

## 5. Matrices

### Exercise 5A

#### 1. Question

$$\text{If } A = \begin{bmatrix} 5 & -2 & 6 & 1 \\ 7 & 0 & 8 & -3 \\ \sqrt{2} & \frac{3}{5} & 4 & 3 \end{bmatrix} \text{ then write}$$

- the number of rows in A,
- the number of columns in A,
- the order of the matrix A,
- the number of all entries in A,
- the elements  $a_{23}$ ,  $a_{31}$ ,  $a_{14}$ ,  $a_{33}$ ,  $a_{22}$  of A.

#### Answer

(i) Number of rows = 3

(ii) Number of columns = 4

(iii) Order of matrix = Number of rows  $\times$  Number of columns = (3  $\times$  4)

(iv) Number of entries = (Number of rows)  $\times$  (Number of columns)

$$= 3 \times 4$$

$$= 12$$

(v)  $a_{ij}$  = element of  $i^{\text{th}}$  row and  $j^{\text{th}}$  column

$$a_{23} = 8$$

$$a_{31} = \sqrt{2}$$

$$a_{14} = 1$$

$$a_{33} = 4$$

$$a_{22} = 0$$

#### 2. Question

Write the order of each of the following matrices:

$$\text{i. } A = \begin{bmatrix} 3 & 5 & 4 & -2 \\ 0 & \sqrt{3} & -1 & \frac{4}{9} \end{bmatrix}$$

$$\text{ii. } B = \begin{bmatrix} 6 & -5 \\ \frac{1}{2} & \frac{3}{4} \\ -2 & -1 \end{bmatrix}$$

$$\text{iii. } C = [7 - \sqrt{2} \quad 5 \quad 0]$$

$$\text{iv. } D = [8 \quad -3]$$

$$\text{v. } E = \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}$$



vi, F = [6]

**Answer**

$$\text{i. } A = \begin{bmatrix} 3 & 5 & 4 & -2 \\ 0 & \sqrt{3} & -1 & \frac{4}{9} \end{bmatrix}$$

Order of matrix = Number of rows x Number of columns

= (2 x 4)

$$\text{ii. } B = \begin{bmatrix} 6 & -5 \\ \frac{1}{2} & \frac{3}{4} \\ -2 & -1 \end{bmatrix}$$

Order of matrix = Number of rows x Number of columns

= (4 x 2)

$$\text{iii. } C = [7 - \sqrt{2} \quad 5 \quad 0]$$

Order of matrix = Number of rows x Number of columns

= (1 x 4)

$$\text{iv. } D = [8 \quad -3]$$

Order of matrix = Number of rows x Number of columns

= (1 x 2)

$$\text{v. } E = \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}$$



Order of matrix = Number of rows x Number of columns

= (3 x 1)

vi, F = [6]

Order of matrix = Number of rows x Number of columns

= (1 x 1)

### 3. Question

If a matrix has 18 elements, what are the possible orders it can have?

**Answer**

Number of entries = (Number of rows) x (Number of columns) = 18

If order is (a x b) then, Number of entries = a x b

So now a x b = 18 (in this case)

Possible cases are (1 x 18), (2 x 9), (3 x 6), (6 x 3), (9 x 2), (18 x 1)

Conclusion: If a matrix has 18 elements, then possible orders are (1 x 18), (2 x 9), (3 x 6), (6 x 3), (9 x 2), (18 x 1)

### 4. Question

Find all possible orders of matrices having 7 elements.

**Answer**

Number of entries = (Number of rows) x (Number of columns) = 7

If order is (a x b) then, Number of entries = a x b

So now a x b = 7 (in this case)

Possible cases are  $(1 \times 7)$ ,  $(7 \times 1)$

Conclusion: If a matrix has 18 elements, then possible orders are  $(1 \times 7)$ ,  $(7 \times 1)$

### 5. Question

Construct a  $3 \times 2$  matrix whose elements are given by  $a_{ij} = (2i - j)$ .

#### Answer

Given:  $a_{ij} = (2i - j)$

$$\text{Now, } a_{11} = (2 \times 1 - 1) = 2 - 1 = 1$$

$$a_{12} = 2 \times 1 - 2 = 2 - 2 = 0$$

$$a_{21} = 2 \times 2 - 1 = 4 - 1 = 3$$

$$a_{22} = 2 \times 2 - 2 = 4 - 2 = 2$$

$$a_{31} = 2 \times 3 - 1 = 6 - 1 = 5$$

$$a_{32} = 2 \times 3 - 2 = 6 - 2 = 4$$

Therefore,

$$A = \begin{bmatrix} 1 & 0 \\ 3 & 2 \\ 5 & 4 \end{bmatrix}$$

### 6. Question

Construct a  $4 \times 3$  matrix whose elements are given by  $a_{ij} = \frac{i}{j}$ .

#### Answer

It is  $(4 \times 3)$  matrix. So it has 4 rows and 3 columns

Given  $a_{ij} = \frac{i}{j}$ .

$$\text{So, } a_{11} = 1, a_{12} = \frac{1}{2}, a_{13} = \frac{1}{3},$$

$$a_{21} = 2, a_{22} = 1, a_{23} = \frac{2}{3},$$

$$a_{31} = 3, a_{32} = \frac{3}{2}, a_{33} = 1,$$

$$a_{41} = 4, a_{42} = 2, a_{43} = \frac{4}{3}$$

$$\text{So, the matrix} = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ 2 & 1 & \frac{2}{3} \\ 3 & \frac{3}{2} & 1 \\ 4 & 2 & \frac{4}{3} \end{bmatrix}$$

$$\text{Conclusion: Therefore, Matrix is } \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ 2 & 1 & \frac{2}{3} \\ 3 & \frac{3}{2} & 1 \\ 4 & 2 & \frac{4}{3} \end{bmatrix}$$

### 7. Question

Construct a  $2 \times 2$  matrix whose elements are  $a_{ij} = \frac{(i+2j)^2}{2}$ .

#### Answer

It is a  $(2 \times 2)$  matrix. So, it has 2 rows and 2 columns.

Given  $a_{ij} = \frac{(i+2j)^2}{2}$

So,  $a_{11} = \frac{9}{2}, a_{12} = \frac{25}{2},$

$a_{21} = 8, a_{22} = 18$

So, the matrix =  $\begin{bmatrix} \frac{9}{2} & \frac{25}{2} \\ 8 & 18 \end{bmatrix}$

Conclusion: Therefore, Matrix is =  $\begin{bmatrix} \frac{9}{2} & \frac{25}{2} \\ 8 & 18 \end{bmatrix}$

**8. Question**

Construct a  $2 \times 3$  matrix whose elements are  $a_{ij} = \frac{(i-2j)^2}{2}$ .

**Answer**

It is a  $(2 \times 3)$  matrix. So, it has 2 rows and 3 columns.

Given  $a_{ij} = \frac{(i-2j)^2}{2}$

So,  $a_{11} = \frac{1}{2}, a_{12} = \frac{9}{2}, a_{13} = \frac{25}{2},$

$a_{21} = 0, a_{22} = 2, a_{23} = 8$

So, the matrix =  $\begin{bmatrix} \frac{1}{2} & \frac{9}{2} & \frac{25}{2} \\ 0 & 2 & 8 \end{bmatrix}$

Conclusion: Therefore, Matrix is  $\begin{bmatrix} \frac{1}{2} & \frac{9}{2} & \frac{25}{2} \\ 0 & 2 & 8 \end{bmatrix}$

**9. Question**

Construct a  $3 \times 4$  matrix whose elements are given by  $a_{ij} = \frac{1}{2} |-3i + j|$ .

**Answer**

It is a  $(3 \times 4)$  matrix. So, it has 3 rows and 4 columns.

Given  $a_{ij} = \frac{|-3i+j|}{2}$

So,  $a_{11} = 1, a_{12} = \frac{1}{2}, a_{13} = 0, a_{14} = \frac{1}{2},$

$a_{21} = \frac{5}{2}, a_{22} = 2, a_{23} = \frac{3}{2}, a_{24} = 1,$

$a_{31} = 4, a_{32} = \frac{7}{2}, a_{33} = 3, a_{34} = \frac{5}{2}$

So, the matrix =  $\begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$

Conclusion: Therefore, Matrix is  $\begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$

**Exercise 5B**

**1. Question**

If  $A = \begin{bmatrix} 2 & -3 & 5 \\ -1 & 0 & 3 \end{bmatrix}$  and  $B = \begin{bmatrix} 3 & 2 & -2 \\ 4 & -3 & 1 \end{bmatrix}$ , verify that  $(A + B) = (B + A)$ .

**Answer**

$$A + B = \begin{bmatrix} 2 & -3 & 5 \\ -1 & 0 & 3 \end{bmatrix} + \begin{bmatrix} 3 & 2 & -2 \\ 4 & -3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 3 \\ 3 & -3 & 4 \end{bmatrix}$$

$$B + A = \begin{bmatrix} 3 & 2 & -2 \\ 4 & -3 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -3 & 5 \\ -1 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 3 \\ 3 & -3 & 4 \end{bmatrix} = B + A$$

Therefore,  $A + B = B + A$

This is true for any matrix

Conclusion:  $A + B = B + A$

**2. Question**

If  $A = \begin{bmatrix} 3 & 5 \\ -2 & 0 \\ 6 & -1 \end{bmatrix}$ ,  $B = \begin{bmatrix} -1 & -3 \\ 4 & 2 \\ -2 & 3 \end{bmatrix}$  and  $C = \begin{bmatrix} 0 & 2 \\ 3 & -4 \\ 1 & 6 \end{bmatrix}$ , verify that  $(A + B) + C = A + (B + C)$ .

**Answer**

$$(A+B)+C = \left( \begin{bmatrix} 3 & 5 \\ -2 & 0 \\ 6 & -1 \end{bmatrix} + \begin{bmatrix} -1 & -3 \\ 4 & 2 \\ -2 & 3 \end{bmatrix} \right) + \begin{bmatrix} 0 & 2 \\ 3 & -4 \\ 1 & 6 \end{bmatrix}$$

$$= \left( \begin{bmatrix} 2 & 2 \\ 2 & 2 \\ 4 & 2 \end{bmatrix} \right) + \begin{bmatrix} 0 & 2 \\ 3 & -4 \\ 1 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 \\ 5 & -2 \\ 5 & 8 \end{bmatrix}$$

$$A+(B+C) = \begin{bmatrix} 3 & 5 \\ -2 & 0 \\ 6 & -1 \end{bmatrix} + \left( \begin{bmatrix} -1 & -3 \\ 4 & 2 \\ -2 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 3 & -4 \\ 1 & 6 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 3 & 5 \\ -2 & 0 \\ 6 & -1 \end{bmatrix} + \begin{bmatrix} -1 & -1 \\ 7 & -2 \\ -1 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 \\ 5 & -2 \\ 5 & 8 \end{bmatrix}$$

Therefore,  $(A+B)+C = A+(B+C)$

It is true for any matrix

Conclusion:  $(A+B)+C = A+(B+C)$

**3. Question**

If  $A = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & -3 \end{bmatrix}$  and  $B = \begin{bmatrix} -2 & 0 & 4 \\ 5 & -3 & 2 \end{bmatrix}$ , find  $(2A - B)$ .

