Now dividing both side by cosx we get,

$2\sin x - 4\cos^2 x + 3 = 0$

We know that,

$\cos^2 x + \sin^2 x = 1$

or $\cos^2 x = 1 - \sin^2 x$

$\Rightarrow 2\sin x - 4(1 - \sin^2 x) + 3 = 0$

$\Rightarrow 2\sin x - 4 + 4\sin^2 x + 3 = 0$

$\Rightarrow 2\sin x + 4\sin^2 x - 1 = 0$

We can write it as,

$4\sin^2 x + 2\sin x - 1 = 0$

Now applying formula

Here, $ax^2 + bx + c = 0$

So, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Now applying it in the equation

$$\sin x = \frac{-2 \pm \sqrt{2^2 - 4(4)(-1)}}{2}$$

$$\sin x = \frac{-2 \pm \sqrt{4 + 16}}{8}$$

$$\sin x = \frac{-2 \pm \sqrt{20}}{8}$$

$$\sin x = \frac{(-2 \pm 2.\sqrt{5})}{8}$$

$\sin x = \frac{2(-1 \pm \sqrt{5})}{8}$

$\sin x = \frac{-1 \pm \sqrt{5}}{4}$

$\sin 18° = \frac{-1 \pm \sqrt{5}}{4}$

Now sin 18° is positive, as 18° lies in first quadrant.

$\therefore \sin 18° = \frac{\sqrt{5}-1}{4}$

Putting the value in eq. (i), we get

= sin 30° sin 18°

$= \frac{1}{2} \times \frac{\sqrt{5}-1}{4}$

$= \frac{\sqrt{5}-1}{8}$

= RHS

∴ LHS = RHS

Hence Proved

Q. 19. B. Prove that

$$\sin^2 72^0 - \cos^2 30^0 = \frac{(\sqrt{5}-1)}{8}$$

Answer :

To Prove: $\sin^2 72° - \cos^2 30° = \frac{\sqrt{5}-1}{8}$

Taking LHS,

= $\sin^2 72°$ - $\cos^2 30°$

$= \sin^2(90° - 18°) - \cos^2 30°$

$= \cos^2 18° - \cos^2 30°$ …(i)

Here, we don't know the value of cos 18°. So, we have to find the value of cos 18°

Let x = 18°

so, 5x = 90°

Now, we can write

2x + 3x = 90°

so 2x = 90° - 3x

Now taking sin both the sides, we get

sin2x = sin(90° - 3x)

sin2x = cos3x [as we know, sin(90°- 3x) = Cos3x]

We know that,

sin2x = 2sinxcosx

$\text{Cos}3x = 4\cos^3 x - 3\cos x$

$2\sin x\cos x = 4\cos^3 x - 3\cos x$

$\Rightarrow 2\sin x\cos x - 4\cos^3 x + 3\cos x = 0$

$\Rightarrow \cos x\,(2\sin x - 4\cos^2 x + 3) = 0$

Now dividing both side by cosx we get,

$2\sin x - 4\cos^2 x + 3 = 0$

We know that,

$\cos^2 x + \sin^2 x = 1$

or $\cos^2 x = 1 – \sin^2 x$

$\Rightarrow 2\sin x – 4(1 – \sin^2 x) + 3 = 0$

$\Rightarrow 2\sin x - 4 + 4\sin^2 x + 3 = 0$

$\Rightarrow 2\sin x + 4\sin^2 x - 1 = 0$

We can write it as,

$4\sin^2 x + 2\sin x - 1 = 0$

Now applying formula

Here, $ax^2 + bx + c = 0$

So, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

now applying it in the equation

$$\sin x = \frac{-2 \pm \sqrt{2^2 - 4(4)(-1)}}{2}$$

$$\sin x = \frac{-2 \pm \sqrt{4 + 16}}{8}$$

$$\sin x = \frac{-2 \pm \sqrt{20}}{8}$$

$$\sin x = \frac{(-2 \pm 2.\sqrt{5})}{8}$$

$$\sin x = \frac{2(-1 \pm \sqrt{5})}{8}$$

$$\sin x = \frac{-1 \pm \sqrt{5}}{4}$$

$$\sin 18° = \frac{-1 \pm \sqrt{5}}{4}$$

Now sin 18° is positive, as 18° lies in first quadrant.

