$$A = \begin{bmatrix} -1 & 5 & 1 \\ 2 & 3 & 4 \\ 7 & 0 & 9 \end{bmatrix}$$

11. Question

Express the matrix A as the sum of a symmetric and a skew-symmetric matrix, where $A = \begin{bmatrix} 3 & -1 & 0 \\ 2 & 0 & 3 \\ 1 & -1 & 2 \end{bmatrix}$.

Answer

Given $A = \begin{bmatrix} 3 & -1 & 0 \\ 2 & 0 & 3 \\ 1 & -1 & 2 \end{bmatrix}$, to express as sum of symmetric matrix P and skew symmetric matrix Q

A = P + Q

Where $P = \frac{1}{2}(A + A')$ and $Q = \frac{1}{2}(A - A')$.

First we will find A',

$$A' = \begin{bmatrix} 3 & 2 & 1 \\ -1 & 0 & -1 \\ 0 & 3 & 2 \end{bmatrix}$$

Now using above mentioned formulas,

$$P = \frac{1}{2}(A + A')$$

$$\Rightarrow \frac{1}{2}\left(\begin{bmatrix} 3 & -1 & 0 \\ 2 & 0 & 3 \\ 1 & -1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 2 & 1 \\ -1 & 0 & -1 \\ 0 & 3 & 2 \end{bmatrix}\right)$$

$$\Rightarrow \frac{1}{2}\begin{bmatrix} 6 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & 1 \\ \frac{1}{2} & 1 & 2 \end{bmatrix}$$

$$Q = \frac{1}{2}(A - A')$$

$$\Rightarrow \frac{1}{2}\left(\begin{bmatrix} 3 & -1 & 0 \\ 2 & 0 & 3 \\ 1 & -1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 & 1 \\ -1 & 0 & -1 \\ 0 & 3 & 2 \end{bmatrix}\right)$$

$$\Rightarrow \frac{1}{2}\begin{bmatrix} 0 & -3 & -1 \\ 3 & 0 & 4 \\ 1 & -4 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & \frac{-3}{2} & \frac{-1}{2} \\ \frac{3}{2} & 0 & 2 \\ \frac{1}{2} & -2 & 0 \end{bmatrix}$$

Now A = P + Q

$$\Rightarrow \begin{bmatrix} 3 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & 1 \\ \frac{1}{2} & 1 & 2 \end{bmatrix} + \begin{bmatrix} 0 & \frac{-3}{2} & \frac{-1}{2} \\ \frac{3}{2} & 0 & 2 \\ \frac{1}{2} & -2 & 0 \end{bmatrix}$$ [Matrix A as sum of P and Q]

$$\Rightarrow \begin{bmatrix} 3 & \frac{-2}{2} & 0 \\ \frac{4}{2} & 0 & 3 \\ 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 0 \\ 2 & 0 & 3 \\ 1 & -1 & 2 \end{bmatrix}$$

12. Question

Express the matrix $A = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$ as sum of two matrices such that one is symmetric and the other is skew-symmetric.

Answer

Given $A = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$, to express as sum of symmetric matrix P and skew symmetric matrix Q.

A = P + Q

Where $P = \frac{1}{2}(A + A')$ and $Q = \frac{1}{2}(A - A')$,

First we will find A'

$$A' = \begin{bmatrix} 3 & 4 & 0 \\ 2 & 1 & 6 \\ 5 & 3 & 7 \end{bmatrix}$$

Now using above mentioned formulas

$$P = \frac{1}{2}(A + A')$$

$$\Rightarrow \frac{1}{2}\left(\begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix} + \begin{bmatrix} 3 & 4 & 0 \\ 2 & 1 & 6 \\ 5 & 3 & 7 \end{bmatrix}\right)$$

$$\Rightarrow \frac{1}{2}\begin{bmatrix} 6 & 6 & 5 \\ 6 & 2 & 9 \\ 5 & 9 & 14 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & 3 & \frac{5}{2} \\ 3 & 1 & \frac{9}{2} \\ \frac{5}{2} & \frac{9}{2} & 7 \end{bmatrix}$$

$$Q = \frac{1}{2}(A - A')$$

$$\Rightarrow \frac{1}{2}\left(\begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix} - \begin{bmatrix} 3 & 4 & 0 \\ 2 & 1 & 6 \\ 5 & 3 & 7 \end{bmatrix}\right)$$

$$\Rightarrow \frac{1}{2}\begin{bmatrix} 0 & -2 & 5 \\ 2 & 0 & -3 \\ -5 & 3 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & -1 & \frac{5}{2} \\ 1 & 0 & \frac{-3}{2} \\ \frac{-5}{2} & \frac{3}{2} & 0 \end{bmatrix}$$

Now A = P + Q

$$\Rightarrow \begin{bmatrix} 3 & 3 & \frac{5}{2} \\ 3 & 1 & \frac{9}{2} \\ \frac{5}{2} & \frac{9}{2} & 7 \end{bmatrix} + \begin{bmatrix} 0 & -1 & \frac{5}{2} \\ 1 & 0 & \frac{-3}{2} \\ \frac{-5}{2} & \frac{3}{2} & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$$

13 A. Question

For each of the following pairs of matrices A and B, verify that (AB)′ = (B′ A′) :

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}$$

Answer

Let us take C = AB

$$C = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1+6 & 4+15 \\ 2+8 & 8+20 \end{bmatrix}$$

$$C = \begin{bmatrix} 7 & 19 \\ 10 & 28 \end{bmatrix}$$

$\mathrm{LHS} \Rightarrow C' = \begin{bmatrix} 7 & 10 \\ 19 & 28 \end{bmatrix}$

To find RHS we will find transpose of matrix A and B,

$A' = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ And $B' = \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}$

RHS = B'A'

$\Rightarrow \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1+6 & 2+8 \\ 3+16 & 8+20 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 7 & 10 \\ 19 & 28 \end{bmatrix}$

LHS = RHS

Hence proved.

13 B. Question

For each of the following pairs of matrices A and B, verify that (AB)' = (B' A') :

$A = \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -3 \\ 2 & -1 \end{bmatrix}$

Answer

Let us take C = AB

$C = \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ 2 & -1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 3+(-2) & -9+1 \\ 2+(-4) & -6+2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & -8 \\ -2 & -4 \end{bmatrix}$

$\mathrm{LHS} \Rightarrow C' = \begin{bmatrix} 1 & -2 \\ -8 & -4 \end{bmatrix}$

To find RHS we will find transpose of matrix A and B,

$B' = \begin{bmatrix} 1 & 2 \\ -3 & -1 \end{bmatrix}$ And $A' = \begin{bmatrix} 3 & 2 \\ -1 & -2 \end{bmatrix}$

RHS = B'A'

$\Rightarrow \begin{bmatrix} 1 & 2 \\ -3 & -1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ -1 & -2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 3+(-2) & 2+(-4) \\ -9+1 & -6+2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & -2 \\ -8 & -4 \end{bmatrix}$

LHS = RHS

Hence proved.

