EXERCISE 30.1

Find the derivative of f(x) = 3x at x = 2 Solution:
Given:
f(x) = 3x
By using the derivative formula,

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$ {Where, h is a small positive number}

Derivative of f(x) = 3x at x = 2 is given as

$$f'(2) = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h\to 0} \frac{3(2 + h) - 3 \times 2}{h}$$

$$= \lim_{h\to 0} \frac{3h + 6 - 6}{h} = \lim_{h\to 0} \frac{3h}{h}$$

$$= \lim_{h\to 0} 3 = 3$$

Hence,
Derivative of f(x) = 3x at x = 2 is 3

1. Find the derivative of f(x) = x^2 – 2 at x = 10
Solution:
Given:
f(x) = x^2 – 2
By using the derivative formula,
Derivative of x^2 – 2 at x = 10 is given as

$$f'(10) = \lim_{h\to 0} \frac{f(10+h) - f(10)}{h}$$

$$= \lim_{h\to 0} \frac{(10 + h)^2 - 2 - (10^2 - 2)}{h}$$

$$= \lim_{h\to 0} \frac{100 + h^2 + 20h - 2 - 100 + 2}{h} = \lim_{h\to 0} \frac{h^2 + 20h}{h}$$

$$= \lim_{h\to 0} \frac{h(h + 20)}{h} = \lim_{h\to 0} (h + 20)$$

= 0 + 20 = 20

Hence,

Derivative of f(x) = $x^2 - 2$ at x = 10 is 20

2. Find the derivative of f(x) = 99x at x = 100.
Solution:
Given:
f(x) = 99x
By using the derivative formula,

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$ {Where h is a very small positive number}

Derivative of 99x at x = 100 is given as

$$f'(100) = \lim_{h \to 0} \frac{f(100+h) - f(100)}{h}$$
$$= \lim_{h \to 0} \frac{99(100 + h) - 99 \times 100}{h}$$
$$= \lim_{h \to 0} \frac{9900 + 99h - 9900}{h} = \lim_{h \to 0} \frac{99h}{h}$$
$$= \lim_{h \to 0} 99 = 99$$

Hence,
Derivative of f(x) = 99x at x = 100 is 99

3. Find the derivative of f(x) = x at x = 1
Solution:
Given:
f(x) = x
By using the derivative formula,

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$ {Where h is a very small positive number}

Derivative of x at x = 1 is given as

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$
$$= \lim_{h \to 0} \frac{(1 + h) - 1}{h}$$
$$= \lim_{h \to 0} \frac{1 + h - 1}{h} = \lim_{h \to 0} \frac{h}{h}$$
$$= \lim_{h \to 0} 1 = 1$$

Hence,
Derivative of f(x) = x at x = 1 is 1

4. Find the derivative of f(x) = cos x at x = 0

Solution:

Given:

f(x) = cos x

By using the derivative formula,

$$f'(a) = \lim_{h\to0} \frac{f(a+h)-f(a)}{h}$$ {Where h is a very small positive number}

Derivative of cos x at x = 0 is given as

$$f'(0) = \lim_{h\to0} \frac{f(0+h)-f(0)}{h}$$

$$= \lim_{h\to0} \frac{\cos(h)-\cos 0}{h}$$

$$= \lim_{h\to0} \frac{\cos h - 1}{h}$$

Let us try and evaluate the limit.

We know that 1 – cos x = 2 sin²(x/2)

So,

$$= \lim_{h\to0} \frac{-(1-\cos h)}{h} = -\lim_{h\to0} \frac{2\sin^2\frac{h}{2}}{h}$$

Divide the numerator and denominator by 2 to get the form (sin x)/x to apply sandwich theorem.

$$= -\lim_{h\to0} \frac{\frac{2\sin^2\frac{h}{2}}{2}}{\frac{h^2}{2}} \times h$$

By using algebra of limits we get

$$= -\lim_{h\to0} \left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right) \times \lim_{h\to0} h$$

[By using the formula: $\lim_{x\to0} \frac{\sin x}{x} = 1$]

f' (0) = – 1×0 = 0

∴ Derivative of f(x) = cos x at x = 0 is 0

5. Find the derivative of f(x) = tan x at x = 0

Solution:

Given:

f(x) = tan x

By using the derivative formula,

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$ {Where h is a small positive number}

Derivative of cos x at x = 0 is given as

$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{\tan(h) - \tan 0}{h}$$

$$= \lim_{h \to 0} \frac{\tan h}{h}$$ [Since it is of indeterminate form]

By using the formula: $\lim_{x \to 0} \frac{\tan x}{x} = 1$ {i.e., sandwich theorem}

f' (0) = 1

∴ Derivative of f(x) = tan x at x = 0 is 1

6. Find the derivatives of the following functions at the indicated points:

(i) sin x at x = π/2

(ii) x at x = 1

(iii) 2 cos x at x = π/2

(iv) sin 2xat x = π/2

Solution:

(i) sin x at x = π/2

Given:

f (x) = sin x

By using the derivative formula,

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$ {Where h is a small positive number}

Derivative of sin x at x = π/2 is given as

$$f'\left(\frac{\pi}{2}\right) = \lim_{h \to 0} \frac{f(\frac{\pi}{2}+h) - f(\frac{\pi}{2})}{h}$$

$$= \lim_{h \to 0} \frac{\sin\left(\frac{\pi}{2} + h\right) - \sin\frac{\pi}{2}}{h}$$

$$= \lim_{h \to 0} \frac{\cos h - 1}{h}$$ {∵ sin (π/2 + x) = cos x}

[Since it is of indeterminate form. Let us try to evaluate the limit.]

We know that 1 – cos x = 2 $\sin^2(x/2)$

$$= \lim_{h \to 0} \frac{-(1 - \cos h)}{h} = -\lim_{h \to 0} \frac{2\sin^2 \frac{h}{2}}{h}$$

Divide the numerator and denominator by 2 to get the form (sin x)/x to apply sandwich theorem.

$$= -\lim_{h \to 0} \frac{\frac{2\sin^2 \frac{h}{2}}{2}}{\frac{h^2}{2}} \times h$$

Using algebra of limits we get

$$= -\lim_{h \to 0} \left(\frac{\sin \frac{h}{2}}{\frac{h}{2}} \right) \times \lim_{h \to 0} h$$

[By using the formula: $\lim_{x \to 0} \frac{\sin x}{x} = 1$]

f' (π/2) = – 1×0 = 0

∴ Derivative of f(x) = sin x at x = π/2 is 0

(ii) x at x = 1

Given:

f (x) = x

By using the derivative formula,

$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ {Where h is a very small positive number}

Derivative of x at x = 1 is given as

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{(1 + h) - 1}{h}$$

$$= \lim_{h \to 0} \frac{1 + h - 1}{h} = \lim_{h \to 0} \frac{h}{h}$$

$$= \lim_{h \to 0} 1 = 1$$

Hence,

Derivative of f(x) = x at x = 1 is 1

(iii) 2 cos x at x = π/2

Given:

f (x) = 2 cos x

By using the derivative formula,

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$ {Where h is a small positive number}

Derivative of 2cos x at x = π/2 is given as

$$f'\left(\frac{\pi}{2}\right) = \lim_{h \to 0} \frac{f(\frac{\pi}{2}+h) - f(\frac{\pi}{2})}{h}$$

$$= \lim_{h \to 0} \frac{2\cos\left(\frac{\pi}{2} + h\right) - 2\cos\frac{\pi}{2}}{h}$$

$$= \lim_{h \to 0} \frac{-2\sin h}{h}$$ {∵ cos (π/2 + x) = – sin x}

[Since it is of indeterminate form]

$$= -2 \lim_{h \to 0} \frac{\sin h}{h}$$

By using the formula: $\lim_{x \to 0} \frac{\sin x}{x} = 1$

f' (π/2) = – 2×1 = – 2

∴ Derivative of f(x) = 2cos x at x = π/2 is – 2

(iv) sin 2xat x = π/2

Solution:

Given:

f (x) = sin 2x

By using the derivative formula,

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$ {Where h is a small positive number}

Derivative of sin 2x at x = π/2 is given as

$$f'\left(\frac{\pi}{2}\right) = \lim_{h \to 0} \frac{f(\frac{\pi}{2}+h) - f(\frac{\pi}{2})}{h}$$

$$= \lim_{h \to 0} \frac{\sin\left\{2 \times \left(\frac{\pi}{2} + h\right)\right\} - \sin 2 \times \frac{\pi}{2}}{h}$$

$$= \lim_{h \to 0} \frac{\sin(\pi + 2h) - \sin \pi}{h}$$ {∵ sin (π + x) = – sin x & sin π = 0}

$$= \lim_{h \to 0} \frac{-\sin 2h - 0}{h}$$

$$= -\lim_{h \to 0} \frac{\sin 2h}{h}$$

[Since it is of indeterminate form. We shall apply sandwich theorem to evaluate the limit.]

Now, multiply numerator and denominator by 2, we get

$$= -\lim_{h \to 0} \frac{\sin 2h}{2h} \times 2 = -2 \lim_{h \to 0} \frac{\sin 2h}{2h}$$

By using the formula: $\lim_{x \to 0} \frac{\sin x}{x} = 1$

f' (π/2) = – 2×1 = – 2

∴ Derivative of f(x) = sin 2x at x = π/2 is – 2