$\therefore \sin 18° = \frac{\sqrt{5}-1}{4}$

Now, we know that

$\cos^2x + \sin^2x = 1$

or $\cos x = \sqrt{1 - \sin^2x}$

$\therefore \cos 18° = \sqrt{1 - \sin^2 18°}$

$\Rightarrow \cos 18° = \sqrt{1 - \left(\frac{\sqrt{5}-1}{4}\right)^2}$

$\Rightarrow \cos 18° = \sqrt{\frac{16-(5+1-2\sqrt{5})}{16}}$

$\Rightarrow \cos 18° = \sqrt{\frac{16-6+2\sqrt{5}}{16}}$

$\Rightarrow \cos 18° = \frac{1}{4}\sqrt{10+2\sqrt{5}}$

Putting the value in eq. (i), we get

$= \cos^2 18° - \cos^2 30°$

$= \left(\frac{1}{4}\sqrt{10+2\sqrt{5}}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 \quad \left[\because \cos 30° = \frac{\sqrt{3}}{2}\right]$

$= \frac{1}{16}(10+2\sqrt{5}) - \frac{3}{4}$

$= \frac{10+2\sqrt{5}-12}{16}$

$= \frac{2\sqrt{5}-2}{16}$

$= \frac{2(\sqrt{5}-1)}{16}$

$= \frac{\sqrt{5}-1}{8}$

= RHS

∴ LHS = RHS

Hence Proved

Q. 20. Prove that tan 6⁰ tan 42⁰ tan 66⁰ tan 78⁰ = 1

Answer : To Prove: tan 6° tan 42° tan 66° tan 78° = 1

Taking LHS,

= tan 6° tan 42° tan 66° tan 78°

Multiply and divide by tan 54° tan 18°

$= \frac{\tan 6° \tan 42° \tan 66° \tan 78°}{\tan 54° \tan 18°} \times \tan 54° \tan 18°$

$= \frac{(\tan 6° \tan 54° \tan 66°)(\tan 18° \tan 42° \tan 72°)}{\tan 54° \tan 18°}$ ...(i)

We know that,

tan x tan(60° – x) tan (60° + x) = tan 3x

In first x = 6°
tan 6° tan (60° – 6°) tan (60° + 6°)
= tan 6° tan 54° tan 66°
and
In second x = 18°
tan 18° tan (60° – 18°) tan (60° + 18°)
= tan 18° tan 42° tan 78°

So, eq. (i) becomes

$= \frac{[\tan 3(6°)][\tan 3(18°)]}{\tan 54° \tan 18°}$

$= \frac{\tan 18° \tan 54°}{\tan 54° \tan 18°}$

= 1

= RHS

∴ LHS = RHS

Hence Proved

Q. 21. If tan $\theta = \frac{a}{b}$, prove that $a \sin 2\theta + b \cos 2\theta = b$

Answer : Given: $\theta = \frac{a}{b}$

To Prove: a sin 2θ + b cos 2θ = b

Given: $\theta = \frac{a}{b}$

We know that,

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{a}{b}$$

By Pythagoras Theorem,

$(\text{Perpendicular})^2 + (\text{Base})^2 = (\text{Hypotenuse})^2$

$\Rightarrow (a)^2 + (b)^2 = (H)^2$

$\Rightarrow a^2 + b^2 = (H)^2$

$$\Rightarrow H = \sqrt{a^2 + b^2}$$

So,

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{a}{\sqrt{a^2+b^2}}$$

$$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{b}{\sqrt{a^2+b^2}}$$

Taking LHS,

= a sin 2θ + b cos 2θ

We know that,

sin 2θ = 2 sin θ cos θ

and cos 2θ = 1 – 2 $\sin^2\theta$

= a(2 sin θ cos θ) + b(1 – 2 $\sin^2\theta$)

Putting the values of sinθ and cosθ, we get

$$= a \times 2 \times \frac{a}{\sqrt{a^2+b^2}} \times \frac{b}{\sqrt{a^2+b^2}} + b\left[1 - 2 \times \left(\frac{a}{\sqrt{a^2+b^2}}\right)^2\right]$$

$$= \frac{2a^2b}{a^2+b^2} + b\left[1 - 2 \times \frac{a^2}{a^2+b^2}\right]$$

$$= \frac{2a^2b}{a^2+b^2} + b - \frac{2a^2b}{a^2+b^2}$$

= b

= RHS

∴ LHS = RHS

Hence Proved

Exercise 15E

Q. 1.

If $\sin x = \frac{\sqrt{5}}{3}$ and $\frac{\pi}{2} < x < \pi$, **find the values of**

(i) $\sin\frac{x}{2}$ **(ii)** $\cos\frac{x}{2}$

(iii) $\tan\frac{x}{2}$

Answer : Given: sin x = $\frac{\sqrt{5}}{3}$ and $\frac{\pi}{2}$<x<π i.e, x lies in the Quadrant II .