**Answer**

$$2A = 2 \left( \begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & -3 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 6 & 2 & 4 \\ 2 & 4 & -6 \end{bmatrix}$$

$$(2A-B) = \begin{bmatrix} 6 & 2 & 4 \\ 2 & 4 & -6 \end{bmatrix} - \begin{bmatrix} -2 & 0 & 4 \\ 5 & -3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 2 & 0 \\ -3 & 7 & -8 \end{bmatrix}$$

$$\text{Conclusion: } (2A-B) = \begin{bmatrix} 8 & 2 & 0 \\ -3 & 7 & -8 \end{bmatrix}$$



#### 4. Question

Let  $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$  and  $C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$ . Find:

i.  $A + 2B$

ii.  $B - 4C$

iii.  $A - 2B + 3C$

#### Answer

$$\begin{aligned} A + 2B &= \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} + 2\left(\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}\right) \\ &= \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 6 \\ -4 & 10 \end{bmatrix} \\ &= \begin{bmatrix} 4 & 10 \\ -1 & 12 \end{bmatrix} \end{aligned}$$

Conclusion:  $(A+2B) = \begin{bmatrix} 4 & 10 \\ -1 & 12 \end{bmatrix}$

ii.  $B - 4C$

$$\begin{aligned} B-4C &= \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} - 4\left(\begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}\right) \\ &= \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} - \begin{bmatrix} -8 & 20 \\ 12 & 16 \end{bmatrix} \\ &= \begin{bmatrix} 9 & -17 \\ -14 & -11 \end{bmatrix} \end{aligned}$$

Conclusion:  $B-4C = \begin{bmatrix} 9 & -17 \\ -14 & -11 \end{bmatrix}$

iii.  $A - 2B + 3C$

$$\begin{aligned} A-2B+3C &= \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - 2\left(\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}\right) + 3\left(\begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}\right) \\ &= \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 6 \\ -4 & 10 \end{bmatrix} + \begin{bmatrix} -6 & 15 \\ 9 & 12 \end{bmatrix} \\ &= \begin{bmatrix} -6 & 13 \\ 16 & 4 \end{bmatrix} \end{aligned}$$

Conclusion:  $A-2B+3C = \begin{bmatrix} -6 & 13 \\ 16 & 4 \end{bmatrix}$

#### 5. Question

Let  $A = \begin{bmatrix} 0 & 1 & -2 \\ 5 & -1 & -4 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & -3 & -1 \\ 0 & -2 & 5 \end{bmatrix}$  and  $C = \begin{bmatrix} 2 & -5 & 1 \\ -4 & 0 & 6 \end{bmatrix}$ . Compute  $5A - 3B + 4C$ .

#### Answer

$$\begin{aligned} 5A-3B+4C &= 5\left(\begin{bmatrix} 0 & 1 & -2 \\ 5 & -1 & -4 \end{bmatrix}\right) - 3\left(\begin{bmatrix} 1 & -3 & -1 \\ 0 & -2 & 5 \end{bmatrix}\right) + 4\left(\begin{bmatrix} 2 & -5 & 1 \\ -4 & 0 & 6 \end{bmatrix}\right) \\ &= \left(\begin{bmatrix} 0 & 5 & -10 \\ 25 & -5 & -20 \end{bmatrix}\right) - \left(\begin{bmatrix} 3 & -9 & -3 \\ 0 & -6 & 15 \end{bmatrix}\right) + \left(\begin{bmatrix} 8 & -20 & 4 \\ -16 & 0 & 24 \end{bmatrix}\right) \\ &= \begin{bmatrix} -3 & 14 & -7 \\ 25 & 1 & -35 \end{bmatrix} + \begin{bmatrix} 8 & -20 & 4 \\ -16 & 0 & 24 \end{bmatrix} \\ &= \begin{bmatrix} 5 & -6 & -3 \\ 9 & 1 & -11 \end{bmatrix} \end{aligned}$$

Conclusion:  $5A-3B+4C = \begin{bmatrix} 5 & -6 & -3 \\ 9 & 1 & -11 \end{bmatrix}$

#### 6. Question



If  $5A = \begin{bmatrix} 5 & 10 & -15 \\ 2 & 3 & 4 \\ 1 & 0 & -5 \end{bmatrix}$ , find A.

**Answer**

$$5A = \begin{bmatrix} 5 & 10 & -15 \\ 2 & 3 & 4 \\ 1 & 0 & -5 \end{bmatrix}$$

$$A = \begin{bmatrix} \frac{5}{5} & \frac{10}{5} & \frac{-15}{5} \\ \frac{2}{5} & \frac{3}{5} & \frac{4}{5} \\ \frac{1}{5} & \frac{0}{5} & \frac{-5}{5} \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & -3 \\ \frac{2}{5} & \frac{3}{5} & \frac{4}{5} \\ \frac{1}{5} & 0 & -1 \end{bmatrix}$$

Conclusion:  $A = \begin{bmatrix} 1 & 2 & -3 \\ \frac{2}{5} & \frac{3}{5} & \frac{4}{5} \\ \frac{1}{5} & 0 & -1 \end{bmatrix}$

### 7. Question

Find matrices A and B, if  $A + B = \begin{bmatrix} 1 & 0 & 2 \\ 5 & 4 & -6 \\ 7 & 3 & 8 \end{bmatrix}$  and  $A - B = \begin{bmatrix} -5 & -4 & 8 \\ 11 & 2 & 0 \\ -1 & 7 & 4 \end{bmatrix}$ .

**Answer**

Add (A+B) and (A-B)

We get  $(A+B)+(A-B) = \begin{bmatrix} 1 & 0 & 2 \\ 5 & 4 & -6 \\ 7 & 3 & 8 \end{bmatrix} + \begin{bmatrix} -5 & -4 & 8 \\ 11 & 2 & 0 \\ -1 & 7 & 4 \end{bmatrix}$

$$2A = \begin{bmatrix} -4 & -4 & 10 \\ 16 & 6 & -6 \\ 6 & 10 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} -2 & -2 & 5 \\ 8 & 3 & -3 \\ 3 & 5 & 6 \end{bmatrix}$$

Now Subtract (A-B) from (A+B)

$$(A+B)-(A-B) = \begin{bmatrix} 1 & 0 & 2 \\ 5 & 4 & -6 \\ 7 & 3 & 8 \end{bmatrix} - \begin{bmatrix} -5 & -4 & 8 \\ 11 & 2 & 0 \\ -1 & 7 & 4 \end{bmatrix}$$

$$(2B) = \begin{bmatrix} 6 & 4 & -6 \\ -6 & 2 & -6 \\ 8 & -4 & 4 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 & 2 & -3 \\ -3 & 1 & -3 \\ 4 & -2 & 2 \end{bmatrix}$$

Conclusion:  $A = \begin{bmatrix} -2 & -2 & 5 \\ 8 & 3 & -3 \\ 3 & 5 & 6 \end{bmatrix}$ ,  $B = \begin{bmatrix} 3 & 2 & -3 \\ -3 & 1 & -3 \\ 4 & -2 & 2 \end{bmatrix}$

### 8. Question

Find matrices A and B, if  $2A - B = \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix}$  and  $2B + A = \begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix}$

**Answer**

Add  $2(2A-B)$  and  $(2B+A)$



$$2(2A-B)+(2B+A) = 2\left(\begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix}\right) + \begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix}$$

$$5A = \left(\begin{bmatrix} 12 & -12 & 0 \\ -8 & 4 & 2 \end{bmatrix}\right) + \begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix}$$

$$5A = \begin{bmatrix} 15 & -10 & 5 \\ -10 & 5 & -5 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & -2 & 1 \\ -2 & 1 & -1 \end{bmatrix}$$

$$B = 2\left(\begin{bmatrix} 3 & -2 & 1 \\ -2 & 1 & -1 \end{bmatrix}\right) - \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -4 & 2 \\ -4 & 2 & -2 \end{bmatrix} - \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 2 & 2 \\ 0 & 0 & -3 \end{bmatrix}$$

$$\text{Conclusion: } A = \begin{bmatrix} 3 & -2 & 1 \\ -2 & 1 & -1 \end{bmatrix}, B = \begin{bmatrix} 0 & 2 & 2 \\ 0 & 0 & -3 \end{bmatrix}$$

(GIVEN ANSWER IS WRONG for question 8)

### 9. Question

$$\text{Find matrix X, if } \begin{bmatrix} 3 & 5 & -9 \\ -1 & 4 & -7 \end{bmatrix} + X = \begin{bmatrix} 6 & 2 & 3 \\ 4 & 8 & 6 \end{bmatrix}.$$

### Answer

$$\text{Given } \begin{bmatrix} 3 & 5 & -9 \\ -1 & 4 & -7 \end{bmatrix} + x = \begin{bmatrix} 6 & 2 & 3 \\ 4 & 8 & 6 \end{bmatrix}$$

$$x = \begin{bmatrix} 6 & 2 & 3 \\ 4 & 8 & 6 \end{bmatrix} - \begin{bmatrix} 3 & 5 & -9 \\ -1 & 4 & -7 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -3 & 12 \\ 5 & 4 & 13 \end{bmatrix}$$

$$\text{Conclusion : } x = \begin{bmatrix} 3 & -3 & 12 \\ 5 & 4 & 13 \end{bmatrix}$$



### 10. Question

$$\text{If } A = \begin{bmatrix} -2 & 3 \\ 4 & 5 \\ 1 & -6 \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 & 2 \\ -7 & 3 \\ 6 & 4 \end{bmatrix}, \text{ find a matrix C such that } A + B - C = O.$$

### Answer

$$\text{Given } A + B - C = 0$$

$$\begin{bmatrix} -2 & 3 \\ 4 & 5 \\ 1 & -6 \end{bmatrix} + \begin{bmatrix} 5 & 2 \\ -7 & 3 \\ 6 & 4 \end{bmatrix} - C = 0$$

$$C = \begin{bmatrix} -2 & 3 \\ 4 & 5 \\ 1 & -6 \end{bmatrix} + \begin{bmatrix} 5 & 2 \\ -7 & 3 \\ 6 & 4 \end{bmatrix}$$

$$C = \begin{bmatrix} 3 & 5 \\ -3 & 8 \\ 7 & -2 \end{bmatrix}$$

$$\text{Conclusion: } C = \begin{bmatrix} 3 & 5 \\ -3 & 8 \\ 7 & -2 \end{bmatrix}$$

### 11. Question

Find the matrix X such that  $2A - B + X = O$ ,

$$\text{where } A = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 & 1 \\ 0 & 3 \end{bmatrix}.$$

**Answer**

Given  $2A - B + X = 0$

$$2\left(\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}\right) - \begin{bmatrix} -2 & 1 \\ 0 & 3 \end{bmatrix} + X = 0$$

$$X = \begin{bmatrix} -2 & 1 \\ 0 & 3 \end{bmatrix} - 2\left(\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}\right)$$

$$= \begin{bmatrix} -2 & 1 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 6 & 2 \\ 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -8 & -1 \\ 0 & -1 \end{bmatrix}$$

Conclusion:  $X = \begin{bmatrix} -8 & -1 \\ 0 & -1 \end{bmatrix}$

**12. Question**

If  $A = \begin{bmatrix} 1 & -3 & 2 \\ 2 & 0 & 2 \end{bmatrix}$  and  $B = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$ , find a matrix  $C$  such that  $(A + B + C)$  is a zero matrix.