13 C. Question

For each of the following pairs of matrices A and B, verify that (AB)′ = (B′ A′) :

$$A = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$ and B = [-2 -1 -4]

Answer

Let us take C = AB

$$C = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} -2 & -1 & -4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 1 & 4 \\ -4 & -2 & -8 \\ -6 & -3 & -12 \end{bmatrix}$$

LHS = C′

$$\Rightarrow \begin{bmatrix} 2 & -4 & -6 \\ 1 & -2 & -3 \\ 4 & -8 & -12 \end{bmatrix}$$

To find RHS we will find transpose of matrix A and B,

$$A' = \begin{bmatrix} -1 & 2 & 3 \end{bmatrix}$$ And $$B' = \begin{bmatrix} -2 \\ -1 \\ -4 \end{bmatrix}$$

RHS = B′A′

$$\Rightarrow \begin{bmatrix} -2 \\ -1 \\ -4 \end{bmatrix} \begin{bmatrix} -1 & 2 & 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & -4 & -6 \\ 1 & -2 & -3 \\ 4 & -8 & -12 \end{bmatrix}$$

LHS = RHS

Hence proved.

13 D. Question

For each of the following pairs of matrices A and B, verify that (AB)′ = (B′ A′) :

$$A = \begin{bmatrix} -1 & 2 & -3 \\ 4 & -5 & 6 \end{bmatrix}$$ and $$B = \begin{bmatrix} 3 & -4 \\ 2 & 1 \\ -1 & 0 \end{bmatrix}$$

Answer

Let us take C = AB

$$C = \begin{bmatrix} -1 & 2 & -3 \\ 4 & -5 & 6 \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 2 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -3+4+3 & 4+2+0 \\ 12+(-10)+(-6) & -16+(-5) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & 6 \\ -4 & -21 \end{bmatrix}$$

LHS = C'

$$\Rightarrow \begin{bmatrix} 4 & -4 \\ 6 & -21 \end{bmatrix}$$

To find RHS we will find transpose of matrix A and B,

$$A' = \begin{bmatrix} -1 & 4 \\ 2 & -5 \\ -3 & 6 \end{bmatrix} \text{ And } B' = \begin{bmatrix} 3 & 2 & -1 \\ -4 & 1 & 0 \end{bmatrix}$$

RHS = B'A'

$$\Rightarrow \begin{bmatrix} 3 & 2 & -1 \\ -4 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 4 \\ 2 & -5 \\ -3 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -3+4+3 & 12+(-10)+(-6) \\ 4+2 & -16+(-5) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & -4 \\ 6 & -21 \end{bmatrix}$$

LHS = RHS

Hence proved.

14. Question

If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, show that A'A = I.

Answer

Given $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, We will find A'

$$A' = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

LHS = A'A

$$\Rightarrow \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos^2\alpha + \sin^2\alpha & \cos\alpha.\sin\alpha + (-\sin\alpha.\cos\alpha) \\ \sin\alpha.\cos\alpha + (-\cos\alpha.\sin\alpha) & \sin^2\alpha + \cos^2\alpha \end{bmatrix}$$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ [Using $\cos^2\alpha + \sin^2\alpha = 1$ and commutative law a.b = b.a i.e. $\sin\alpha.\cos\alpha = \cos\alpha.\sin\alpha$)]

RHS = I $\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

LHS = RHS

Hence proved.

15. Question

If matrix A = [1 2 3], write AA'.

Answer

Given $A = [1\ 2\ 3]$

We will find A' to calculate AA',

$$A' = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Now

$$AA' = [123]\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\Rightarrow$[1 + 4 + 9]

$\Rightarrow$[14]

Exercise 5E

1. Question

Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$$

Answer

Let, A = $\begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$\text{Aug}[A|I] = \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 7 & 0 & 1 \end{array}\right]$, where I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 7 & 0 & 1 \end{array}\right] \xrightarrow{R_2 - 3R_1} \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & -3 & 1 \end{array}\right] \xrightarrow{R_1 - 2R_2} \left[\begin{array}{cc|cc} 1 & 0 & 7 & -2 \\ 0 & 1 & -3 & 1 \end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$A^{-1} = \begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}$ [Answer]

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

2. Question

Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$

Answer

Let, $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$\text{Aug}\left[A \mid I\right] = \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 2 & -1 & 0 & 1 \end{array}\right]$, where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 2 & -1 & 0 & 1 \end{array}\right] \xrightarrow{R_2-2R_1} \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -5 & -2 & 1 \end{array}\right] \xrightarrow{-\frac{1}{5}R_2} \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & \frac{2}{5} & -\frac{1}{5} \end{array}\right] \xrightarrow{R_1-2R_2} \left[\begin{array}{cc|cc} 1 & 0 & \frac{1}{5} & \frac{2}{5} \\ 0 & 1 & \frac{2}{5} & -\frac{1}{5} \end{array}\right]$$

Here, the matrix A is converted into the Identity matrix. Therefore, we get the A^{-1} as,

$A^{-1} = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & -\frac{1}{5} \end{bmatrix}$ [Answer]

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

3. Question

Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$$

Answer

Let, $A = \begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$\text{Aug}\left[A \mid I\right] = \left[\begin{array}{cc|cc} 2 & 5 & 1 & 0 \\ -3 & 1 & 0 & 1 \end{array}\right]$, where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{cc|cc} 2 & 5 & 1 & 0 \\ -3 & 1 & 0 & 1 \end{array}\right] \xrightarrow{R_2+R_1} \left[\begin{array}{cc|cc} 2 & 5 & 1 & 0 \\ -1 & 6 & 1 & 1 \end{array}\right] \xrightarrow{R_1+R_2} \left[\begin{array}{cc|cc} 1 & 11 & 2 & 1 \\ -1 & 6 & 1 & 1 \end{array}\right] \xrightarrow{R_2+R_1} \left[\begin{array}{cc|cc} 1 & 11 & 2 & 1 \\ 0 & 17 & 3 & 2 \end{array}\right]$$

$$\xrightarrow{\frac{1}{17}R_2} \left[\begin{array}{cc|cc} 1 & 11 & 2 & 1 \\ 0 & 1 & \frac{3}{17} & \frac{2}{17} \end{array}\right] \xrightarrow{R_1-11R_2} \left[\begin{array}{cc|cc} 1 & 0 & \frac{1}{17} & -\frac{5}{17} \\ 0 & 1 & \frac{3}{17} & \frac{2}{17} \end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$A^{-1} = \begin{bmatrix} \frac{1}{17} & -\frac{5}{17} \\ \frac{3}{17} & \frac{2}{17} \end{bmatrix} = \frac{1}{17}\begin{bmatrix} 1 & -5 \\ 3 & 2 \end{bmatrix}$ [Answer]