To Find: i) sin $\frac{x}{2}$ ii) cos $\frac{x}{2}$ iii) tan $\frac{x}{2}$

Now, since sin x = $\frac{\sqrt{5}}{3}$

We know that cos x = $\pm\sqrt{1 - \sin^2 x}$

cos x = $\pm\sqrt{1 - (\frac{\sqrt{5}}{3})^2}$

cos x = $\pm\sqrt{1 - \frac{5}{9}}$

cos x = $\pm\sqrt{\frac{4}{9}} = \pm\frac{2}{3}$

since cos x is negative in II quadrant, hence cos x = $-\frac{2}{3}$

i) sin $\frac{x}{2}$

Formula used:

sin $\frac{x}{2}$ = $\pm\sqrt{\frac{1-\cos x}{2}}$

Now, sin $\frac{x}{2}$ = $\pm\sqrt{\frac{1-(\frac{-2}{3})}{2}}$ = $\pm\sqrt{\frac{\frac{5}{3}}{2}}$ = $\pm\sqrt{\frac{5}{6}}$

Since sinx is positive in II quadrant, hence sin $\frac{x}{2} = \sqrt{\frac{5}{6}}$

ii) cos $\frac{x}{2}$

Formula used:

cos $\frac{x}{2}$ = $\pm\sqrt{\frac{1+\cos x}{2}}$

now, cos $\frac{x}{2}$ = $\pm\sqrt{\frac{1+(\frac{-2}{3})}{2}}$ = = $\pm\sqrt{\frac{\frac{1}{3}}{2}}$ = = $\pm\sqrt{\frac{1}{6}}$

since cosx is negative in II quadrant, hence cos $\frac{x}{2}$ = $-\frac{1}{\sqrt{6}}$

iii) tan $\frac{x}{2}$

Formula used:

tan x = $\frac{\sin x}{\cos x}$

hence, tan $\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{\sqrt{\frac{5}{6}}}{-\frac{1}{\sqrt{6}}} = \frac{\sqrt{5}}{\sqrt{6}} \times \frac{\sqrt{6}}{-1}$ = -√5

Here, tan x is negative in II quadrant.

Q. 2.

If $\cos x = \frac{-3}{5}$ and $\frac{\pi}{2} < x < \pi$, **find the values of**

(i) $\sin\frac{x}{2}$ **(ii)** $\cos\frac{x}{2}$

(iii) $\tan\frac{x}{2}$

Answer :

Given: cos x = = $-\frac{3}{5}$ and $\frac{\pi}{2}$<x< π.i.e, x lies in II quadrant

To Find: i)sin $\frac{x}{2}$ ii)cos $\frac{x}{2}$ iii)tan $\frac{x}{2}$

i) sin $\frac{x}{2}$

Formula used:

$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$

Now, $\sin\frac{x}{2} = \pm\sqrt{\frac{1-(\frac{-3}{5})}{2}} = \pm\sqrt{\frac{\frac{8}{5}}{2}} = \pm\frac{2}{\sqrt{5}}$

Since sinx is positive in II quadrant, hence $\sin\frac{x}{2} = \frac{2}{\sqrt{5}}$

ii) $\cos\frac{x}{2}$

Formula used:

$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$

now, $\cos\frac{x}{2} = \pm\sqrt{\frac{1+(\frac{-3}{5})}{2}} = = \pm\sqrt{\frac{\frac{2}{5}}{2}} = = \pm\sqrt{\frac{1}{5}}$

since cosx is negative in II quadrant, hence $\cos\frac{x}{2} = -\frac{1}{\sqrt{5}}$

iii) $\tan\frac{x}{2}$

Formula used:

$\tan x = \frac{\sin x}{\cos x}$

hence, $\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{\frac{2}{\sqrt{5}}}{-\frac{1}{\sqrt{5}}} = \frac{2}{\sqrt{5}} \times \frac{\sqrt{5}}{-1} = -2$

Here, tanx is negative in II quadrant.

Q. 3. If Sin X = $\frac{-1}{2}$ and X lies in Quadrant IV, find the values of

(i) Sin $\frac{X}{2}$

(ii) Cos $\frac{X}{2}$

(iii) tan $\frac{X}{2}$

Answer :

Given: sin x = $\frac{-1}{2}$ and x lies in Quadrant IV.