**Answer**

Given  $A+B+C$  is zero matrix i.e  $A+B+C = 0$

$$\begin{bmatrix} 1 & -3 & 2 \\ 2 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \end{bmatrix} + C = 0$$

$$C = -\begin{bmatrix} 1 & -3 & 2 \\ 2 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 4 & -1 \\ -3 & 0 & -1 \end{bmatrix}$$

Conclusion:  $C = \begin{bmatrix} -3 & 4 & -1 \\ -3 & 0 & -1 \end{bmatrix}$

**13. Question**

If  $A = \text{diag}[2, -5, 9]$ ,  $B = \text{diag}[-3, 7, 14]$  and  $C = \text{diag}[4, -6, 3]$ , find:

(i)  $A + 2B$

(ii)  $B + C - A$

**Answer**

If  $Z = \text{diag}[a,b,c]$ , then we can write it as

$$Z = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

$$\text{So, } A+2B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 9 \end{bmatrix} + 2\left(\begin{bmatrix} -3 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 14 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 9 \end{bmatrix} + \begin{bmatrix} -6 & 0 & 0 \\ 0 & 14 & 0 \\ 0 & 0 & 28 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 37 \end{bmatrix}$$

$$= \text{diag}[4,9,37]$$

Conclusion:  $A + 2B = \text{diag}[4,9,37]$

(Given answer is wrong)

ii.  $B + C - A$

If  $Z = \text{diag}[a,b,c]$ , then we can write it as

$$Z = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

$$B+C-A = \begin{bmatrix} -3 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 14 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

$$= \text{diag}[-1,6,8]$$

Conclusion:  $B+C-A = \text{diag}[-1,6,8]$

iii.  $2A + B - 5C$

If  $Z = \text{diag}[a,b,c]$ , then we can write it as

$$Z = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

$$2A+B-5C = 2 \left( \begin{bmatrix} 2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 9 \end{bmatrix} \right) + \begin{bmatrix} -3 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 14 \end{bmatrix} - 5 \left( \begin{bmatrix} 4 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & 3 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 4 & 0 & 0 \\ 0 & -10 & 0 \\ 0 & 0 & 18 \end{bmatrix} + \begin{bmatrix} -3 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 14 \end{bmatrix} - \begin{bmatrix} 20 & 0 & 0 \\ 0 & -30 & 0 \\ 0 & 0 & 15 \end{bmatrix}$$

$$= \begin{bmatrix} -19 & 0 & 0 \\ 0 & 27 & 0 \\ 0 & 0 & 17 \end{bmatrix}$$

$$= \text{diag}[-19,27,17]$$

Conclusion:  $2A + B - 5C = \text{diag}[-19,27,17]$

(Given answer is wrong)

#### 14. Question

Find the value of  $x$  and  $y$ , when

$$i. \begin{bmatrix} x+y \\ x-y \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \end{bmatrix}$$

#### Answer

$$\text{If } \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} e & f \\ g & h \end{bmatrix},$$

Then  $a=e, b=f, c=g, d=h$

$$\text{Given } \begin{bmatrix} x+y \\ x-y \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \end{bmatrix}$$

So,  $x + y = 8$  and  $x - y = 4$

Adding these two gives  $2x = 12$

$$\Rightarrow x = 6$$

$$y = 2$$

Conclusion :  $x = 6$  and  $y = 2$

$$ii. \begin{bmatrix} 2x+5 & 7 \\ 0 & 3y-7 \end{bmatrix} = \begin{bmatrix} x-3 & 7 \\ 0 & -5 \end{bmatrix}$$

$$\text{Given, } \begin{bmatrix} 2x+5 & 7 \\ 0 & 3y-7 \end{bmatrix} = \begin{bmatrix} x-3 & 7 \\ 0 & -5 \end{bmatrix}$$

So,  $2x+5 = x-3$  and  $3y-7 = -5$

$$\Rightarrow 3y = 2 \Rightarrow y = \frac{2}{3}$$

$$\Rightarrow 2x + 5 = x - 3 \Rightarrow x = -8$$



Conclusion :  $x = -8$  and  $y = \frac{2}{3}$

$$\text{iii. } 2 \begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$

$$2 \begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$

$$\begin{bmatrix} 2x+3 & 6 \\ 15 & 2y-4 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$

$$2x+3 = 7 \Rightarrow x = 2$$

$$2y-4 = 14 \Rightarrow y = 9$$

Conclusion :  $x = 2$  and  $y = 9$

(Given answer is wrong)

### 15. Question

Find the value of  $(x + y)$  from the following equation :

$$2 \begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

### Answer

Given

$$2 \begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

So,  $2+y = 5$  and  $2x+2 = 8$

i.e  $y = 3$  and  $x = 3$

Therefore,  $x+y=6$

Conclusion: Therefore  $x+y = 6$

### 16. Question

$$\text{If } \begin{bmatrix} x-y & 2y \\ 2y+z & x+y \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 9 & 5 \end{bmatrix} \text{ then write the value of } (x + y).$$

### Answer

$$\text{If } \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} e & f \\ g & h \end{bmatrix},$$

Then  $a=e$ ,  $b=f$ ,  $c=g$ ,  $d=h$

$$\text{Given, } \begin{bmatrix} x-y & 2y \\ 2y+z & x+y \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 9 & 5 \end{bmatrix},$$

So,  $x-y = 1$ ,  $x+y = 5$ ,  $2y = 4$  and  $2y+z = 9$

Therefore,  $x+y = 5$

Conclusion:  $x+y = 5$

(Given answer is wrong)

### Exercise 5C

#### 1 A. Question

Compute  $AB$  and  $BA$ , which ever exists when



$$A = \begin{bmatrix} 2 & -1 \\ 3 & 0 \\ -1 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 & 3 \\ 0 & 4 \end{bmatrix}$$

**Answer**

$$\text{Given : } A = \begin{bmatrix} 2 & -1 \\ 3 & 0 \\ -1 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 & 3 \\ 0 & 4 \end{bmatrix}$$

Matrix A is of order  $3 \times 2$ , and Matrix B is of order  $2 \times 2$

To find : matrix AB and BA

Formula used :

$$\begin{array}{c} \text{row } i \leftarrow \\ \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \cdot \begin{array}{c} \text{column } j \\ \downarrow \\ \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1j} & \dots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \dots & b_{ij} & \dots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nj} & \dots & b_{nn} \end{bmatrix} \end{array} = \\ \\ = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix} \end{array}$$

entry on row  $i$   
column  $j$

Where  $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

If A is a matrix of order  $a \times b$  and B is a matrix of order  $c \times d$ , then matrix AB exists and is of order  $a \times d$ , if and only if  $b = c$

If A is a matrix of order  $a \times b$  and B is a matrix of order  $c \times d$ , then matrix BA exists and is of order  $c \times b$ , if and only if  $d = a$

For matrix AB,  $a = 3, b = c = 2, d = 2$ , thus matrix AB is of order  $3 \times 2$

$$\text{Matrix AB} = \begin{bmatrix} 2 & -1 \\ 3 & 0 \\ -1 & 4 \end{bmatrix} \times \begin{bmatrix} -2 & 3 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 2(-2) + (-1)(0) & 2(3) + (-1)(4) \\ 3(-2) + 0(0) & 3(3) + 0(4) \\ -1(-2) + 4(0) & -1(3) + 4(4) \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} -4 + 0 & 6 - 4 \\ -6 + 0 & 9 + 0 \\ 2 + 0 & -3 + 16 \end{bmatrix} = \begin{bmatrix} -4 & 2 \\ -6 & 9 \\ 2 & 13 \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} -4 & 2 \\ -6 & 9 \\ 2 & 13 \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} -4 & 2 \\ -6 & 9 \\ 2 & 13 \end{bmatrix}$$

For matrix BA,  $a = 3, b = c = 2, d = 2$ , thus matrix BA exists, if and only if  $d = a$

But  $3 \neq 2$

Thus matrix BA does not exist

### 1 B. Question

Compute AB and BA, which ever exists when

$$A = \begin{bmatrix} -1 & 1 \\ -2 & 2 \\ -3 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 & -2 & 1 \\ 0 & 1 & 2 \\ -3 & 4 & -5 \end{bmatrix}$$

**Answer**

$$\text{Given : } A = \begin{bmatrix} -1 & 1 \\ -2 & 2 \\ -3 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 & -2 & 1 \\ 0 & 1 & 2 \\ -3 & 4 & -5 \end{bmatrix}$$

Matrix A is of order  $3 \times 2$ , and Matrice B is of order  $3 \times 3$

To find : matrix AB and BA

Formula used :

$$\begin{array}{c} \text{row } i \leftarrow \\ \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \cdot \begin{array}{c} \text{column } j \\ \downarrow \\ \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1j} & \dots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \dots & b_{ij} & \dots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nj} & \dots & b_{nn} \end{bmatrix} \end{array} = \\ \\ = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix} \end{array}$$

entry on row  $i$   
column  $j$

Where  $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

If A is a matrix of order  $a \times b$  and B is a matrice of order  $c \times d$ , then matrice AB exists and is of order  $a \times d$ , if and only if  $b = c$

If A is a matrix of order  $a \times b$  and B is a matrice of order  $c \times d$ , then matrice BA exists and is of order  $c \times b$ , if and only if  $d = a$

For matrix AB,  $a = 3, b = 2, c = 3, d = 3$ , thus matrix AB does not exist, as  $d \neq b$

But  $2 \neq 3$

Thus matrix AB does not exist

For matrix BA,  $a = 3, b = 2, c = 3, d = 3$ , thus matrix BA is of order  $3 \times 2$

as  $d = a = 3$

$$\text{Matrix BA} = \begin{bmatrix} 3 & -2 & 1 \\ 0 & 1 & 2 \\ -3 & 4 & -5 \end{bmatrix} \times \begin{bmatrix} -1 & 1 \\ -2 & 2 \\ -3 & 3 \end{bmatrix} = \begin{bmatrix} 3(-1) - 2(-2) + 1(-3) & 3(1) - 2(2) + 1(3) \\ 0(-1) + 1(-2) + 2(-3) & 0(1) + 1(2) + 2(3) \\ -3(-1) + 4(-2) - 5(-3) & -3(1) + 4(2) - 5(3) \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} -3 + 4 - 3 & 3 - 4 + 3 \\ 0 - 2 - 6 & 0 + 2 + 6 \\ 3 - 8 + 15 & -3 + 8 - 15 \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ -8 & 8 \\ 10 & -10 \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} -2 & 2 \\ -8 & 8 \\ 10 & -10 \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} -2 & 2 \\ -8 & 8 \\ 10 & -10 \end{bmatrix}$$