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

4. Question

Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix}$$

Answer

Let, $A = \begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$\text{Aug}\left[A \mid I\right] = \left[\begin{array}{cc|cc} 2 & -3 & 1 & 0 \\ 4 & 5 & 0 & 1 \end{array}\right]$, where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{cc|cc} 2 & -3 & 1 & 0 \\ 4 & 5 & 0 & 1 \end{array}\right] \xrightarrow{R_2 - 2R_1} \left[\begin{array}{cc|cc} 2 & -3 & 1 & 0 \\ 0 & 11 & -2 & 1 \end{array}\right] \xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{cc|cc} 1 & -\frac{3}{2} & \frac{1}{2} & 0 \\ 0 & 11 & -2 & 1 \end{array}\right] \xrightarrow{\frac{1}{11}R_2} \left[\begin{array}{cc|cc} 1 & -\frac{3}{2} & \frac{1}{2} & 0 \\ 0 & 1 & -\frac{2}{11} & \frac{1}{11} \end{array}\right]$$

$$\xrightarrow{R_1 + \frac{3}{2}R_2} \left[\begin{array}{cc|cc} 1 & 0 & \frac{5}{22} & \frac{3}{22} \\ 0 & 1 & -\frac{2}{11} & \frac{1}{11} \end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$A^{-1} = \begin{bmatrix} \frac{5}{22} & \frac{3}{22} \\ -\frac{2}{11} & \frac{1}{11} \end{bmatrix}$ [Answer]

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

5. Question

Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix} 4 & 0 \\ 2 & 5 \end{bmatrix}$$

Answer

Let, $A = \begin{bmatrix} 4 & 0 \\ 2 & 5 \end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$\text{Aug}\left[A \mid I\right] = \left[\begin{array}{cc|cc} 4 & 0 & 1 & 0 \\ 2 & 5 & 0 & 1 \end{array}\right]$, where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{cc|cc}4 & 0 & 1 & 0\\2 & 5 & 0 & 1\end{array}\right]\xrightarrow{R_1-2R_2}\left[\begin{array}{cc|cc}0 & -10 & 1 & -2\\2 & 5 & 0 & 1\end{array}\right]\xrightarrow{R_1\leftrightarrow R_2}\left[\begin{array}{cc|cc}2 & 5 & 0 & 1\\0 & -10 & 1 & -2\end{array}\right]\xrightarrow{\frac{1}{2}R_1}\left[\begin{array}{cc|cc}1 & \frac{5}{2} & 0 & \frac{1}{2}\\0 & -10 & 1 & -2\end{array}\right]$$

$$\xrightarrow{-\frac{1}{10}R_2}\left[\begin{array}{cc|cc}1 & \frac{5}{2} & 0 & \frac{1}{2}\\0 & 1 & -\frac{1}{10} & \frac{1}{5}\end{array}\right]\xrightarrow{R_1-\frac{5}{2}R_2}\left[\begin{array}{cc|cc}1 & 0 & \frac{1}{4} & 0\\0 & 1 & -\frac{1}{10} & \frac{1}{5}\end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$A^{-1} = \begin{bmatrix}\frac{1}{4} & 0\\-\frac{1}{10} & \frac{1}{5}\end{bmatrix}$ [Answer]

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

6. Question

Using elementary row transformations, find the inverse of each of the following matrices:

$\begin{bmatrix}6 & 7\\8 & 9\end{bmatrix}$

Answer

Let, $A = \begin{bmatrix}6 & 7\\8 & 9\end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$\text{Aug}\left[A|I\right]=\left[\begin{array}{cc|cc}6 & 7 & 1 & 0\\8 & 9 & 0 & 1\end{array}\right]$, where $I = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{cc|cc}6 & 7 & 1 & 0\\8 & 9 & 0 & 1\end{array}\right]\xrightarrow{R_2-R_1}\left[\begin{array}{cc|cc}6 & 7 & 1 & 0\\2 & 2 & -1 & 1\end{array}\right]\xrightarrow{R_1\leftrightarrow R_2}\left[\begin{array}{cc|cc}2 & 2 & -1 & 1\\6 & 7 & 1 & 0\end{array}\right]\xrightarrow{R_2-3R_1}\left[\begin{array}{cc|cc}2 & 2 & -1 & 1\\0 & 1 & 4 & -3\end{array}\right]$$

$$\xrightarrow{\frac{1}{2}R_1}\left[\begin{array}{cc|cc}1 & 1 & -\frac{1}{2} & \frac{1}{2}\\0 & 1 & 4 & -3\end{array}\right]\xrightarrow{R_1-R_2}\left[\begin{array}{cc|cc}1 & 0 & -\frac{9}{2} & \frac{7}{2}\\0 & 1 & 4 & -3\end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$A^{-1} = \begin{bmatrix}-\frac{9}{2} & \frac{7}{2}\\4 & -3\end{bmatrix}$ [Answer]

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

7. Question

Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

Answer

Let, A = $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\text{Aug}\left[A\,|\,I\right] = \left[\begin{array}{ccc|ccc} 0 & 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array}\right], \text{ where I = } \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{ccc|ccc} 0 & 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3 - 3R_1} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & -5 & -8 & 0 & -3 & 1 \end{array}\right]$$

$$\xrightarrow{R_3 + 4R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & -1 & 0 & 4 & -3 & 1 \end{array}\right] \xrightarrow{R_3 + R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & 5 & -3 & 1 \end{array}\right] \xrightarrow{R_1 - 2R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -2 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & 5 & -3 & 1 \end{array}\right]$$

$$\xrightarrow{R_2 - R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -2 & 1 & 0 \\ 0 & 1 & 0 & -4 & 3 & -1 \\ 0 & 0 & 2 & 5 & -3 & 1 \end{array}\right] \xrightarrow{\frac{1}{2}R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -2 & 1 & 0 \\ 0 & 1 & 0 & -4 & 3 & -1 \\ 0 & 0 & 1 & \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{array}\right] \xrightarrow{R_1 + R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 0 & -4 & 3 & -1 \\ 0 & 0 & 1 & \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$$A^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & -1 & 1 \\ -8 & 6 & -2 \\ 5 & -3 & 1 \end{bmatrix} \text{ [Answer]}$$

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

8. Question

Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$$

Answer

Let, A = $\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\text{Aug}\left[A\,|\,I\right]=\left[\begin{array}{ccc|ccc}2 & -3 & 3 & 1 & 0 & 0\\ 2 & 2 & 3 & 0 & 1 & 0\\ 3 & -2 & 2 & 0 & 0 & 1\end{array}\right],\ \text{where } I=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{ccc|ccc}2 & -3 & 3 & 1 & 0 & 0\\ 2 & 2 & 3 & 0 & 1 & 0\\ 3 & -2 & 2 & 0 & 0 & 1\end{array}\right]\xrightarrow{R_2-R_1}\left[\begin{array}{ccc|ccc}2 & -3 & 3 & 1 & 0 & 0\\ 0 & 5 & 0 & -1 & 1 & 0\\ 3 & -2 & 2 & 0 & 0 & 1\end{array}\right]\xrightarrow{R_3-R_1}\left[\begin{array}{ccc|ccc}2 & -3 & 3 & 1 & 0 & 0\\ 0 & 5 & 0 & -1 & 1 & 0\\ 1 & 1 & -1 & -1 & 0 & 1\end{array}\right]$$

$$\xrightarrow{R_1-R_3}\left[\begin{array}{ccc|ccc}1 & -4 & 4 & 2 & 0 & -1\\ 0 & 5 & 0 & -1 & 1 & 0\\ 1 & 1 & -1 & -1 & 0 & 1\end{array}\right]\xrightarrow{R_3-R_1}\left[\begin{array}{ccc|ccc}1 & -4 & 4 & 2 & 0 & -1\\ 0 & 5 & 0 & -1 & 1 & 0\\ 0 & 5 & -5 & -3 & 0 & 2\end{array}\right]\xrightarrow{R_3-R_2}\left[\begin{array}{ccc|ccc}1 & -4 & 4 & 2 & 0 & -1\\ 0 & 5 & 0 & -1 & 1 & 0\\ 0 & 0 & -5 & -2 & -1 & 2\end{array}\right]$$

