Exercise 7(C)

Solution 1:

(i) $9^{\frac{5}{2}} - 3 \times 8^{0} - \left(\frac{1}{81}\right)^{-\frac{1}{2}}$

$= \left(3^2\right)^{\frac{5}{2}} - 3 \times 1 - \left(\frac{1}{3^4}\right)^{-\frac{1}{2}}$

$= 3^{2 \times \frac{5}{2}} - 3 - 3^{-4 \times \left(-\frac{1}{2}\right)}$

$= 3^5 - 3 - 3^2$

$= 243 - 3 - 9$

$= 231$

(ii) $(64)^{\frac{2}{3}} - \sqrt[3]{125} - \frac{1}{2^{-5}} + (27)^{-\frac{2}{3}} \times \left(\frac{25}{9}\right)^{-\frac{1}{2}}$

$= \left(4^3\right)^{\frac{2}{3}} - \sqrt[3]{5^3} - 2^5 + \left(3^3\right)^{-\frac{2}{3}} \times \left(\frac{5^2}{3^2}\right)^{-\frac{1}{2}}$

$= 4^2 - 5 - 2^5 + 3^{-2} \times \left(\frac{5}{3}\right)^{2 \times \left(-\frac{1}{2}\right)}$

$= 16 - 5 - 32 + \frac{1}{3^2} \times \left(\frac{5}{3}\right)^{-1}$

$= -21 + \frac{1}{9} \times \frac{3}{5}$

$= -21 + \frac{1}{15}$

$= \frac{-315 + 1}{15}$

$= \frac{-314}{15}$

$= -20\frac{14}{15}$

(iii) $\left[\left(-\frac{2}{3}\right)^{-2}\right]^{3} \times \left(\frac{1}{3}\right)^{-4} \times 3^{-1} \times \frac{1}{6}$

$$= \left[\left(-\frac{3}{2}\right)^{2}\right]^{3} \times (3)^{4} \times \frac{1}{3} \times \frac{1}{3 \times 2}$$

$$= \left(-\frac{3}{2}\right)^{6} \times (3)^{2} \times \frac{1}{2}$$

$$= \frac{3^{6+2}}{2^{6+1}}$$

$$= \frac{3^{8}}{2^{7}}$$

Solution 2:

$$\frac{3 \times 9^{n+1} - 9 \times 3^{2n}}{3 \times 3^{2n+3} - 9^{n+1}}$$

$$= \frac{3 \times \left(3^{2}\right)^{n+1} - 3^{2} \times 3^{2n}}{3 \times 3^{2n+3} - \left(3^{2}\right)^{n+1}}$$

$$= \frac{3^{1+2n+2} - 3^{2+2n}}{3^{1+2n+3} - 3^{2n+2}}$$

$$= \frac{3^{3+2n} - 3^{2+2n}}{3^{4+2n} - 3^{2n+2}}$$

$$= \frac{3^{2n}\left(3^{3} - 3^{2}\right)}{3^{2n}\left(3^{4} - 3^{2}\right)}$$

$$= \frac{27 - 9}{81 - 9}$$

$$= \frac{18}{72}$$

$$= \frac{1}{4}$$

Solution 3:

$3^{x-1} \times 5^{2y-3} = 225$

$\Rightarrow 3^{x-1} \times 5^{2y-3} = 3^2 \times 5^2$

$\Rightarrow x - 1 = 2 \text{ and } 2y - 3 = 2$

$\Rightarrow x = 3 \text{ and } 2y = 5$

$\Rightarrow x = 3 \text{ and } y = \frac{5}{2}$

$\Rightarrow x = 3 \text{ and } y = 2\frac{1}{2}$

Solution 4:

$\left(\frac{a^{-1}b^2}{a^2b^{-4}}\right)^7 \div \left(\frac{a^3b^{-5}}{a^{-2}b^3}\right)^{-5} = a^x \cdot b^y$

$\Rightarrow \left(\frac{b^6}{a^3}\right)^7 \div \left(\frac{a^5}{b^8}\right)^{-5} = a^x \cdot b^y$

$\Rightarrow \left(\frac{b^6}{a^3}\right)^7 \div \left(\frac{b^8}{a^5}\right)^5 = a^x \cdot b^y$

$\Rightarrow \frac{b^{42}}{a^{21}} \div \frac{b^{40}}{a^{25}} = a^x \cdot b^y$

$\Rightarrow \frac{b^{42}}{a^{21}} \times \frac{a^{25}}{b^{40}} = a^x \cdot b^y$

$\Rightarrow b^2 \times a^4 = a^x \times b^y$

$\Rightarrow x = 4 \text{ and } y = 2$

$\Rightarrow x + y = 4 + 2 = 6$

Solution 5:

$3^{x+1} = 9^{x-3}$

$\Rightarrow 3^x \times 3 = (3^2)^{x-3}$

$\Rightarrow 3^x \times 3 = 3^{2x-6}$

$\Rightarrow 3^x \times 3 = \frac{3^{2x}}{3^6}$

$\Rightarrow 3^6 \times 3 = \frac{3^{2x}}{3^x}$

$\Rightarrow 3^7 = 3^x$

$\Rightarrow x = 7$

$\Rightarrow 2^{1+x} = 2^{1+7} = 2^8 = 256$

Solution 6:

$2^x = 4^y = 8^z$

$\Rightarrow 2^x = 2^{2y} = 2^{3z}$

$\Rightarrow x = 2y = 3z$

$\Rightarrow y = \frac{x}{2}$ and $z = \frac{x}{3}$

Now, $\frac{1}{2x} + \frac{1}{4y} + \frac{1}{8z} = 4$

$\Rightarrow \frac{1}{2x} + \frac{1}{\frac{4x}{2}} + \frac{1}{\frac{8x}{3}} = 4$

$\Rightarrow \frac{1}{2x} + \frac{2}{4x} + \frac{3}{8x} = 4$

$\Rightarrow \frac{1}{2x} + \frac{1}{2x} + \frac{3}{8x} = 4$

$\Rightarrow \frac{4+4+3}{8x} = 4$

$\Rightarrow \frac{11}{8x} = 4$

$\Rightarrow x = \frac{11}{32}$

Solution 7:

$\frac{9^n \cdot 3^2 \cdot 3^n - (27)^n}{(3^m \cdot 2)^3} = 3^{-3}$

$\Rightarrow \frac{3^{2n} \cdot 3^2 \cdot 3^n - 3^{3n}}{3^{3m} \cdot 2^3} = \frac{1}{3^3}$

$\Rightarrow \frac{3^{3n} \cdot 3^2 - 3^{3n}}{3^{3m} \cdot 2^3} = \frac{1}{3^3}$

$\Rightarrow \frac{3^{3n}(3^2 - 1)}{3^{3m} \times 8} = \frac{1}{3^3}$

$\Rightarrow \frac{3^{3n} \times 8}{3^{3m} \times 8} = \frac{1}{3^3}$

$\Rightarrow \frac{1}{3^{3(m-n)}} = \frac{1}{3^{3\times1}}$

$\Rightarrow m - n = 1$ (proved)

Solution 8:

$$(13)^{\sqrt{x}} = 4^4 - 3^4 - 6$$

$$\Rightarrow (13)^{\sqrt{x}} = 256 - 81 - 6$$

$$\Rightarrow (13)^{\sqrt{x}} = 169$$

$$\Rightarrow (13)^{\sqrt{x}} = 13^2$$

$$\Rightarrow \sqrt{x} = 2$$

$$\Rightarrow x = 4$$

Solution 9:

$$3^{4x} = (81)^{-1} \text{ and } (10)^{\frac{1}{y}} = 0.0001$$

$$\Rightarrow 3^{4x} = \left(3^4\right)^{-1} \text{ and } (10)^{\frac{1}{y}} = \frac{1}{10000}$$

$$\Rightarrow 3^{4x} = 3^{-4} \text{ and } (10)^{\frac{1}{y}} = \frac{1}{10^4}$$

$$\Rightarrow 4x = -4 \text{ and } (10)^{\frac{1}{y}} = 10^{-4}$$

$$\Rightarrow x = -1 \text{ and } \frac{1}{y} = -4$$

$$\Rightarrow x = -1 \text{ and } y = -\frac{1}{4}$$

$$\therefore 2^{-x} \times 16^{y} = 2^{-(-1)} \times 16^{-\frac{1}{4}}$$

$$= 2 \times 2^{4x\left(-\frac{1}{4}\right)}$$

$$= 2 \times 2^{-1}$$

$$= 2^{1-1}$$

$$= 2^0$$

$$= 1$$

Solution 10:

$3(2^x + 1) - 2^{x+2} + 5 = 0$

$\Rightarrow 3 \times 2^x + 3 - 2^x \times 2^2 + 5 = 0$

$\Rightarrow 2^x(3 - 2^2) + 8 = 0$

$\Rightarrow 2^x(3 - 4) = -8$

$\Rightarrow 2^x \times (-1) = -8$

$\Rightarrow 2^x = 8$

$\Rightarrow 2^x = 2^3$

$\Rightarrow x = 3$

Solution 11:

$(a^m)^n = a^m \cdot a^n$

$\Rightarrow a^{mn} = a^{m+n}$

$\Rightarrow mn = m + n \quad \ldots.(1)$

Now,

$m(n-1) - (n-1)$

$= mn - m - n + 1$

$= m + n - m - n + 1 \quad \ldots.[\text{From } (1)]$

$= 1$

Solution 12:

$m = \sqrt[3]{15}$ and $n = \sqrt[3]{14}$

$\Rightarrow m^3 = 15$ and $n^3 = 14$

$$\therefore m - n - \frac{1}{m^2 + mn + n^2} = \frac{(m^3 + m^2n + mn^2) - (m^2n + mn^2 + n^3) - 1}{m^2 + mn + n^2}$$

$$= \frac{m^3 + m^2n + mn^2 - m^2n - mn^2 - n^3 - 1}{m^2 + mn + n^2}$$

$$= \frac{m^3 - n^3 - 1}{m^2 + mn + n^2}$$

$$= \frac{15 - 14 - 1}{m^2 + mn + n^2}$$

$$= \frac{1 - 1}{m^2 + mn + n^2}$$

$$= 0$$