### 1 C. Question

Compute AB and BA, which ever exists when

$$A = \begin{bmatrix} 0 & 1 & -5 \\ 2 & 4 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 3 \\ -1 & 0 \\ 0 & 5 \end{bmatrix}$$

**Answer**

Given :  $A = \begin{bmatrix} 0 & 1 & -5 \\ 2 & 4 & 0 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & 3 \\ -1 & 0 \\ 0 & 5 \end{bmatrix}$

Matrix A is of order  $2 \times 3$  and Matrix B is of order  $3 \times 2$

To find : matrices AB and BA

Formula used :

$$\begin{array}{c} \text{row } i \leftarrow \\ \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \cdot \begin{array}{c} \text{column } j \\ \downarrow \\ \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1j} & \dots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \dots & b_{ij} & \dots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nj} & \dots & b_{nn} \end{bmatrix} \end{array} = \\ \\ = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix} \begin{array}{c} \text{entry on row } i \\ \text{column } j \end{array}
 \end{array}$$

Where  $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

If A is a matrix of order  $a \times b$  and B is a matrix of order  $c \times d$ , then matrix AB exists and is of order  $a \times d$ , if and only if  $b = c$

If A is a matrix of order  $a \times b$  and B is a matrix of order  $c \times d$ , then matrix BA exists and is of order  $c \times b$ , if and only if  $d = a$

For matrix AB,  $a = 2, b = 3, c = 3, d = 2$ , matrix AB exists and is of order  $2 \times 2$ , as

$b = c = 3$

Matrix AB =  $\begin{bmatrix} 0 & 1 & -5 \\ 2 & 4 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 3 \\ -1 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 0(1) + 1(-1) - 5(0) & 0(3) + 1(0) - 5(5) \\ 2(1) + 4(-1) + 0(0) & 2(3) + 4(0) + 0(5) \end{bmatrix}$

Matrix AB =  $\begin{bmatrix} 0 - 1 - 0 & 0 + 0 - 25 \\ 2 - 4 + 0 & 6 + 0 + 0 \end{bmatrix} = \begin{bmatrix} -1 & -25 \\ -2 & 6 \end{bmatrix}$

Matrix AB =  $\begin{bmatrix} -1 & -25 \\ -2 & 6 \end{bmatrix}$

Matrix AB =  $\begin{bmatrix} -1 & -25 \\ -2 & 6 \end{bmatrix}$

For matrix BA,  $a = 2, b = 3, c = 3, d = 2$ , matrix BA exists and is of order  $3 \times 3$ , as

$d = a = 2$

Matrix BA =  $\begin{bmatrix} 1 & 3 \\ -1 & 0 \\ 0 & 5 \end{bmatrix} \times \begin{bmatrix} 0 & 1 & -5 \\ 2 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 1(0) + 3(2) & 1(1) + 3(4) & 1(-5) + 3(0) \\ -1(0) + 0(2) & -1(1) + 0(4) & -1(-5) + 0(0) \\ 0(0) + 5(2) & 0(1) + 5(4) & 0(-5) + 5(0) \end{bmatrix}$

Matrix BA =  $\begin{bmatrix} 0 + 6 & 1 + 12 & -5 + 0 \\ 0 + 0 & -1 + 0 & 5 + 0 \\ 0 + 10 & 0 + 20 & 0 + 0 \end{bmatrix}$

Matrix BA =  $\begin{bmatrix} 6 & 13 & -5 \\ 0 & -1 & 5 \\ 10 & 20 & 0 \end{bmatrix}$

Matrix BA =  $\begin{bmatrix} 6 & 13 & -5 \\ 0 & -1 & 5 \\ 10 & 20 & 0 \end{bmatrix}$

**1 D. Question**

Compute AB and BA, which ever exists when

$$A = [1 \ 2 \ 3 \ 4] \text{ and } B = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

**Answer**

$$\text{Given : } A = [1 \ 2 \ 3 \ 4] \text{ and } B = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

Matrix A is of order  $1 \times 4$  and Matrix B is of order  $4 \times 1$

To find : matrices AB and BA

Formula used :

$$\begin{array}{c} \text{row } i \leftarrow \\ \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \cdot \begin{array}{c} \text{column } j \\ \downarrow \\ \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1j} & \dots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \dots & b_{ij} & \dots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nj} & \dots & b_{nn} \end{bmatrix} \end{array} = \\ \\ = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix} \begin{array}{c} \text{entry on row } i \\ \text{column } j \end{array}
 \end{array}$$

Where  $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

If A is a matrix of order  $a \times b$  and B is a matrix of order  $c \times d$ , then matrix AB exists and is of order  $a \times d$ , if and only if  $b = c$

If A is a matrix of order  $a \times b$  and B is a matrix of order  $c \times d$ , then matrix BA exists and is of order  $c \times b$ , if and only if  $d = a$

For matrix AB,  $a = 1, b = 4, c = 4, d = 1$ , matrix AB exists and is of order  $1 \times 1$ , as

$$b = c = 4$$

$$\text{Matrix AB} = [1 \ 2 \ 3 \ 4] \times \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = [1(1) + 2(2) + 3(3) + 4(4)]$$

$$\text{Matrix AB} = [1 + 4 + 9 + 16] = [30]$$

$$\text{Matrix AB} = [30]$$

$$\text{Matrix AB} = [30]$$

For matrix BA,  $a = 1, b = 4, c = 4, d = 1$ , matrix BA exists and is of order  $4 \times 4$ , as

$$d = a = 1$$

$$\text{Matrix BA} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \times [1 \ 2 \ 3 \ 4] = \begin{bmatrix} 1(1) & 1(2) & 1(3) & 1(4) \\ 2(1) & 2(2) & 2(3) & 2(4) \\ 3(1) & 3(2) & 3(3) & 3(4) \\ 4(1) & 4(2) & 4(3) & 4(4) \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \\ 4 & 8 & 12 & 16 \end{bmatrix}$$



$$\text{Matrix BA} = \begin{bmatrix} 2+0-1 & 1+0+1 \\ -2+6-1 & -1+4+1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

**2 A. Question**

Show that  $AB \neq BA$  in each of the following cases :

$$A = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$

**Answer**

$$\text{Given : } A = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$

Matrix A is of order  $2 \times 2$  and Matrix B is of order  $2 \times 2$

To show : matrix  $AB \neq BA$

Formula used :

The diagram shows the multiplication of two matrices, A and B, to produce matrix C. Matrix A is  $\begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix}$ . Matrix B is  $\begin{bmatrix} b_{11} & b_{12} & \dots & b_{1j} & \dots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \dots & b_{ij} & \dots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nj} & \dots & b_{nn} \end{bmatrix}$ . The resulting matrix C is  $\begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix}$ . The entry  $c_{ij}$  is the dot product of row i of A and column j of B.

$$\text{Where } c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$$

If A is a matrix of order  $a \times b$  and B is a matrix of order  $c \times d$ , then matrix AB exists and is of order  $a \times d$ , if and only if  $b = c$

If A is a matrix of order  $a \times b$  and B is a matrix of order  $c \times d$ , then matrix BA exists and is of order  $c \times b$ , if and only if  $d = a$

For matrix AB,  $a = 2, b = c = 2, d = 2$ , thus matrix AB is of order  $2 \times 2$

$$\text{Matrix AB} = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 5(2) - 1(3) & 5(1) - 1(4) \\ 6(2) + 7(3) & 6(1) + 7(4) \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} 10 - 3 & 5 - 4 \\ 12 + 21 & 6 + 28 \end{bmatrix} = \begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix}$$

For matrix BA,  $a = 2, b = c = 2, d = 2$ , thus matrix BA is of order  $2 \times 2$

$$\text{Matrix BA} = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} = \begin{bmatrix} 2(5) + 1(6) & 2(-1) + 1(7) \\ 3(5) + 4(6) & 3(-1) + 4(7) \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} 10 + 6 & -2 + 7 \\ 15 + 24 & -3 + 28 \end{bmatrix} = \begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix} \text{ and Matrix AB} = \begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix}$$

Matrix  $AB \neq BA$

**2 B. Question**

Show that  $AB \neq BA$  in each of the following cases :

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$$

**Answer**

$$\text{Given : } A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$$

Matrix A is of order  $3 \times 3$ , and Matrix B is of order  $3 \times 3$

To show : matrix  $AB \neq BA$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

The formula used :

$$= \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} & a_{11}b_{13} + a_{12}b_{23} + a_{13}b_{33} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} & a_{21}b_{13} + a_{22}b_{23} + a_{23}b_{33} \\ a_{31}b_{11} + a_{32}b_{21} + a_{33}b_{31} & a_{31}b_{12} + a_{32}b_{22} + a_{33}b_{32} & a_{31}b_{13} + a_{32}b_{23} + a_{33}b_{33} \end{bmatrix}$$

If A is a matrix of order  $a \times b$  and B is a matrix of order  $c \times d$ , then matrix AB exists and is of order  $a \times d$ , if and only if  $b = c$

If A is a matrix of order  $a \times b$  and B is a matrix of order  $c \times d$ , then matrix BA exists and is of order  $c \times b$ , if and only if  $d = a$

For matrix AB,  $a = 3, b = c = 3, d = 3$ , thus matrix AB is of order  $3 \times 3$

Matrix AB =

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 1(-1) + 2(0) + 3(2) & 1(1) + 2(-1) + 3(3) & 1(0) + 2(1) + 3(4) \\ 0(-1) + 1(0) + 0(2) & 0(1) + 1(-1) + 0(3) & 0(0) + 1(1) + 0(4) \\ 1(-1) + 1(0) + 0(2) & 1(1) + 1(-1) + 0(3) & 1(0) + 1(1) + 0(4) \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} -1 + 0 + 6 & 1 - 2 + 9 & 0 + 2 + 12 \\ 0 + 0 + 0 & 0 - 1 + 0 & 0 + 1 + 0 \\ -1 + 0 + 0 & 1 - 1 + 0 & 0 + 1 + 0 \end{bmatrix} = \begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

For matrix BA,  $a = 3, b = c = 3, d = 3$ , thus matrix AB is of order  $3 \times 3$