$$\xrightarrow{R_1+R_2}\left[\begin{array}{ccc|ccc}1 & 1 & 4 & 1 & 1 & -1\\ 0 & 5 & 0 & -1 & 1 & 0\\ 0 & 0 & -5 & -2 & -1 & 2\end{array}\right]\xrightarrow{\frac{1}{5}R_2}\left[\begin{array}{ccc|ccc}1 & 1 & 4 & 1 & 1 & -1\\ 0 & 1 & 0 & -\frac{1}{5} & \frac{1}{5} & 0\\ 0 & 0 & -5 & -2 & -1 & 2\end{array}\right]\xrightarrow{-\frac{1}{5}R_3}\left[\begin{array}{ccc|ccc}1 & 1 & 4 & 1 & 1 & -1\\ 0 & 1 & 0 & -\frac{1}{5} & \frac{1}{5} & 0\\ 0 & 0 & 1 & \frac{2}{5} & \frac{1}{5} & -\frac{2}{5}\end{array}\right]$$

$$\xrightarrow{R_1-R_2}\left[\begin{array}{ccc|ccc}1 & 0 & 4 & \frac{6}{5} & \frac{4}{5} & -1\\ 0 & 1 & 0 & -\frac{1}{5} & \frac{1}{5} & 0\\ 0 & 0 & 1 & \frac{2}{5} & \frac{1}{5} & -\frac{2}{5}\end{array}\right]\xrightarrow{R_1-4R_3}\left[\begin{array}{ccc|ccc}1 & 0 & 0 & -\frac{2}{5} & 0 & \frac{3}{5}\\ 0 & 1 & 0 & -\frac{1}{5} & \frac{1}{5} & 0\\ 0 & 0 & 1 & \frac{2}{5} & \frac{1}{5} & -\frac{2}{5}\end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$$A^{-1}=\begin{bmatrix}-\frac{2}{5} & 0 & \frac{3}{5}\\ -\frac{1}{5} & \frac{1}{5} & 0\\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5}\end{bmatrix}=-\frac{1}{5}\begin{bmatrix}2 & 0 & -3\\ 1 & -1 & 0\\ -2 & -1 & 2\end{bmatrix}\ \text{[Answer]}$$

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

9. Question

Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix}3 & 0 & 2\\ 1 & 5 & 9\\ 6 & 4 & 7\end{bmatrix}$$

Answer

Let, $A=\begin{bmatrix}3 & 0 & 2\\ 1 & 5 & 9\\ 6 & 4 & 7\end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\text{Aug}\left[A\mid I\right]=\left[\begin{array}{ccc|ccc}3&0&2&1&0&0\\1&5&9&0&1&0\\6&4&7&0&0&1\end{array}\right]\text{, where I = }\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{ccc|ccc}3&0&2&1&0&0\\1&5&9&0&1&0\\6&4&7&0&0&1\end{array}\right]\xrightarrow{R_2\leftrightarrow R_1}\left[\begin{array}{ccc|ccc}1&5&9&0&1&0\\3&0&2&1&0&0\\6&4&7&0&0&1\end{array}\right]\xrightarrow{R_3-2R_2}\left[\begin{array}{ccc|ccc}1&5&9&0&1&0\\3&0&2&1&0&0\\0&4&3&-2&0&1\end{array}\right]$$

$$\xrightarrow{R_2-3R_1}\left[\begin{array}{ccc|ccc}1&5&9&0&1&0\\0&-15&-25&1&-3&0\\0&4&3&-2&0&1\end{array}\right]\xrightarrow{R_2+4R_3}\left[\begin{array}{ccc|ccc}1&5&9&0&1&0\\0&1&-13&-7&-3&4\\0&4&3&-2&0&1\end{array}\right]\xrightarrow{R_1-R_3}\left[\begin{array}{ccc|ccc}1&1&6&2&1&-1\\0&1&-13&-7&-3&4\\0&4&3&-2&0&1\end{array}\right]$$

$$\xrightarrow{R_3-4R_2}\left[\begin{array}{ccc|ccc}1&1&6&2&1&-1\\0&1&-13&-7&-3&4\\0&0&55&26&12&-15\end{array}\right]\xrightarrow{\frac{1}{55}R_3}\left[\begin{array}{ccc|ccc}1&1&6&2&1&-1\\0&1&-13&-7&-3&4\\0&0&1&\frac{26}{55}&\frac{12}{55}&-\frac{15}{55}\end{array}\right]$$

$$\xrightarrow{R_2+13R_3}\left[\begin{array}{ccc|ccc}1&1&6&2&1&-1\\0&1&0&-\frac{47}{55}&-\frac{9}{55}&\frac{25}{55}\\0&0&1&\frac{26}{55}&\frac{12}{55}&-\frac{15}{55}\end{array}\right]\xrightarrow{R_1-R_2}\left[\begin{array}{ccc|ccc}1&0&6&\frac{157}{55}&\frac{64}{55}&-\frac{80}{55}\\0&1&0&-\frac{47}{55}&-\frac{9}{55}&\frac{25}{55}\\0&0&1&\frac{26}{55}&\frac{12}{55}&-\frac{15}{55}\end{array}\right]$$

$$\xrightarrow{R_1-6R_3}\left[\begin{array}{ccc|ccc}1&0&0&\frac{1}{55}&-\frac{8}{55}&\frac{10}{55}\\0&1&0&-\frac{47}{55}&-\frac{9}{55}&\frac{25}{55}\\0&0&1&\frac{26}{55}&\frac{12}{55}&-\frac{15}{55}\end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$$A^{-1}=\begin{bmatrix}\frac{1}{55}&-\frac{8}{55}&\frac{10}{55}\\-\frac{47}{55}&-\frac{9}{55}&\frac{25}{55}\\\frac{26}{55}&\frac{12}{55}&-\frac{15}{55}\end{bmatrix}=-\frac{1}{55}\begin{bmatrix}-1&8&-10\\47&9&-25\\-26&-12&15\end{bmatrix}\text{ [Answer]}$$

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

10. Question

Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix}1&2&-3\\2&3&2\\3&-3&-4\end{bmatrix}$$

