EXERCISE 1.2

Describe the following sets in Roster form:
(i) {x : x is a letter before e in the English alphabet}
(ii) {x ∈ N: $x^2 < 25$}
(iii) {x ∈ N: x is a prime number, 10 < x < 20}
(iv) {x ∈ N: x = 2n, n ∈ N}
(v) {x ∈ R: x > x}
(vi) {x : x is a prime number which is a divisor of 60}
(vii) {x : x is a two digit number such that the sum of its digits is 8}
(viii) The set of all letters in the word 'Trigonometry'
(ix) The set of all letters in the word 'Better.'
Solution:
(i) {x : x is a letter before e in the English alphabet}
So, when we read whole sentence it becomes x is such that x is a letter before 'e' in the English alphabet. Now letters before 'e' are a,b,c,d.
∴ Roster form will be {a,b,c,d}.

(ii) {x ∈ N: $x^2 < 25$}
x ∈ N that implies x is a natural number.
$x^2 < 25$
$x < \pm 5$
As x belongs to the natural number that means x < 5.
All numbers less than 5 are 1,2,3,4.
∴ Roster form will be {1,2,3,4}.

(iii) {x ∈ N: x is a prime number, 10 < x < 20}
X is a natural number and is between 10 and 20.
X is such that X is a prime number between 10 and 20.
Prime numbers between 10 and 20 are 11,13,17,19.
∴ Roster form will be {11,13,17,19}.

(iv) {x ∈ N: x = 2n, n ∈ N}
X is a natural number also x = 2n
∴ Roster form will be {2,4,6,8…..}.
This an infinite set.

(v) {x ∈ R: x > x}
Any real number is equal to its value it is neither less nor greater.

So, Roster form of such real numbers which has value less than itself has no such numbers.
∴ Roster form will be ϕ. This is called a null set.

(vi) {x : x is a prime number which is a divisor of 60}
All numbers which are divisor of 60 are = 1,2,3,4,5,6,10,12,15,20,30,60.
Now, prime numbers are = 2, 3, 5.
∴ Roster form will be {2, 3, 5}.

(vii) {x : x is a two digit number such that the sum of its digits is 8}
Numbers which have sum of its digits as 8 are = 17, 26, 35, 44, 53, 62, 71, 80
∴ Roster form will be {17, 26, 35, 44, 53, 62, 71, 80}.

(viii) The set of all letters in the word 'Trigonometry'
As repetition is not allowed in a set, then the distinct letters are
Trigonometry = t, r, i, g, o, n, m, e, y
∴ Roster form will be {t, r, i, g, o, n, m, e, y}

(ix) The set of all letters in the word 'Better.'
As repetition is not allowed in a set, then the distinct letters are
Better = b, e, t, r
∴ Roster form will be {b, e, t, r}

2. Describe the following sets in set-builder form:
(i) A = {1, 2, 3, 4, 5, 6}
(ii) B = {1, 1/2, 1/3, 1/4, 1/5, …..}
(iii) C = {0, 3, 6, 9, 12,….}
(iv) D = {10, 11, 12, 13, 14, 15}
(v) E = {0}
(vi) {1, 4, 9, 16,…,100}
(vii) {2, 4, 6, 8,….}
(viii) {5, 25, 125, 625}
Solution:
(i) A = {1, 2, 3, 4, 5, 6}
{x : x ∈ N, x<7}
This is read as x is such that x belongs to natural number and x is less than 7. It satisfies all condition of roster form.

(ii) B = {1, 1/2, 1/3, 1/4, 1/5, …}

$\{x : x = 1/n, n \in N\}$
This is read as x is such that x =1/n, where n ∈ N.

(iii) C = {0, 3, 6, 9, 12, ….}
$\{x : x = 3n, n \in Z^+$, the set of positive integers}
This is read as x is such that C is the set of multiples of 3 including 0.

(iv) D = {10, 11, 12, 13, 14, 15}
$\{x : x \in N, 9<x<16\}$
This is read as x is such that D is the set of natural numbers which are more than 9 but less than 16.

(v) E = {0}
$\{x : x = 0\}$
This is read as x is such that E is an integer equal to 0.

(vi) {1, 4, 9, 16,…, 100}
Where,
$1^2 = 1$
$2^2 = 4$
$3^2 = 9$
$4^2 = 16$
.
.
.
$10^2 = 100$
So, above set can be expressed in set-builder form as $\{x^2 : x \in N, 1 \leq x \leq 10\}$

(vii) {2, 4, 6, 8,….}
$\{x: x = 2n, n \in N\}$
This is read as x is such that the given set are multiples of 2.