Matrix BA =

$$\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} -1(1) + 1(0) + 0(1) & -1(2) + 1(1) + 0(1) & -1(3) + 1(0) + 0(0) \\ 0(1) - 1(0) + 1(1) & 0(2) - 1(1) + 1(1) & 0(3) - 1(0) + 1(0) \\ 2(1) + 3(0) + 4(1) & 2(2) + 3(1) + 4(1) & 2(3) + 3(0) + 4(0) \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} -1 + 0 + 0 & -2 + 1 + 0 & -3 + 0 + 0 \\ 0 - 1 + 1 & 0 - 1 + 1 & 0 + 0 + 0 \\ 2 + 0 + 4 & 4 + 3 + 4 & 6 + 0 + 0 \end{bmatrix} = \begin{bmatrix} -1 & -1 & -3 \\ 0 & 0 & 0 \\ 6 & 11 & 6 \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} -1 & -1 & -3 \\ 0 & 0 & 0 \\ 6 & 11 & 6 \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} -1 & -1 & -3 \\ 0 & 0 & 0 \\ 6 & 11 & 6 \end{bmatrix} \text{ and } \text{Matrix AB} = \begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

Matrix  $AB \neq BA$

### 3 A. Question

Show that  $AB = BA$  in each of the following cases:

$$A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \text{ and } B = \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix}$$

**Answer**

$$\text{Given : } A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \text{ and } B = \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix}$$

Matrix A is of order  $2 \times 2$  and Matrix B is of order  $2 \times 2$

To show : matrix  $AB = BA$

Formula used :

$$\begin{array}{c} \text{row } i \leftarrow \\ \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \cdot \begin{array}{c} \text{column } j \\ \downarrow \\ \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1j} & \dots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \dots & b_{ij} & \dots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nj} & \dots & b_{nn} \end{bmatrix} \end{array} = \\ \\ = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix} \end{array}$$

entry on row  $i$   
column  $j$

Where  $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

If A is a matrix of order  $a \times b$  and B is a matrix of order  $c \times d$ , then matrix AB exists and is of order  $a \times d$ , if and only if  $b = c$

If A is a matrix of order  $a \times b$  and B is a matrix of order  $c \times d$ , then matrix BA exists and is of order  $c \times b$ , if and only if  $d = a$

For matrix AB,  $a = 2, b = c = 2, d = 2$ , thus matrix AB is of order  $2 \times 2$

Matrix AB =

$$\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \times \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix} = \begin{bmatrix} \cos \theta \cos \phi - \sin \theta \sin \phi & -\cos \theta \sin \phi - \sin \theta \cos \phi \\ \sin \theta \cos \phi + \cos \theta \sin \phi & -\sin \theta \sin \phi + \cos \theta \cos \phi \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} \cos \theta \cos \phi - \sin \theta \sin \phi & -\cos \theta \sin \phi - \sin \theta \cos \phi \\ \sin \theta \cos \phi + \cos \theta \sin \phi & -\sin \theta \sin \phi + \cos \theta \cos \phi \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} \cos \theta \cos \phi - \sin \theta \sin \phi & -\cos \theta \sin \phi - \sin \theta \cos \phi \\ \sin \theta \cos \phi + \cos \theta \sin \phi & -\sin \theta \sin \phi + \cos \theta \cos \phi \end{bmatrix}$$

For matrix BA,  $a = 2, b = c = 2, d = 2$ , thus matrix BA is of order  $2 \times 2$

Matrix BA =

$$\begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix} \times \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} = \begin{bmatrix} \cos \phi \cos \theta - \sin \phi \sin \theta & -\cos \phi \sin \theta - \sin \phi \cos \theta \\ \sin \phi \cos \theta + \cos \phi \sin \theta & -\sin \phi \sin \theta + \cos \phi \cos \theta \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} \cos \theta \cos \phi - \sin \theta \sin \phi & -\cos \theta \sin \phi - \sin \theta \cos \phi \\ \sin \theta \cos \phi + \cos \theta \sin \phi & -\sin \theta \sin \phi + \cos \theta \cos \phi \end{bmatrix}$$

$$\text{Matrix BA} = \text{Matrix AB} = \begin{bmatrix} \cos \theta \cos \phi - \sin \theta \sin \phi & -\cos \theta \sin \phi - \sin \theta \cos \phi \\ \sin \theta \cos \phi + \cos \theta \sin \phi & -\sin \theta \sin \phi + \cos \theta \cos \phi \end{bmatrix}$$

Thus Matrix  $AB = BA$

### 3 B. Question

Show that  $AB = BA$  in each of the following cases:

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix}$$

### Answer

$$\text{Given : } A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix}$$

Matrix A is of order  $3 \times 3$  and Matrix B is of order  $3 \times 3$

To show : matrix  $AB \neq BA$

$$\begin{aligned} \text{Formula used : } & \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \\ & = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} & a_{11}b_{13} + a_{12}b_{23} + a_{13}b_{33} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} & a_{21}b_{13} + a_{22}b_{23} + a_{23}b_{33} \\ a_{31}b_{11} + a_{32}b_{21} + a_{33}b_{31} & a_{31}b_{12} + a_{32}b_{22} + a_{33}b_{32} & a_{31}b_{13} + a_{32}b_{23} + a_{33}b_{33} \end{bmatrix} \end{aligned}$$

If A is a matrix of order  $a \times b$  and B is a matrix of order  $c \times d$ , then matrix AB exists and is of order  $a \times d$ , if and only if  $b = c$

If A is a matrix of order  $a \times b$  and B is a matrix of order  $c \times d$ , then matrix BA exists and is of order  $c \times b$ , if and only if  $d = a$

For matrix AB,  $a = 3, b = c = 3, d = 3$ , thus matrix AB is of order  $3 \times 3$

$$\text{Matrix AB} = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix} \times \begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} 1(10) + 2(-11) + 1(-9) & 1(-4) + 2(5) + 1(-5) & 1(-1) + 2(0) + 1(1) \\ 3(10) + 4(-11) + 2(-9) & 3(-4) + 4(5) + 2(-5) & 3(-1) + 4(0) + 2(1) \\ 1(10) + 3(-11) + 2(-9) & 1(-4) + 3(5) + 2(-5) & 1(-1) + 3(0) + 2(1) \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} 10 - 22 - 9 & -4 + 10 - 5 & -1 + 0 + 1 \\ 30 - 44 - 18 & -12 + 20 - 10 & -3 + 0 + 2 \\ 10 - 33 - 18 & -4 + 15 - 10 & -1 + 0 + 2 \end{bmatrix} = \begin{bmatrix} -3 & 1 & 0 \\ -32 & -2 & -1 \\ -41 & 1 & 1 \end{bmatrix}$$

For matrix BA,  $a = 3, b = c = 3, d = 3$ , thus matrix BA is of order  $3 \times 3$

Matrix BA =

$$\begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix} = \begin{bmatrix} 10(1) - 4(3) - 1(1) & 10(2) - 4(4) - 1(3) & 10(1) - 4(2) - 1(2) \\ -11(1) + 5(3) + 0(1) & -11(2) + 5(4) + 0(3) & -11(1) + 5(2) + 0(2) \\ 9(1) - 5(3) + 1(1) & 9(2) - 5(4) + 1(3) & 9(1) - 5(2) + 1(2) \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} 10 - 12 - 1 & 20 - 16 - 3 & 10 - 8 - 2 \\ -11 + 15 + 0 & -22 + 20 + 0 & -11 + 10 + 0 \\ 9 - 15 + 1 & 18 - 20 + 3 & 9 - 10 + 2 \end{bmatrix} = \begin{bmatrix} -3 & 1 & 0 \\ -4 & -2 & -1 \\ -5 & 1 & 1 \end{bmatrix}$$

Matrix  $AB \neq BA$

### 3 C. Question

Show that  $AB = BA$  in each of the following cases:

$$A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & -1 \\ 3 & 0 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix}$$

### Answer

$$\text{Given : } A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & -1 \\ 3 & 0 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix}$$

Matrix A is of order  $3 \times 3$  and Matrix B is of order  $3 \times 3$

To show : matrix  $AB = BA$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

Formula used :

$$= \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} & a_{11}b_{13} + a_{12}b_{23} + a_{13}b_{33} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} & a_{21}b_{13} + a_{22}b_{23} + a_{23}b_{33} \\ a_{31}b_{11} + a_{32}b_{21} + a_{33}b_{31} & a_{31}b_{12} + a_{32}b_{22} + a_{33}b_{32} & a_{31}b_{13} + a_{32}b_{23} + a_{33}b_{33} \end{bmatrix}$$

If A is a matrix of order  $a \times b$  and B is a matrix of order  $c \times d$ , then matrix AB exists and is of order  $a \times d$ , if and only if  $b = c$

If A is a matrix of order  $a \times b$  and B is a matrix of order  $c \times d$ , then matrix BA exists and is of order  $c \times b$ , if and only if  $d = a$

For matrix AB,  $a = 3, b = c = 3, d = 3$ , thus matrix AB is of order  $3 \times 3$

$$\text{Matrix AB} = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & -1 \\ 3 & 0 & -1 \end{bmatrix} \times \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix} =$$

$$\begin{bmatrix} 1(-2) + 3(-1) - 1(-6) & 1(3) + 3(2) - 1(9) & 1(-1) + 3(-1) - 1(-4) \\ 2(-2) + 2(-1) - 1(-6) & 2(3) + 2(2) - 1(9) & 2(-1) + 2(-1) - 1(-4) \\ 3(-2) + 0(-1) - 1(-6) & 3(3) + 0(2) - 1(9) & 3(-1) + 0(-1) - 1(-4) \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} -2 - 3 + 6 & 3 + 6 - 9 & -1 - 3 + 4 \\ -4 - 2 + 6 & 6 + 4 - 9 & -2 - 2 + 4 \\ -6 + 0 + 6 & 9 + 0 - 9 & -3 + 0 + 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

For matrix BA,  $a = 3, b = c = 3, d = 3$ , thus matrix AB is of order  $3 \times 3$

$$\text{Matrix BA} = \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix} \times \begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & -1 \\ 3 & 0 & -1 \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} -2(1) + 3(2) - 1(3) & -2(3) + 3(2) - 1(0) & -2(-1) + 3(-1) - 1(-1) \\ -1(1) + 2(2) - 1(3) & -1(3) + 2(2) - 1(0) & -1(-1) + 2(-1) - 1(-1) \\ -6(1) + 9(2) - 4(3) & -6(3) + 9(2) - 4(0) & -6(-1) + 9(-1) - 4(-1) \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} -2 + 6 - 3 & -6 + 6 + 0 & 2 - 3 + 1 \\ -1 + 2 - 3 & -3 + 4 + 0 & 1 - 2 + 1 \\ -6 + 18 - 12 & -18 + 18 + 0 & 6 - 9 + 4 \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Matrix AB} = \text{Matrix BA} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

#### 4. Question

$$\text{If } A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}, \text{ shown that } AB = A \text{ and } BA = B.$$

**Answer**

$$\text{Given : } A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix},$$

Matrix A is of order  $3 \times 3$  and Matrix B is of order  $3 \times 3$

To show : matrix  $AB = A, BA = B$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