Answer

Let, $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\text{Aug}\left[A\,|\,I\right] = \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 2 & 3 & 2 & 0 & 1 & 0 \\ 3 & -3 & -4 & 0 & 0 & 1 \end{array}\right], \text{ where } I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 2 & 3 & 2 & 0 & 1 & 0 \\ 3 & -3 & -4 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3-R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 2 & 3 & 2 & 0 & 1 & 0 \\ 1 & -6 & -6 & 0 & -1 & 1 \end{array}\right] \xrightarrow{R_2-R_1} \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 1 & 1 & 5 & -1 & 1 & 0 \\ 1 & -6 & -6 & 0 & -1 & 1 \end{array}\right]$$

$$\xrightarrow{R_3-R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 1 & 1 & 5 & -1 & 1 & 0 \\ 0 & -7 & -11 & 1 & -2 & 1 \end{array}\right] \xrightarrow{R_2-R_1} \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & -1 & 8 & -2 & 1 & 0 \\ 0 & -7 & -11 & 1 & -2 & 1 \end{array}\right] \xrightarrow{-R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & 1 & -8 & 2 & -1 & 0 \\ 0 & -7 & -11 & 1 & -2 & 1 \end{array}\right]$$

$$\xrightarrow{R_3+7R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & 1 & -8 & 2 & -1 & 0 \\ 0 & 0 & -67 & 15 & -9 & 1 \end{array}\right] \xrightarrow{-\frac{1}{67}R_3} \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & 1 & -8 & 2 & -1 & 0 \\ 0 & 0 & 1 & -\frac{15}{67} & \frac{9}{67} & -\frac{1}{67} \end{array}\right]$$

$$\xrightarrow{R_2+8R_3} \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & 1 & 0 & \frac{14}{67} & \frac{5}{67} & -\frac{8}{67} \\ 0 & 0 & 1 & -\frac{15}{67} & \frac{9}{67} & -\frac{1}{67} \end{array}\right] \xrightarrow{R_1-2R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & -3 & \frac{39}{67} & -\frac{10}{67} & \frac{16}{67} \\ 0 & 1 & 0 & \frac{14}{67} & \frac{5}{67} & -\frac{8}{67} \\ 0 & 0 & 1 & -\frac{15}{67} & \frac{9}{67} & -\frac{1}{67} \end{array}\right]$$

$$\xrightarrow{R_1+3R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{6}{67} & \frac{17}{67} & \frac{13}{67} \\ 0 & 1 & 0 & \frac{14}{67} & \frac{5}{67} & -\frac{8}{67} \\ 0 & 0 & 1 & -\frac{15}{67} & \frac{9}{67} & -\frac{1}{67} \end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$$A^{-1} = \begin{bmatrix} -\frac{6}{67} & \frac{17}{67} & \frac{13}{67} \\ \frac{14}{67} & \frac{5}{67} & -\frac{8}{67} \\ -\frac{15}{67} & \frac{9}{67} & -\frac{1}{67} \end{bmatrix} = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix} \text{ [Answer]}$$

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

11. Question

Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -1 \\ 3 & -5 & 0 \end{bmatrix}$$

Answer

Let, A = $\begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -1 \\ 3 & -5 & 0 \end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\text{Aug}\left[A\,|\,I\right] = \left[\begin{array}{ccc|ccc} 3 & -1 & -2 & 1 & 0 & 0 \\ 2 & 0 & -1 & 0 & 1 & 0 \\ 3 & -5 & 0 & 0 & 0 & 1 \end{array}\right], \text{ where } I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{ccc|ccc} 3 & -1 & -2 & 1 & 0 & 0 \\ 2 & 0 & -1 & 0 & 1 & 0 \\ 3 & -5 & 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3 - R_1} \left[\begin{array}{ccc|ccc} 3 & -1 & -2 & 1 & 0 & 0 \\ 2 & 0 & -1 & 0 & 1 & 0 \\ 0 & -4 & 2 & -1 & 0 & 1 \end{array}\right] \xrightarrow{R_1 - R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & -1 & 0 \\ 2 & 0 & -1 & 0 & 1 & 0 \\ 0 & -4 & 2 & -1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2 - 2R_1} \left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & -1 & 0 \\ 0 & 2 & 1 & -2 & 3 & 0 \\ 0 & -4 & 2 & -1 & 0 & 1 \end{array}\right] \xrightarrow{R_3 + 2R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & -1 & 0 \\ 0 & 2 & 1 & -2 & 3 & 0 \\ 0 & 0 & 4 & -5 & 6 & 1 \end{array}\right] \xrightarrow{\frac{1}{4}R_3} \left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & -1 & 0 \\ 0 & 2 & 1 & -2 & 3 & 0 \\ 0 & 0 & 1 & -\frac{5}{4} & \frac{6}{4} & \frac{1}{4} \end{array}\right]$$

$$\xrightarrow{R_2 - R_3} \left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & -1 & 0 \\ 0 & 2 & 0 & -\frac{3}{4} & \frac{6}{4} & -\frac{1}{4} \\ 0 & 0 & 1 & -\frac{5}{4} & \frac{6}{4} & \frac{1}{4} \end{array}\right] \xrightarrow{\frac{1}{2}R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & -1 & 0 \\ 0 & 1 & 0 & -\frac{3}{8} & \frac{6}{8} & -\frac{1}{8} \\ 0 & 0 & 1 & -\frac{5}{4} & \frac{6}{4} & \frac{1}{4} \end{array}\right]$$

$$\xrightarrow{R_1 + R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & \frac{5}{8} & -\frac{2}{8} & -\frac{1}{8} \\ 0 & 1 & 0 & -\frac{3}{8} & \frac{6}{8} & -\frac{1}{8} \\ 0 & 0 & 1 & -\frac{5}{4} & \frac{6}{4} & \frac{1}{4} \end{array}\right] \xrightarrow{R_1 + R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{5}{8} & \frac{10}{8} & \frac{1}{8} \\ 0 & 1 & 0 & -\frac{3}{8} & \frac{6}{8} & -\frac{1}{8} \\ 0 & 0 & 1 & -\frac{5}{4} & \frac{6}{4} & \frac{1}{4} \end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$$A^{-1} = \begin{bmatrix} -\frac{5}{8} & \frac{10}{8} & \frac{1}{8} \\ -\frac{3}{8} & \frac{6}{8} & -\frac{1}{8} \\ -\frac{5}{4} & \frac{6}{4} & \frac{1}{4} \end{bmatrix} = -\frac{1}{8}\begin{bmatrix} 5 & -10 & -1 \\ 3 & -6 & 1 \\ 10 & -12 & -2 \end{bmatrix} \text{ [Answer]}$$

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

12. Question

Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$$

Answer

Let, A = $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\text{Aug}\left[A\,|\,I\right] = \left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ -3 & 0 & -1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 0 & 1 \end{array}\right], \text{ where } I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ -3 & 0 & -1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3-2R_1} \left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ -3 & 0 & -1 & 0 & 1 & 0 \\ 0 & -5 & 4 & -2 & 0 & 1 \end{array}\right] \xrightarrow{R_2+3R_1} \left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 9 & -7 & 3 & 1 & 0 \\ 0 & -5 & 4 & -2 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2+R_3} \left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 4 & -3 & 1 & 1 & 1 \\ 0 & -5 & 4 & -2 & 0 & 1 \end{array}\right] \xrightarrow{-R_3} \left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 4 & -3 & 1 & 1 & 1 \\ 0 & 5 & -4 & 2 & 0 & -1 \end{array}\right] \xrightarrow{R_3-R_2} \left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 4 & -3 & 1 & 1 & 1 \\ 0 & 1 & -1 & 1 & -1 & -2 \end{array}\right]$$

$$\xrightarrow{R_2-4R_3} \left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & -3 & 5 & 9 \\ 0 & 1 & -1 & 1 & -1 & -2 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & -1 & -2 \\ 0 & 0 & 1 & -3 & 5 & 9 \end{array}\right]$$

$$\xrightarrow{R_2+R_3} \left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 4 & 7 \\ 0 & 0 & 1 & -3 & 5 & 9 \end{array}\right] \xrightarrow{R_1+2R_3} \left[\begin{array}{ccc|ccc} 1 & 3 & 0 & -5 & 10 & 18 \\ 0 & 1 & 0 & -2 & 4 & 7 \\ 0 & 0 & 1 & -3 & 5 & 9 \end{array}\right] \xrightarrow{R_1-3R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & -3 \\ 0 & 1 & 0 & -2 & 4 & 7 \\ 0 & 0 & 1 & -3 & 5 & 9 \end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$A^{-1} = \begin{bmatrix} 1 & -2 & -3 \\ -2 & 4 & 7 \\ -3 & 5 & 9 \end{bmatrix}$ [Answer]

