that $\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\frac{1}{3} = \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$

12. $\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\frac{1}{3} = \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$

Now, L.H.S. $= \frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\frac{1}{3}$

$= \frac{9}{4}\left(\frac{\pi}{2}-\sin^{-1}\frac{1}{3}\right)$

$= \frac{9}{4}\left(\cos^{-1}\frac{1}{3}\right)$…………………..Eq.(1)

Now, Let

$\cos^{-1}\frac{1}{3} = x$. Then $\cos x = \frac{1}{3} \Rightarrow \sin x = \sqrt{1-\left(\frac{1}{3}\right)^2} = \frac{2\sqrt{2}}{3}$

$\therefore x = \sin^{-1}\frac{2\sqrt{2}}{3} \Rightarrow \cos^{-1}\frac{1}{3} = \sin^{-1}\frac{2\sqrt{2}}{3}$

L.H.S. $= \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$

= R.H.S.

Hence Proved

13. Solve for x : 2tan^{-1} (cos x) = tan^{-1} (2cosec x)

13. Given that 2tan^{-1} (cos x) = tan^{-1} (2cosec x)

$\Rightarrow \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1}(2\cos ecx)$ $\left[as 2\tan^{-1} x = \tan^{-1}\frac{2x}{1-x^2}\right]$

$\Rightarrow \frac{2\cos x}{\sin^2 x} = 2\text{cosec}\, x$

$\Rightarrow \frac{2\cos x}{\sin^2 x} = \frac{2}{\sin x} \Rightarrow 2\sin x.\cos x = 2\sin^2 x$

$\Rightarrow 2\sin x.\cos x - 2\sin^2 x = 0$ $\Rightarrow 2\sin x(\cos x - \sin x) = 0$

$\Rightarrow$ 2 sin x = 0 or cos x – sin x = 0

But sin x ≠ 0 as it does not satisfy the equation

$\therefore$ cos x – sin x = 0 $\Rightarrow$ cos x = sin x $\Rightarrow$ tan x = 1

$\therefore x = \frac{\pi}{4}$

14. Solve for x : $\tan^{-1}\frac{1-x}{1+x} = \frac{1}{2}\tan^{-1} x, (x > 0)$

14. $\tan^{-1}\frac{1-\text{x}}{1+\text{x}} = \frac{1}{2}\tan^{-1}\text{x}$

$\Rightarrow \tan^{-1}1 - \tan^{-1}\text{x} = \frac{1}{2}\tan^{-1}\text{x}$

$\Rightarrow \frac{\pi}{4} = \frac{3}{2}\tan^{-1}\text{x}$

$\Rightarrow \tan^{-1}\text{x} = \frac{\pi}{6}$

$\Rightarrow \text{x} = \tan\frac{\pi}{6}$

Hence, $\text{x} = \frac{\pi}{6}$

15. $\sin(\tan^{-1} x), |x| < 1$ is equal to

(A) $\frac{x}{\sqrt{1-x^2}}$ (B) $\frac{1}{\sqrt{1-x^2}}$

(C) $\frac{1}{\sqrt{1+x^2}}$ (D) $\frac{x}{\sqrt{1+x^2}}$

15. Given that : sin (tan^{-1} x)

$= \sin\left(\sin^{-1}\frac{x}{\sqrt{1+x^2}}\right)$ $\left[as \tan^{-1}\frac{a}{b} = \sin^{-1}\frac{a}{\sqrt{a^2+b^2}}\right]$

$=\frac{x}{\sqrt{1+x^2}}$

Hence, the option (D) is correct.

16. $\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$, then x is equal to

(A) $0,\frac{1}{2}$

(B) $1,\frac{1}{2}$

(C) 0

(D) $\frac{1}{2}$

16. The correct option is C

Explanation: $\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$

Now we will put Now, we will put x= siny in the given equation, and we get,

$\sin^{-1}(1-\text{siny})-2\sin^{-1}\text{siny}=\frac{\pi}{2}$

$\Rightarrow \sin^{-1}(1-\text{siny})-2y=\frac{\pi}{2}$

$\Rightarrow \sin^{-1}(1-\text{siny})=\frac{\pi}{2}+2y$

$\Rightarrow 1-\text{siny}=\sin\left(\frac{\pi}{2}+2y\right)$

$\Longrightarrow$ 1 – siny = cos2y as $\sin\left(\frac{\pi}{2}+x\right)=\cos x$

$\Longrightarrow$ 1 - cos2y = sin y

$\Longrightarrow$ $2\sin^2 y$ = sin y

$\Longrightarrow$ siny(2siny - 1) = 0

$\Longrightarrow$ siny = 0 or 1/2

∴ x = 0 or 1/2

Now, if we put $x=\frac{1}{2}$, then we will see that,

L.H.S. $=\sin^{-1}\left(1-\frac{1}{2}\right)-2\sin^{-1}\frac{1}{2}$

$=\sin^{-1}\left(\frac{1}{2}\right)-2\sin^{-1}\frac{1}{2}$

$=-\sin^{-1}\frac{1}{2}$

$=-\frac{\pi}{6}\neq\frac{\pi}{2}\neq$ R.H.S

Hence, $x = \frac{1}{2}$ is not the solution of the given equation.

Thus, x =

17. $\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\frac{x-y}{x+y}$ is equal to

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$ (D) $-\frac{3\pi}{4}$

17. $\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\frac{x-y}{x+y}$

$$= \tan^{-1}\left[\frac{\frac{x}{y} - \frac{x-y}{x+y}}{1 + \frac{x}{y} \times \frac{x-y}{x+y}}\right] \qquad \left[as \tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)\right]$$

$$= \tan^{-1}\left[\frac{\frac{x(x+y) - y(x-y)}{y(x+y)}}{\frac{y(x+y) + x(x-y)}{y(x+y)}}\right]$$

$$= \tan^{-1}\left[\frac{x^2 + xy - xy + y^2}{xy + y^2 + x^2 - xy}\right]$$

$$= \tan^{-1}\left[\frac{x^2 + y^2}{x^2 + y^2}\right]$$

$$= \tan^{-1}1 = \frac{\pi}{4}$$

Hence, the option (C) is correct