Formula used :

$$= \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} & a_{11}b_{13} + a_{12}b_{23} + a_{13}b_{33} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} & a_{21}b_{13} + a_{22}b_{23} + a_{23}b_{33} \\ a_{31}b_{11} + a_{32}b_{21} + a_{33}b_{31} & a_{31}b_{12} + a_{32}b_{22} + a_{33}b_{32} & a_{31}b_{13} + a_{32}b_{23} + a_{33}b_{33} \end{bmatrix}$$

If A is a matrix of order  $a \times b$  and B is a matrix of order  $c \times d$ , then matrix AB exists and is of order  $a \times d$ , if and only if  $b = c$

If A is a matrix of order  $a \times b$  and B is a matrix of order  $c \times d$ , then matrix BA exists and is of order  $c \times b$ , if and only if  $d = a$

For matrix AB,  $a = 3, b = c = 3, d = 3$ , thus matrix AB is of order  $3 \times 3$

$$\text{Matrix AB} = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix} \times \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} =$$

$$\begin{bmatrix} 2(2) - 3(-1) - 5(1) & 2(-2) - 3(3) - 5(-2) & 2(-4) - 3(4) - 5(-3) \\ -1(2) + 4(-1) + 5(1) & -1(-2) + 4(3) + 5(-2) & -1(-4) + 4(4) + 5(-3) \\ 1(2) - 3(-1) - 4(1) & 1(-2) - 3(3) - 4(-2) & 1(-4) - 3(4) - 4(-3) \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} 4 + 3 - 5 & -4 - 9 + 10 & -8 - 12 + 15 \\ -2 - 4 + 5 & +2 + 12 - 10 & 4 + 16 - 15 \\ 2 + 3 - 4 & -2 - 9 + 8 & -4 - 12 + 12 \end{bmatrix} = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix} = \text{Matrix A}$$

Matrix AB = Matrix A

For matrix BA,  $a = 3, b = c = 3, d = 3$ , thus matrix AB is of order  $3 \times 3$

$$\text{Matrix BA} = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} \times \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} 2(2) - 2(-1) - 4(1) & 2(-3) - 2(4) - 4(-3) & 2(-5) - 2(5) - 4(-4) \\ -1(2) + 3(-1) + 4(1) & -1(-3) + 3(4) + 4(-3) & -1(-5) + 3(5) + 4(-4) \\ 1(2) - 2(-1) - 3(1) & 1(-3) - 2(4) - 3(-3) & 1(-5) - 2(5) - 3(-4) \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} 4 + 2 - 4 & -6 - 8 + 12 & -10 - 10 + 16 \\ -2 - 3 + 4 & +3 + 12 - 12 & +5 + 15 - 16 \\ 2 + 2 - 3 & -3 - 8 + 9 & -5 - 10 + 12 \end{bmatrix} = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} = \text{Matrix B}$$

$$\text{Matrix BA} = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} = \text{Matrix B}$$

MATRIX AB = A and MATRIX BA = B

## 5. Question

If  $A = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$  and  $B = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$ , show that AB is a zero matrix.

**Answer**

$$\text{Given : } A = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$$

Matrix A is of order  $3 \times 3$ , matrix B is of order  $3 \times 3$  and matrix C is of order  $3 \times 3$

To show : AB is a zero matrix

Formula used :

$$\begin{array}{c}
 \text{row } i \leftarrow \\
 \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \cdot \begin{array}{c} \text{column } j \\ \downarrow \\ \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1j} & \dots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \dots & b_{ij} & \dots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nj} & \dots & b_{nn} \end{bmatrix} \end{array} = \\
 \\
 = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix} \begin{array}{c} \text{entry on row } i \\ \text{column } j \end{array}
 \end{array}$$

Where  $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

If A is a matrix of order  $a \times b$  and B is a matrix of order  $c \times d$ , then matrix AB exists and is of order  $a \times d$ , if and only if  $b = c$

If A is a matrix of order  $a \times b$  and B is a matrix of order  $c \times d$ , then matrix BA exists and is of order  $c \times b$ , if and only if  $d = a$

$$AB = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix} \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$$

$$\begin{aligned}
 &AB \\
 &= \begin{bmatrix} 0 \times a^2 + c \times ab - b \times ac & 0 \times ab + c \times b^2 - b \times bc & 0 \times ac + c \times bc - b \times c^2 \\ -c \times a^2 + 0 \times ab + a \times ac & -c \times ab + 0 \times b^2 + a \times bc & -c \times ac + 0 \times bc + a \times c^2 \\ b \times a^2 - a \times ab + a \times ac & b \times ab - a \times b^2 + a \times bc & b \times ac - a \times bc + a \times c^2 \end{bmatrix} \\
 &= \begin{bmatrix} abc - abc & b^2c - b^2c & bc^2 - bc^2 \\ -a^2c + a^2c & -abc + abc & -ac^2 + ac^2 \\ a^2b - a^2b & ab^2 - ab^2 & abc - abc \end{bmatrix} \\
 &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
 \end{aligned}$$

= 0 matrix



Hence, Proved

**16 A. Question**

For the following matrices, verify that  $A(BC) = (AB)C$  :

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 3 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 0 & 4 \\ 1 & -1 & 2 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix}$$

**Answer**

$$\text{Given : } A = \begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 3 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 0 & 4 \\ 1 & -1 & 2 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix}$$

Matrix A is of order  $2 \times 3$ , matrix B is of order  $3 \times 3$  and matrix C is of order  $3 \times 1$

To show : matrix  $A(BC) = (AB)C$

Formula used :

$$\begin{array}{c}
 \text{column } j \\
 \downarrow \\
 \begin{array}{c}
 \text{row } i \leftarrow \\
 \left[ \begin{array}{cccc} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{array} \right] \cdot \left[ \begin{array}{cccc} b_{11} & b_{12} & \dots & b_{1j} & \dots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \dots & b_{ij} & \dots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nj} & \dots & b_{nn} \end{array} \right] = \\
 \\
 = \left[ \begin{array}{cccc} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{array} \right] \begin{array}{l} \\ \\ \text{entry on row } i \\ \text{column } j \\ \end{array}
 \end{array}
 \end{array}$$

Where  $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

If A is a matrix of order  $a \times b$  and B is a matrix of order  $c \times d$ , then matrix AB exists and is of order  $a \times d$ , if and only if  $b = c$

If A is a matrix of order  $a \times b$  and B is a matrix of order  $c \times d$ , then matrix BA exists and is of order  $c \times b$ , if and only if  $d = a$

For matrix BC,  $a = 3, b = c = 3, d = 1$ , thus matrix BC is of order  $3 \times 1$

$$\text{Matrix BC} = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 0 & 4 \\ 1 & -1 & 2 \end{bmatrix} \times \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 2(1) + 3(4) + 0(5) \\ 1(1) + 0(4) + 4(5) \\ 1(1) - 1(4) + 2(5) \end{bmatrix} = \begin{bmatrix} 2 + 12 + 0 \\ 1 + 0 + 20 \\ 1 - 4 + 10 \end{bmatrix}$$

$$\text{Matrix BC} = \begin{bmatrix} 14 \\ 21 \\ 7 \end{bmatrix}$$

For matrix A(BC),  $a = 2, b = c = 3, d = 1$ , thus matrix A(BC) is of order  $2 \times 1$

$$\text{Matrix A(BC)} = \begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 3 \end{bmatrix} \times \begin{bmatrix} 14 \\ 21 \\ 7 \end{bmatrix} = \begin{bmatrix} 1(14) + 2(21) + 5(7) \\ 0(14) + 1(21) + 3(7) \end{bmatrix} = \begin{bmatrix} 14 + 42 + 35 \\ 0 + 21 + 21 \end{bmatrix}$$

$$\text{Matrix A(BC)} = \begin{bmatrix} 91 \\ 42 \end{bmatrix}$$

$$\text{Matrix A(BC)} = \begin{bmatrix} 91 \\ 42 \end{bmatrix}$$

For matrix AB,  $a = 2, b = c = 3, d = 3$ , thus matrix AB is of order  $2 \times 3$

$$\text{Matrix AB} = \begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 3 \end{bmatrix} \times \begin{bmatrix} 2 & 3 & 0 \\ 1 & 0 & 4 \\ 1 & -1 & 2 \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} 1(2) + 2(1) + 5(1) & 1(3) + 2(0) + 5(-1) & 1(0) + 2(4) + 5(2) \\ 0(2) + 1(1) + 3(1) & 0(3) + 1(0) + 3(-1) & 0(0) + 1(4) + 3(2) \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} 2 + 2 + 5 & 3 + 0 - 5 & 0 + 8 + 10 \\ 0 + 1 + 3 & 0 + 0 - 3 & 0 + 4 + 6 \end{bmatrix} = \begin{bmatrix} 9 & -2 & 18 \\ 4 & -3 & 10 \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} 9 & -2 & 18 \\ 4 & -3 & 10 \end{bmatrix}$$

For matrix (AB)C,  $a = 2, b = c = 3, d = 1$ , thus matrix (AB)C is of order  $2 \times 1$

$$\text{Matrix (AB)C} = \begin{bmatrix} 9 & -2 & 18 \\ 4 & -3 & 10 \end{bmatrix} \times \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 9(1) - 2(4) + 18(5) \\ 4(1) - 3(4) + 10(5) \end{bmatrix}$$

$$\text{Matrix (AB)C} = \begin{bmatrix} 9 - 8 + 90 \\ 4 - 12 + 50 \end{bmatrix} = \begin{bmatrix} 91 \\ 42 \end{bmatrix}$$

$$\text{Matrix (AB)C} = \begin{bmatrix} 91 \\ 42 \end{bmatrix}$$

$$\text{Matrix A(BC)} = \text{(AB)C} = \begin{bmatrix} 91 \\ 42 \end{bmatrix}$$

## 6 B. Question

For the following matrices, verify that  $A(BC) = (AB)C$  :

$$A = \begin{bmatrix} 2 & 3 & -1 \\ 3 & 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \text{ and } C = [1 \ -2]$$

**Answer**

$$\text{Given : } A = \begin{bmatrix} 2 & 3 & -1 \\ 3 & 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \text{ and } C = [1 \ -2]$$

Matrix A is of order  $2 \times 3$ , matrix B is of order  $3 \times 1$  and matrix C is of order  $1 \times 2$

To show : matrix  $A(BC) = (AB)C$

Formula used :

$$\begin{array}{c} \text{column } j \\ \downarrow \\ \text{row } i \leftarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \cdot \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1j} & \dots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \dots & b_{ij} & \dots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nj} & \dots & b_{nn} \end{bmatrix} = \\ \\ = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix} \end{array}$$

entry on row  $i$   
column  $j$

Where  $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

If A is a matrix of order  $a \times b$  and B is a matrix of order  $c \times d$ , then matrix AB exists and is of order  $a \times d$ , if and only if  $b = c$