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

13. Question

Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$$

Answer

Let, A = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$Aug\left[A|I\right]=\left[\begin{array}{ccc|ccc}1 & 2 & 3 & 1 & 0 & 0\\ 2 & 5 & 7 & 0 & 1 & 0\\ -2 & -4 & -5 & 0 & 0 & 1\end{array}\right]\text{, where I = }\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{ccc|ccc}1 & 2 & 3 & 1 & 0 & 0\\ 2 & 5 & 7 & 0 & 1 & 0\\ -2 & -4 & -5 & 0 & 0 & 1\end{array}\right]\xrightarrow{R_2-2R_1}\left[\begin{array}{ccc|ccc}1 & 2 & 3 & 1 & 0 & 0\\ 0 & 1 & 1 & -2 & 1 & 0\\ -2 & -4 & -5 & 0 & 0 & 1\end{array}\right]\xrightarrow{R_3+2R_1}\left[\begin{array}{ccc|ccc}1 & 2 & 3 & 1 & 0 & 0\\ 0 & 1 & 1 & -2 & 1 & 0\\ 0 & 0 & 1 & 2 & 0 & 1\end{array}\right]$$

$$\xrightarrow{R_2-R_3}\left[\begin{array}{ccc|ccc}1 & 2 & 3 & 1 & 0 & 0\\ 0 & 1 & 0 & -4 & 1 & -1\\ 0 & 0 & 1 & 2 & 0 & 1\end{array}\right]\xrightarrow{R_1-2R_2}\left[\begin{array}{ccc|ccc}1 & 0 & 3 & 9 & -2 & 2\\ 0 & 1 & 0 & -4 & 1 & -1\\ 0 & 0 & 1 & 2 & 0 & 1\end{array}\right]\xrightarrow{R_1-3R_3}\left[\begin{array}{ccc|ccc}1 & 0 & 0 & 3 & -2 & -1\\ 0 & 1 & 0 & -4 & 1 & -1\\ 0 & 0 & 1 & 2 & 0 & 1\end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$$A^{-1}=\begin{bmatrix}3 & -2 & -1\\ -4 & 1 & -1\\ 2 & 0 & 1\end{bmatrix}\text{ [Answer]}$$

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

14. Question

Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix}3 & 0 & -1\\ 2 & 3 & 0\\ 0 & 4 & 1\end{bmatrix}$$

Answer

Let, $A=\begin{bmatrix}3 & 0 & -1\\ 2 & 3 & 0\\ 0 & 4 & 1\end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$Aug\left[A|I\right]=\left[\begin{array}{ccc|ccc}3 & 0 & -1 & 1 & 0 & 0\\ 2 & 3 & 0 & 0 & 1 & 0\\ 0 & 4 & 1 & 0 & 0 & 1\end{array}\right]\text{, where I = }\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{ccc|ccc}3 & 0 & -1 & 1 & 0 & 0\\ 2 & 3 & 0 & 0 & 1 & 0\\ 0 & 4 & 1 & 0 & 0 & 1\end{array}\right]\xrightarrow{R_1-R_2}\left[\begin{array}{ccc|ccc}1 & -3 & -1 & 1 & -1 & 0\\ 2 & 3 & 0 & 0 & 1 & 0\\ 0 & 4 & 1 & 0 & 0 & 1\end{array}\right]\xrightarrow{R_2-2R_1}\left[\begin{array}{ccc|ccc}1 & -3 & -1 & 1 & -1 & 0\\ 0 & 9 & 2 & -2 & 3 & 0\\ 0 & 4 & 1 & 0 & 0 & 1\end{array}\right]$$

$$\xrightarrow{R_2-2R_3}\left[\begin{array}{ccc|ccc}1 & -3 & -1 & 1 & -1 & 0\\ 0 & 1 & 0 & -2 & 3 & -2\\ 0 & 4 & 1 & 0 & 0 & 1\end{array}\right]\xrightarrow{R_3-4R_2}\left[\begin{array}{ccc|ccc}1 & -3 & -1 & 1 & -1 & 0\\ 0 & 1 & 0 & -2 & 3 & -2\\ 0 & 0 & 1 & 8 & -12 & 9\end{array}\right]\xrightarrow{R_1+R_3}\left[\begin{array}{ccc|ccc}1 & -3 & 0 & 9 & -13 & 9\\ 0 & 1 & 0 & -2 & 3 & -2\\ 0 & 0 & 1 & 8 & -12 & 9\end{array}\right]$$

$$\xrightarrow{R_1+3R_2}\left[\begin{array}{ccc|ccc}1 & 0 & 0 & 3 & -4 & 3\\ 0 & 1 & 0 & -2 & 3 & -2\\ 0 & 0 & 1 & 8 & -12 & 9\end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$$A^{-1} = \begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix} \text{[Answer]}$$

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

15. Question

Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

Answer

$$\text{Let, } A = \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\text{Aug}\left[A\,|\,I\right] = \left[\begin{array}{ccc|ccc} -1 & 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array}\right], \text{ where } I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{ccc|ccc} -1 & 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1+R_2} \left[\begin{array}{ccc|ccc} 0 & 3 & 5 & 1 & 1 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3-3R_2} \left[\begin{array}{ccc|ccc} 0 & 3 & 5 & 1 & 1 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & -5 & -8 & 0 & -3 & 1 \end{array}\right]$$

$$\xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 3 & 5 & 1 & 1 & 0 \\ 0 & -5 & -8 & 0 & -3 & 1 \end{array}\right] \xrightarrow{R_2+R_3} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & -2 & -3 & 1 & -2 & 1 \\ 0 & -5 & -8 & 0 & -3 & 1 \end{array}\right] \xrightarrow{R_2+R_1} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & -5 & -8 & 0 & -3 & 1 \end{array}\right]$$

$$\xrightarrow{R_1-R_2} \left[\begin{array}{ccc|ccc} 0 & 2 & 3 & -1 & 2 & -1 \\ 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & -5 & -8 & 0 & -3 & 1 \end{array}\right] \xrightarrow{R_3+3R_1} \left[\begin{array}{ccc|ccc} 0 & 2 & 3 & -1 & 2 & -1 \\ 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & 1 & -3 & 3 & -2 \end{array}\right] \xrightarrow{R_1-2R_3} \left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 5 & -4 & 3 \\ 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & 1 & -3 & 3 & -2 \end{array}\right]$$

$$\xrightarrow{R_3-R_1} \left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 5 & -4 & 3 \\ 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & 0 & -8 & 7 & -5 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 5 & -4 & 3 \\ 0 & 1 & 0 & -8 & 7 & -5 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & 0 & -8 & 7 & -5 \\ 0 & 0 & 1 & 5 & -4 & 3 \end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$$A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix} \text{[Answer]}$$