(viii) {5, 25, 125, 625}
Where,
$5^1 = 5$
$5^2 = 25$
$5^3 = 125$
$5^4 = 625$
So, above set can be expressed in set-builder form as $\{5^n : n \in N, 1 \leq n \leq 4\}$

3. List all the elements of the following sets:
(i) A={x : $x^2 \leq 10$, x ∈ Z}
(ii) B = {x : x = 1/(2n-1), 1 ≤ n ≤ 5}
(iii) C = {x : x is an integer, -1/2 < x < 9/2}
(iv) D={x : x is a vowel in the word "EQUATION"}
(v) E = {x : x is a month of a year not having 31 days}
(vi) F={x : x is a letter of the word "MISSISSIPPI"}
Solution:
(i) A={x : $x^2 \leq 10$, x ∈ Z}
First of all, x is an integer hence it can be positive and negative also.
$x^2 \leq 10$
$(-3)^2 = 9 < 10$
$(-2)^2 = 4 < 10$
$(-1)^2 = 1 < 10$
$0^2 = 0 < 10$
$1^2 = 1 < 10$
$2^2 = 4 < 10$
$3^2 = 9 < 10$
Square root of next integers are greater than 10.
$x \leq \sqrt{10}$
x = 0, ±1, ±2, ±3
A = {0, ±1, ±2, ±3}

(ii) B = {x : x = 1/(2n-1), 1 ≤ n ≤ 5}
Let us substitute the value of n to find the values of x.
At n=1, x = 1/(2(1)-1) = 1/1
At n=2, x = 1/(2(2)-1) = 1/3
At n=3, x = 1/(2(3)-1) = 1/5
At n=4, x = 1/(2(4)-1) = 1/7
At n=5, x = 1/(2(5)-1) = 1/9
x = 1, 1/3, 1/5, 1/7, 1/9
∴ B = {1, 1/3, 1/5, 1/7, 1/9}

(iii) C = {x : x is an integer, -1/2 < x < 9/2}
Given, x is an integer between -1/2 and 9/2
So all integers between -0.5<x<4.5 are = 0, 1, 2, 3, 4
∴ C = {0, 1, 2, 3, 4}

(iv) D={x : x is a vowel in the word "EQUATION"}
All vowels in the word 'EQUATION' are E, U, A, I, O
∴ D = {A, E, I, O, U}

(v) E = {x : x is a month of a year not having 31 days}
A month has either 28, 29, 30, 31 days.
Out of 12 months in a year which are not having 31 days are:
February, April, June, September, November.
∴ E: {February, April, June, September, November}

(vi) F = {x : x is a letter of the word "MISSISSIPPI"}
Letters in word 'MISSISSIPPI' are M, I, S, P.
∴ F = {M, I, S, P}.

4. Match each of the sets on the left in the roster form with the same set on the right described in the set-builder form:

(i) {A,P,L,E}	**(i) {x : x+5=5, x ∈ z}**
(ii) {5,-5}	**(ii) {x : x is a prime natural number and a divisor of 10}**
(iii) {0}	**(iii) {x : x is a letter of the word "RAJASTHAN"}**
(iv) {1, 2, 5, 10}	**(iv) {x : x is a natural and divisor of 10}**
(v) {A, H, J, R, S, T, N}	**(v) {x : $x^2 - 25 = 0$}**
(vi) {2,5}	**(vi) {x : x is a letter of word "APPLE"}**

Solution:

(i) {A, P, L, E} ⇔ {x: x is a letter of word "APPLE"}

(ii) {5,-5} ⇔ {x: $x^2 - 25 = 0$}
The solution set of $x^2 - 25 = 0$ is $x = \pm 5$

(iii) {0} ⇔ {x: x+5=5, x ∈ z}
The solution set of x + 5 = 5 is x = 0.

(iv) {1, 2, 5, 10} ⇔ {x: x is a natural and divisor of 10}
The natural numbers which are divisor of 10 are 1, 2, 5, 10.

(v) {A, H, J, R, S, T, N} ⇔ {x: x is a letter of the word "RAJASTHAN"}
The distinct letters of word "RAJASTHAN" are A, H, j, R, S, T, N.

(vi) {2, 5} ⇔ {x: x is a prime natural number and a divisor of 10}
The prime natural numbers which are divisor of 10 are 2, 5.

5. Write the set of all vowels in the English alphabet which precede q.
Solution:
Set of all vowels which precede q are
A, E, I, O these are the vowels they come before q.
∴ B = {A, E, I, O}.

6. Write the set of all positive integers whose cube is odd.
Solution:
Every odd number has an odd cube
Odd numbers can be represented as 2n+1.
$\{2n+1 : n \in Z, n>0\}$ or
{1,3,5,7,……}

7. Write the set {1/2, 2/5, 3/10, 4/17, 5/26, 6/37, 7/50} in the set-builder form.
Solution:
Where,
$2 = 1^2 + 1$
$5 = 2^2 + 1$
$10 = 3^2 + 1$
.
.
$50 = 7^2 + 1$
Here we can see denominator is square of numerator +1.
So, we can write the set builder form as
$\{n/(n^2+1) : n \in N, 1 \leq n \leq 7\}$