If A is a matrix of order  $a \times b$  and B is a matrix of order  $c \times d$ , then matrix BA exists and is of order  $c \times b$ , if and only if  $d = a$

For matrix BC,  $a = 3, b = c = 1, d = 2$ , thus matrix BC is of order  $3 \times 2$

$$\text{Matrix BC} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \times [1 \ -2] = \begin{bmatrix} 1(1) & 1(-2) \\ 1(1) & 1(-2) \\ 2(1) & 2(-2) \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 1 & -2 \\ 2 & -4 \end{bmatrix}$$

$$\text{Matrix BC} = \begin{bmatrix} 1 & -2 \\ 1 & -2 \\ 2 & -4 \end{bmatrix}$$

For matrix A(BC),  $a = 2, b = c = 3, d = 2$ , thus matrix A(BC) is of order  $2 \times 2$

$$\text{Matrix A(BC)} = \begin{bmatrix} 2 & 3 & -1 \\ 3 & 0 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & -2 \\ 1 & -2 \\ 2 & -4 \end{bmatrix} = \begin{bmatrix} 2(1) + 3(1) - 1(2) & 2(-2) + 3(-2) - 1(-4) \\ 3(1) + 0(1) + 2(2) & 3(-2) + 0(-2) + 2(-4) \end{bmatrix}$$

$$\text{Matrix A(BC)} = \begin{bmatrix} 2 + 3 - 2 & -4 - 6 + 4 \\ 3 + 0 + 4 & -6 + 0 - 8 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ 7 & -14 \end{bmatrix}$$

$$\text{Matrix A(BC)} = \begin{bmatrix} 3 & -6 \\ 7 & -14 \end{bmatrix}$$

$$\text{Matrix A(BC)} = \begin{bmatrix} 3 & -6 \\ 7 & -14 \end{bmatrix}$$

For matrix AB,  $a = 2, b = c = 3, d = 1$ , thus matrix AB is of order  $2 \times 1$

$$\text{Matrix AB} = \begin{bmatrix} 2 & 3 & -1 \\ 3 & 0 & 2 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2(1) + 3(1) - 1(2) \\ 3(1) + 0(1) + 2(2) \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} 2 + 3 - 2 \\ 3 + 0 + 4 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

For matrix  $(AB)C$ ,  $a = 2, b = c = 1, d = 2$ , thus matrix  $(AB)C$  is of order  $2 \times 2$

$$\text{Matrix } (AB)C = \begin{bmatrix} 3 \\ 7 \end{bmatrix} \times \begin{bmatrix} 1 & -2 \end{bmatrix} = \begin{bmatrix} 3(1) & 3(-2) \\ 7(1) & 7(-2) \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ 7 & -14 \end{bmatrix}$$

$$\text{Matrix } (AB)C = \begin{bmatrix} 3 & -6 \\ 7 & -14 \end{bmatrix}$$

$$\text{Matrix } A(BC) = (AB)C = \begin{bmatrix} 3 & -6 \\ 7 & -14 \end{bmatrix}$$

**7 A. Question**

Verify that  $A(B + C) = (AB + AC)$ , when

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 2 & 0 \\ 1 & -3 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}.$$

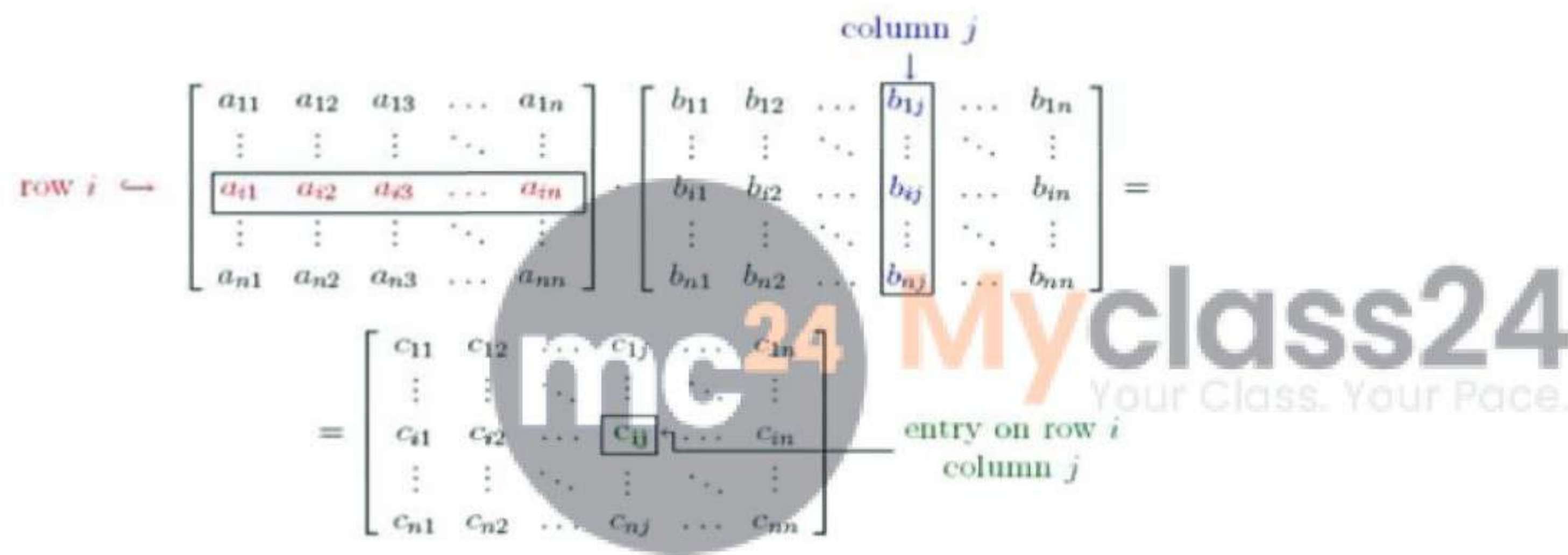
**Answer**

$$\text{Given : } A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 2 & 0 \\ 1 & -3 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}.$$

Matrix A is of order  $2 \times 2$ , matrix B is of order  $2 \times 2$  and matrix C is of order  $2 \times 2$

To verify :  $A(B + C) = (AB + AC)$

Formula used :



$$\text{Where } c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$$

If A is a matrix of order  $a \times b$  and B is a matrix of order  $c \times d$ , then matrix AB exists and is of order  $a \times d$ , if and only if  $b = c$

If A is a matrix of order  $a \times b$  and B is a matrix of order  $c \times d$ , then matrix BA exists and is of order  $c \times b$ , if and only if  $d = a$

$$B + C = \begin{bmatrix} 2 & 0 \\ 1 & -3 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2+1 & 0-1 \\ 1+0 & -3+1 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & -2 \end{bmatrix}$$

$$B + C = \begin{bmatrix} 3 & -1 \\ 1 & -2 \end{bmatrix}$$

Matrix  $A(B + C)$  is of order  $2 \times 2$

$$A(B + C) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 3 & -1 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 1(3) + 2(1) & 1(-1) + 2(-2) \\ 3(3) + 4(1) & 3(-1) + 4(-2) \end{bmatrix}$$

$$A(B + C) = \begin{bmatrix} 3+2 & -1-4 \\ 9+4 & -3-8 \end{bmatrix} = \begin{bmatrix} 5 & -5 \\ 13 & -11 \end{bmatrix}$$

$$A(B + C) = \begin{bmatrix} 5 & -5 \\ 13 & -11 \end{bmatrix}$$

For matrix AB,  $a = b = c = d = 2$ , matrix AB is of order  $2 \times 2$

$$\text{Matrix } AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 2 & 0 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} 1(2) + 2(1) & 1(0) + 2(-3) \\ 3(2) + 4(1) & 3(0) + 4(-3) \end{bmatrix}$$

$$\text{Matrix } AB = \begin{bmatrix} 2+2 & 0-6 \\ 6+4 & 0-12 \end{bmatrix} = \begin{bmatrix} 4 & -6 \\ 10 & -12 \end{bmatrix}$$

$$\text{Matrix } AB = \begin{bmatrix} 4 & -6 \\ 10 & -12 \end{bmatrix}$$

For matrix AC, a = b = c = d = 2, matrix AC is of order 2 x 2

$$\text{Matrix } AC = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1(1) + 2(0) & 1(-1) + 2(1) \\ 3(1) + 4(0) & 3(-1) + 4(1) \end{bmatrix}$$

$$\text{Matrix } AC = \begin{bmatrix} 1+0 & -1+2 \\ 3+0 & -3+4 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix}$$

$$\text{Matrix } AC = \begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix}$$

$$\text{Matrix } AB + AC = \begin{bmatrix} 4 & -6 \\ 10 & -12 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 4+1 & -6+1 \\ 10+3 & -12+1 \end{bmatrix} = \begin{bmatrix} 5 & -5 \\ 13 & -11 \end{bmatrix}$$

$$\text{Matrix } AB + AC = A(B + C) = \begin{bmatrix} 5 & -5 \\ 13 & -11 \end{bmatrix}$$

$$A(B + C) = (AB + AC)$$

### 7 B. Question

Verify that  $A(B + C) = (AB + AC)$ , when

$$A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 5 & -3 \\ 2 & 1 \end{bmatrix} \text{ and } C = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}.$$

### Answer

$$\text{Given : } A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 5 & -3 \\ 2 & 1 \end{bmatrix} \text{ and } C = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}.$$

Matrix A is of order  $3 \times 2$ , matrix B is of order  $2 \times 2$  and matrix C is of order  $2 \times 2$

To verify :  $A(B + C) = (AB + AC)$

Formula used :

$$\begin{array}{c} \text{row } i \leftarrow \\ \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \cdot \begin{array}{c} \text{column } j \\ \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1j} & \dots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \dots & b_{ij} & \dots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nj} & \dots & b_{nn} \end{bmatrix} \end{array} = \\ \\ = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix} \end{array}$$

entry on row  $i$   
column  $j$

$$\text{Where } c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$$

If A is a matrix of order  $a \times b$  and B is a matrix of order  $c \times d$ , then matrix AB exists and is of order  $a \times d$ , if and only if  $b = c$