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

Exercise 5F

1. Question

Construct a 3 × 2 matrix whose elements are given by

$a_{ij} = \frac{1}{2}(i-2j)^2$

Answer

Here, i is the subscript for a row, and j is the subscript for column

And the given matrix is 3×2, so $1 \le i \le 3$ and $1 \le j \le 2$

Hence for i=1, j=1, $a_{11} = \frac{1}{2}(1-(2\times 1))^2 = \frac{1}{2}$

For i=1, j=2, $a_{12} = \frac{1}{2}(1-(2\times 2))^2 = \frac{9}{2}$

For i=2, j=1 $a_{21} = \frac{1}{2}(2-(2\times 1))^2 = 0$

For i=2, j=2 $a_{22} = \frac{1}{2}(2-(2\times 2))^2 = 2$

For i=3, j=1 $a_{31} = \frac{1}{2}(3-(2\times 1))^2 = \frac{1}{2}$

For i=3, j=2 $a_{32} = \frac{1}{2}(3-(2\times 2))^2 = \frac{1}{2}$

Hence the required matrix is :- $\begin{bmatrix} \frac{1}{2} & \frac{9}{2} \\ 0 & 2 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

2. Question

Construct a 2 × 3 matrix whose elements are given by

$a_{ij} = \frac{1}{2}|-3i+j|.$

Answer

The elements of the matrix are given by, $a_{ij} = \frac{1}{2}|-3j+j|$

Matrix is 2× 3 hence, $1 \le i \le 2, 1 \le j \le 3$

Here, i is the subscript for a row, and j is the subscript for column

For i=1, j=1, $a_{11} = \frac{1}{2}|-3(1)+1| = 1$

For i=1, j=2, $a_{12} = \frac{1}{2}|-3(1)+2| = \frac{1}{2}$

For i=1, j=3, $a_{13} = \frac{1}{2}|-3(1)+3| = 0$

For i=2, j=1, $a_{21} = \frac{1}{2}|-3(2)+1| = \frac{5}{2}$

For i=2, j=2, $a_{22} = \frac{1}{2}|-3(2)+2| = 2$

For i=2, j=3, $a_{23} = \frac{1}{2}|-3(2)+3| = \frac{3}{2}$

Hence the required matrix is :-

$$\begin{bmatrix} 1 & \frac{1}{2} & 0 \\ \frac{5}{2} & 2 & \frac{3}{2} \end{bmatrix}$$

3. Question

If $\begin{bmatrix} x+2y & -y \\ 3x & 4 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 6 & 4 \end{bmatrix}$, find the values of x and y.

Answer

On comparing L.H.S. and R. H.S we get,

$\begin{bmatrix} x+2y & -y \\ 3x & 4 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 6 & 4 \end{bmatrix}$

On comparing each term we get,

$x + 2y = -4$ ....(i)

$-y = 3$ ...(ii)

$3x = 6$ .....(iii)

From (i), (ii) and (iii), we get,

$y = -3 \ and \ x = 2$

4. Question

Find the values of x and y, if

$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$.

Answer

Given,

$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

Using the property of matrix multiplication such that h is scalar, $h\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ah & bh \\ ch & dh \end{bmatrix}$

Using the matrix property of matrix addition, when two matrices are of the same order then, each element gets added to the corresponding element,

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$

$\begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix}$

$\begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

Comparing each element we get,

2+y=5, ⇒ y=3

2x+2=8, ⇒x=3

5. Question

If $x.\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y.\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$, find the values of x and y.

Answer

Given, $x.\ \begin{bmatrix} 2 \\ 3 \end{bmatrix} \ + \ y.\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

$\begin{bmatrix} 2x \\ 3x \end{bmatrix} + \begin{bmatrix} -y \\ y \end{bmatrix} = \begin{bmatrix} 2x-y \\ 3x+y \end{bmatrix}$

And we have,

$\begin{bmatrix} 2x-y \\ 3x+y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

Solving the linear equations, we get,

$x = 3, y = -4$

6. Question

If $\begin{bmatrix} x & 3x-y \\ 2x+z & 3y-w \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 4 & 7 \end{bmatrix}$, find the values of x, y, z, ω.

Answer

Given,

$\begin{bmatrix} x & 3x-y \\ 2x+z & 3y-w \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 4 & 7 \end{bmatrix}$

On comparing each element of the two matrices we get,

x=3,

3x-y=2

y=7

2x+z=4,

z=-2,

3y-w=7,

w=14

7. Question

If $\begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix} = 3\begin{bmatrix} x & y \\ z & w \end{bmatrix}$, find the values of x, y, z, ω.

Answer

Given,

$\begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix} = 3\begin{bmatrix} x & y \\ z & w \end{bmatrix}$

First applying matrix addition then, comparing each element of the matrix with the corresponding element we get,

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$

$\begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix} = \begin{bmatrix} 3x & 3y \\ 3z & 3w \end{bmatrix}$

$\begin{bmatrix} x+4 & 6+x+y \\ -1+z+w & 2w+3 \end{bmatrix} = \begin{bmatrix} 3x & 3y \\ 3z & 3w \end{bmatrix}$

We now have, $x + 4 = 3x$, ...(i)

x=2

$2w + 3 = 3w$, ...(ii)

$w = 3$

6+x+y=3y, substituting x from (i) we get,

y = 4,

And -1+z+w=3z, substituting w from (ii), we get,

z=1

8. Question

If A = diag (3 -2, 5) and B = diag (1 3 -4), find (A + B).

Answer

We are given two diagonal matrices A and B,

On adding the two diagonal matrices of order (3× 3) we get an diagonal matrix of order (3×3)

Each of the elements get added to the corresponding element hence, we get after adding,

$$\begin{bmatrix}3 & 0 & 0\\0 & -2 & 0\\0 & 0 & 5\end{bmatrix}+\begin{bmatrix}1 & 0 & 0\\0 & 3 & 0\\0 & 0 & -4\end{bmatrix}=\begin{bmatrix}4 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

Hence, we get A+B = diag(4 1 1)

9. Question

Show that

$$\cos\theta\cdot\begin{bmatrix}\cos\theta & \sin\theta\\-\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\cdot$$

$$\begin{bmatrix}\sin\theta & -\cos\theta\\\cos\theta & \sin\theta\end{bmatrix}=I$$

Answer

We have to show that

$$\cos\theta.\begin{bmatrix}\cos\theta & \sin\theta\\-\sin\theta & \cos\theta\end{bmatrix}+sin\theta.\begin{bmatrix}\sin\theta & -\cos\theta\\\cos\theta & \sin\theta\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

Multiplying the scalars with we get,

$$\begin{bmatrix}\cos\theta\times\cos\theta & \cos\theta\times\sin\theta\\\cos\theta\times(-\sin\theta) & \cos\theta\times\cos\theta\end{bmatrix}+\begin{bmatrix}\sin\theta\times\sin\theta & \sin\theta\times(-\cos\theta)\\\sin\theta\times\cos\theta & \sin\theta\times\sin\theta\end{bmatrix}$$

$$\begin{bmatrix}cos^2\theta+sin^2\theta & 0\\0 & cos^2\theta+sin^2\theta\end{bmatrix}$$

And we know that $cos^2\theta+sin^2\theta=1$

$$\begin{bmatrix}cos^2\theta+sin^2\theta & 0\\0 & cos^2\theta+sin^2\theta\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

Hence, proved.