If A is a matrix of order  $a \times b$  and B is a matrix of order  $c \times d$ , then matrix BA exists and is of order  $c \times b$ , if and only if  $d = a$

$$B + C = \begin{bmatrix} 5 & -3 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 5-1 & -3+2 \\ 2+3 & 1+4 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 5 & 5 \end{bmatrix}$$

$$B + C = \begin{bmatrix} 4 & -1 \\ 5 & 5 \end{bmatrix}$$

For Matrix  $A(B + C)$ , a = 3, b = c = d = 2, thus matrix  $A(B + C)$  is of order  $3 \times 2$

$$A(B + C) = \begin{bmatrix} 2 & 3 \\ -1 & 4 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 4 & -1 \\ 5 & 5 \end{bmatrix} = \begin{bmatrix} 2(4) + 3(5) & 2(-1) + 3(5) \\ -1(4) + 4(5) & -1(-1) + 4(5) \\ 0(4) + 1(5) & 0(-1) + 1(5) \end{bmatrix}$$

$$A(B + C) = \begin{bmatrix} 8 + 15 & -2 + 15 \\ -4 + 20 & 1 + 20 \\ 0 + 5 & 0 + 5 \end{bmatrix} = \begin{bmatrix} 23 & 13 \\ 16 & 21 \\ 5 & 5 \end{bmatrix}$$

$$A(B + C) = \begin{bmatrix} 23 & 13 \\ 16 & 21 \\ 5 & 5 \end{bmatrix}$$

For matrix AB, a = 3, b = c = d = 2, matrix AB is of order 3 x 2

$$\text{Matrix AB} = \begin{bmatrix} 2 & 3 \\ -1 & 4 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 5 & -3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 2(5) + 3(2) & 2(-3) + 3(1) \\ -1(5) + 4(2) & -1(-3) + 4(1) \\ 0(5) + 1(2) & 0(-3) + 1(1) \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} 10 + 6 & -6 + 3 \\ -5 + 8 & 3 + 4 \\ 0 + 2 & 0 + 1 \end{bmatrix} = \begin{bmatrix} 16 & -3 \\ 3 & 7 \\ 2 & 1 \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} 16 & -3 \\ 3 & 7 \\ 2 & 1 \end{bmatrix}$$

For matrix AC, a = 3, b = c = d = 2, matrix AC is of order 3 x 2

$$\text{Matrix AC} = \begin{bmatrix} 2 & 3 \\ -1 & 4 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2(-1) + 3(3) & 2(2) + 3(4) \\ -1(-1) + 4(3) & -1(2) + 4(4) \\ 0(-1) + 1(3) & 0(2) + 1(4) \end{bmatrix}$$

$$\text{Matrix AC} = \begin{bmatrix} -2 + 9 & 4 + 12 \\ 1 + 12 & -2 + 16 \\ 0 + 3 & 0 + 4 \end{bmatrix} = \begin{bmatrix} 7 & 16 \\ 13 & 14 \\ 3 & 4 \end{bmatrix}$$

$$\text{Matrix AC} = \begin{bmatrix} 7 & 16 \\ 13 & 14 \\ 3 & 4 \end{bmatrix}$$

$$\text{Matrix AB} + \text{AC} = \begin{bmatrix} 16 & -3 \\ 3 & 7 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 7 & 16 \\ 13 & 14 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 16 + 7 & 16 - 3 \\ 3 + 13 & 7 + 21 \\ 2 + 3 & 1 + 4 \end{bmatrix} = \begin{bmatrix} 23 & 13 \\ 16 & 21 \\ 5 & 5 \end{bmatrix}$$

$$\text{Matrix AB} + \text{AC} = A(B + C) = \begin{bmatrix} 23 & 13 \\ 16 & 21 \\ 5 & 5 \end{bmatrix}$$

$$A(B + C) = (AB + AC)$$

## 8. Question

$$\text{If } A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ 1 & 0 & 2 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}; \text{ verify that } A(B - C) = (AB - AC).$$

## Answer

$$\text{Given : } A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ 1 & 0 & 2 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix};$$

Matrix A is of order  $3 \times 3$ ; matrix B is of order  $3 \times 3$  and matrix C is of order  $3 \times 3$

To verify :  $A(B - C) = (AB - AC)$ .

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

The formula used :

$$= \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} & a_{11}b_{13} + a_{12}b_{23} + a_{13}b_{33} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} & a_{21}b_{13} + a_{22}b_{23} + a_{23}b_{33} \\ a_{31}b_{11} + a_{32}b_{21} + a_{33}b_{31} & a_{31}b_{12} + a_{32}b_{22} + a_{33}b_{32} & a_{31}b_{13} + a_{32}b_{23} + a_{33}b_{33} \end{bmatrix}$$

If A is a matrix of order  $a \times b$  and B is a matrix of order  $c \times d$ , then matrix AB exists and is of order  $a \times d$ , if and only if  $b = c$

If A is a matrix of order  $a \times b$  and B is a matrix of order  $c \times d$ , then matrix BA exists and is of order  $c \times b$ , if and only if  $d = a$

$$B - C = \begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ 1 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 0-1 & 5-5 & -4-2 \\ -2+1 & 1-1 & 3-0 \\ 1-0 & 0+1 & 2-1 \end{bmatrix}$$

$$B - C = \begin{bmatrix} -1 & 0 & -6 \\ -1 & 0 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

For Matrix A(B - C), a = 3, b = c = d = 3, thus matrix A(B + C) is of order 3 x 3

$$A(B - C) = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} -1 & 0 & -6 \\ -1 & 0 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$A(B - C) = \begin{bmatrix} 1(-1) + 0(-1) - 2(1) & 1(0) + 0(0) - 2(1) & 1(-6) + 0(3) - 2(1) \\ 3(-1) - 1(-1) + 0(1) & 3(0) - 1(0) + 0(1) & 3(-6) - 1(3) + 0(1) \\ -2(-1) + 1(-1) + 1(1) & -2(0) + 1(0) + 1(1) & -2(-6) + 1(3) + 1(1) \end{bmatrix}$$

$$A(B - C) = \begin{bmatrix} -1+0-2 & 0+0-2 & -6+0-2 \\ -3+1+0 & 0+0+0 & -18-3+0 \\ 2-1+1 & 0+0+1 & 12+3+1 \end{bmatrix} = \begin{bmatrix} -3 & -2 & -8 \\ -2 & 0 & -21 \\ 2 & 1 & 16 \end{bmatrix}$$

$$A(B - C) = \begin{bmatrix} -3 & -2 & -8 \\ -2 & 0 & -21 \\ 2 & 1 & 16 \end{bmatrix}$$

For matrix AB, a = 3, b = c = d = 3, matrix AB is of order 3 x 3

$$\text{Matrix AB} = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ 1 & 0 & 2 \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} 1(0) + 0(-2) - 2(1) & 1(5) + 0(1) - 2(0) & 1(-4) + 0(3) - 2(2) \\ 3(0) - 1(-2) + 0(1) & 3(5) - 1(1) + 0(0) & 3(-4) - 1(3) + 0(2) \\ -2(0) + 1(-2) + 1(1) & -2(5) + 1(1) + 1(0) & -2(-4) + 1(3) + 1(2) \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} 0+0-2 & 5+0+0 & -4+0-4 \\ 0+2+0 & 15-1+0 & -12-3+0 \\ 0-2+1 & -10+1+0 & 8+3+2 \end{bmatrix} = \begin{bmatrix} -2 & 5 & -8 \\ 2 & 14 & -15 \\ -1 & -9 & 13 \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} -2 & 5 & -8 \\ 2 & 14 & -15 \\ -1 & -9 & 13 \end{bmatrix}$$

For matrix AC, a = 3, b = c = d = 3, matrix AC is of order 3 x 3

$$\text{Matrix AC} = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\text{Matrix AC} = \begin{bmatrix} 1(1) + 0(-1) - 2(0) & 1(5) + 0(1) - 2(-1) & 1(2) + 0(0) - 2(1) \\ 3(1) - 1(-1) + 0(0) & 3(5) - 1(1) + 0(-1) & 3(2) - 1(0) + 0(1) \\ -2(1) + 1(-1) + 1(0) & -2(5) + 1(1) + 1(-1) & -2(2) + 1(0) + 1(1) \end{bmatrix}$$

$$\text{Matrix AC} = \begin{bmatrix} 1+0+0 & 5+0+2 & 2+0-2 \\ 3+1+0 & 15+1+0 & 6+0+0 \\ -2-1+0 & -10+1-1 & -4+0+1 \end{bmatrix} = \begin{bmatrix} 1 & 7 & 0 \\ 4 & 16 & 6 \\ -3 & -10 & -3 \end{bmatrix}$$

$$\text{Matrix AC} = \begin{bmatrix} 1 & 7 & 0 \\ 4 & 16 & 6 \\ -3 & -10 & -3 \end{bmatrix}$$

$$\text{Matrix AB} - \text{AC} = \begin{bmatrix} -2 & 5 & -8 \\ 2 & 14 & -15 \\ -1 & -9 & 13 \end{bmatrix} - \begin{bmatrix} 1 & 7 & 0 \\ 4 & 16 & 6 \\ -3 & -10 & -3 \end{bmatrix} = \begin{bmatrix} -2-1 & 5-7 & -8-0 \\ 2-4 & 14-16 & -15-6 \\ -1+3 & -9+10 & 13+3 \end{bmatrix}$$

$$\text{Matrix AB} - \text{AC} = \begin{bmatrix} -3 & -2 & -8 \\ -2 & 0 & -21 \\ 2 & 1 & 16 \end{bmatrix}$$

$$A(B - C) = (AB - AC) = \begin{bmatrix} -3 & -2 & -8 \\ -2 & 0 & -21 \\ 2 & 1 & 16 \end{bmatrix}$$

## 9. Question

If  $A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$ , show that  $A^2 = O$ .

**Answer**

Given :  $A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$ ,

Matrix A is of order  $2 \times 2$

To show :  $A^2 = O$

Formula used :

$$\begin{matrix} \text{column } j \\ \downarrow \\ \text{ROW } i \leftarrow \end{matrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \cdot \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1j} & \dots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \dots & b_{ij} & \dots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nj} & \dots & b_{nn} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix}$$

entry on row  $i$  column  $j$

Where  $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

If A is a matrix of order  $a \times b$  and B is a matrix of order  $c \times d$ , then matrix AB exists and is of order  $a \times d$ , if and only if  $b = c$

$$A^2 = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix} \times \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix} = \begin{bmatrix} ab(ab) + b^2(-a^2) & ab(b^2) + b^2(-ab) \\ -a^2(ab) - ab(-a^2) & -a^2(b^2) - ab(-ab) \end{bmatrix}$$

$$A^2 = \begin{bmatrix} a^2b^2 - a^2b^2 & ab^3 - ab^3 \\ -a^3b + a^3b & -a^2b^2 + a^2b^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A^2 = O$$

**10. Question**

If  $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ , show that  $A^2 = A$ .

**Answer**

Given :  $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ ,

Matrix A is of order  $3 \times 3$

To show :  $A^2 = A$

Formula used :

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} & a_{11}b_{13} + a_{12}b_{23} + a_{13}b_{33} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} & a_{21}b_{13} + a_{22}b_{23} + a_{23}b_{33} \\ a_{31}b_{11} + a_{32}b_{21} + a_{33}b_{31} & a_{31}b_{12} + a_{32}b_{22} + a_{33}b_{32} & a_{31}b_{13} + a_{32}b_{23} + a_{33}b_{33} \end{bmatrix}$$

If A is a matrix of order  $a \times b$  and B is a matrix of order  $c \times d$ , then matrix AB exists and is of order  $a \times d$ , if and only if  $b = c$