10. Question

If $A=\begin{bmatrix}1 & -5\\-3 & 2\\4 & -2\end{bmatrix}$ and $B=\begin{bmatrix}3 & 1\\2 & -1\\-2 & 3\end{bmatrix}$, find the matrix C such that A + B + C is a zero matrix

Answer

Given, A+B+C = zero matrix

We know that zero matrix is a matrix whose all elements are zero, so we have,

$$A=\begin{bmatrix}1 & -5\\-3 & 2\\4 & -2\end{bmatrix},B=\begin{bmatrix}3 & 1\\2 & -1\\-2 & 3\end{bmatrix}$$

WE have A+B+C=0,

So C = -A+B,

$$-C=\begin{bmatrix}1 & -5\\-3 & 2\\4 & -2\end{bmatrix}+\begin{bmatrix}3 & 1\\2 & -1\\-2 & 3\end{bmatrix}$$

$$C=\begin{bmatrix}-4 & 4\\1 & -1\\-2 & -1\end{bmatrix}$$

11. Question

If $A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\\sin\alpha & \cos\alpha\end{bmatrix}$ then find the least value of α for which A + A' = I.

Answer

Given, $A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\\sin\alpha & \cos\alpha\end{bmatrix}$

Here, A' i.e. A transpose is $\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

We are given that A+A'=I

So, $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

After doing addition of matrices, we get,

$\begin{bmatrix} \cos\alpha + \cos\alpha & \sin\alpha - \sin\alpha \\ \sin\alpha - \sin\alpha & \cos\alpha + \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

On comparing the elements we get,

$2\cos\alpha = 1$

This implies, $\cos\alpha = \frac{1}{2}$

For α belongs 0 to π, $\alpha = \frac{\pi}{3}$

12. Question

Find the value of x and y for which

$$\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Answer

Given,

$\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

Applying matrix multiplication we get,

$\begin{bmatrix} 2x - 3y \\ x + y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

On comparing the elements we get, 2x-3y = 1,

x+y = 3,

On solving the equations we get, x=2, y=1

13. Question

Find the value of x and y for which

$$\begin{bmatrix} x & y \\ 3y & x \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}.$$

Answer

Given,

$\begin{bmatrix} x & y \\ 3y & x \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

Applying matrix multiplication we have, $\begin{bmatrix} x + 2y \\ 3y + 2x \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

On comparing the elements with each other we get,

The linear equations, x+2y=3, 3y+2x=5

On solving these equations we get x = 1, y = 1

14. Question

If $A = \begin{bmatrix} 4 & 5 \\ 1 & 8 \end{bmatrix}$, show that (A + A') is symmetric

Answer

Given, $A = \begin{bmatrix} 4 & 5 \\ 1 & 8 \end{bmatrix} and\ A' = \begin{bmatrix} 4 & 1 \\ 5 & 8 \end{bmatrix}$

Then, (A +A') will be, $\begin{bmatrix} 4 & 5 \\ 1 & 8 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ 5 & 8 \end{bmatrix} = \begin{bmatrix} 8 & 6 \\ 6 & 16 \end{bmatrix}$

The matrix $\begin{bmatrix} 8 & 6 \\ 6 & 16 \end{bmatrix}$ is a symmetrical matrix.

15. Question

If $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, and show that (A - A') is skew-symmetric

Answer

Given,

$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, and

$A' = \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}$

(A - A') = $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

The matrix $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ is skew-symmetric.

16. Question

If $A = \begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix}$, find a matrix X such that A + 2B + X = O.

Answer

Given, $A = \begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix}, B = \begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix}$

We need to a matrix X such that, A +2B + X = 0

We have, X = -(A + 2B),

$X = - \begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix} + 2\begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix}$

$X = - \begin{bmatrix} 2 + (-2) & -3 + (2 \times 2) \\ 4 + 0 & 5 + (2 \times 3) \end{bmatrix}$

$X = \begin{bmatrix} 0 & -1 \\ -4 & -11 \end{bmatrix}$

17. Question

If $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix}$, find a matrix X such that

3 A – 2B + X = O.

Answer

Given, $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}\ and\ B = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix}$

We have 3A - 2B + X = 0

So X = -(3A - 2B)

Thus,

$X = - 3\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix} - 2\begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix}$

$$X = -\begin{bmatrix} 3\times 4+2\times 2 & 3\times 2-2\times 1 \\ 3\times 1-2\times 3 & 3\times 3-2\times 2 \end{bmatrix}$$

$$X = \begin{bmatrix} -16 & -4 \\ 3 & -5 \end{bmatrix}$$

18. Question

If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, show that A' A = I.

Answer

Given, $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$A' = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

Then , $AA' = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

Applying matrix multiplication we get,

$$AA' = \begin{bmatrix} \cos\alpha\times\cos\alpha+\sin\alpha\times\sin\alpha & \cos\alpha\times(-\sin\alpha)+\sin\alpha\times\cos\alpha \\ (-\sin\alpha)\times\cos\alpha+\cos\alpha\times\sin\alpha & (-\sin\alpha)\times(-\sin\alpha)+\cos\alpha\times\cos\alpha \end{bmatrix}$$

$$AA' = \begin{bmatrix} cos^2\alpha+sin^2\alpha & 0 \\ 0 & cos^2\alpha+sin^2\alpha \end{bmatrix}$$

Hence, $AA' = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

As we know that $cos^2\alpha + sin^2\alpha = 1$

19. Question

If A and B are symmetric matrices of the same order, show that (AB – BA) is a skew symmetric matrix.

Answer

We are given that A and B are symmetric matrices of the same order then, we need to show that (AB – BA) is a skew symmetric matrix.

Let us consider P is a matrix of the same order as A and B

And let P = (AB – BA),

we have A = A' and B = B'

then, P' = (AB - BA)'

P' = ((AB)' - (BA)') ....using reversal law we have (CD)'=D'C'

P' = (B'A' – A'B')

P' = (BA - AB)

P' = -P

Hence, P is a skew symmetric matrix.

20. Question

If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ and f(x) = x^2 – 4x + 1, find f(A).

Answer

Given, $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

f(x) = x^2 - 4x + 1,

f(A) = A^2 - 4A + I,

$$f(A) = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} - 4\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$f(A) = \begin{bmatrix} 4+3-8+1 & 6+6-12+0 \\ 2+2-4+0 & 3+4-8+1 \end{bmatrix}$$

$$f(A) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

21. Question

If the matrix A is both symmetric and skew-symmetric, show that A is a zero matrix.

Answer

Given that matrix A is both symmetric and skew symmetric, then,

We have A = A′ ......(i)

And A = -A′ ......(ii)

From (i) and (ii) we get,

A′ = -A′,

2A′ = 0

A′ = 0

Then, A = 0

Hence proved.