To Find: i)sin $\frac{x}{2}$ ii)cos $\frac{x}{2}$ iii)tan $\frac{x}{2}$

Now, since sin x = $\frac{-1}{2}$

We know that cos x = $\pm\sqrt{1 - \sin^2 x}$

$\cos x = \pm\sqrt{1 - (\frac{-1}{2})^2}$

$\cos x = \pm\sqrt{1 - \frac{1}{4}}$

$\cos x = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2}$

since cos x is positive in IV quadrant, hence cos x = $\frac{\sqrt{3}}{2}$

i) sin $\frac{x}{2}$

Formula used:

$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$

Now, $\sin\frac{x}{2} = \pm\sqrt{\frac{1-(\frac{\sqrt{3}}{2})}{2}} = \pm\sqrt{\frac{\frac{2-\sqrt{3}}{2}}{2}} = \pm\sqrt{\frac{2-\sqrt{3}}{4}} = = \pm\frac{\sqrt{2-\sqrt{3}}}{2}$

Since sinx is negative in IV quadrant, hence $\sin\frac{x}{2} = -\frac{\sqrt{2-\sqrt{3}}}{2}$

ii)cos $\frac{x}{2}$

Formula used:

$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$

now, $\cos\frac{x}{2} = \pm\sqrt{\frac{1+(\frac{\sqrt{3}}{2})}{2}} = = \pm\sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}} = = \pm\frac{\sqrt{2+\sqrt{3}}}{2}$

since cosx is positive in IV quadrant, hence $\cos\frac{x}{2} = \frac{\sqrt{2+\sqrt{3}}}{2}$

iii)$\tan\frac{x}{2}$

Formula used:

$\tan x = \frac{\sin x}{\cos x}$

Q. 4. If Cos $\frac{X}{2} = \frac{12}{13}$ and X lies in Quadrant I, find the values of

(i) sin x
(ii) cos x
(iii) cot x

Answer : Given: $\cos\frac{X}{2} = \frac{12}{13}$ and x lies in Quadrant I i.e, All the trigonometric ratios are positive in I quadrant

To Find: (i) sin x ii) cos x iii) cot x

(i) sin x

Formula used:

We have, Sin x = $\sqrt{1-\cos^2 x}$

We know that, cos $\frac{x}{2}$ = $\sqrt{\frac{1+\cos x}{2}}$ (∵cos x is positive in I quadrant)

$\Rightarrow 2\cos^2\frac{x}{2} - 1 = \cos x$

$\Rightarrow 2\times\left(\frac{12}{13}\right)^2 - 1 = \cos x$

$\Rightarrow 2\times\left(\frac{144}{169}\right) - 1 = \cos x$

$\Rightarrow \cos x = \frac{119}{169}$

Since, Sin x = $\sqrt{1-\cos^2 x}$

$\Rightarrow$ Sin x = $\sqrt{1-(\frac{119}{169})^2}$

$\Rightarrow$ Sin x = $\frac{120}{169}$

Hence, we have Sin x = $\frac{120}{169}$.

ii)cos x

Formula used:

We know that, cos $\frac{x}{2} = \sqrt{\frac{1+\cos x}{2}}$ ($\because$cos x is positive in I quadrant)

$\Rightarrow 2\cos^2\frac{x}{2} - 1 = \cos x$

$\Rightarrow 2\times\left(\frac{12}{13}\right)^2 - 1 = \cos x$

$\Rightarrow 2\times\left(\frac{144}{169}\right) - 1 = \cos x$

$\Rightarrow \cos x = \frac{119}{169}$

iii) cot x

Formula used:

$\cot x = \frac{\cos x}{\sin x}$

$\cot x = \frac{\frac{119}{169}}{\frac{120}{169}} = \frac{119}{169} \times \frac{169}{120} = \frac{119}{120}$

Hence, we have $\cot x = \frac{119}{120}$

Q. 5. **If** $\sin x = \frac{3}{5}$ and $0 < x < \frac{\pi}{2}$, **find the value of** $\tan\frac{x}{2}$.

Answer : Given: $\sin x = \frac{3}{5}$ and $0 < x < \frac{\pi}{2}$ i.e, x lies in Quadrant I and all the trigonometric ratios are positive in quadrant I.

To Find: $\tan\frac{x}{2}$

Formula used:

$\tan\frac{x}{2} = \frac{\sin x}{1+\cos x}$

Now, $\cos x = \sqrt{1-\sin^2 x}$ ($\because$cos x is positive in I quadrant)

$\Rightarrow \cos x = \sqrt{1-(\frac{3}{5})^2} = \sqrt{1-\frac{9}{25}} = \frac{4}{5}$

Since, $\tan\frac{x}{2} = \frac{\sin x}{1+\cos x} = \frac{\frac{3}{5}}{1+\frac{4}{5}} = \frac{3}{5} \times \frac{5}{9} = \frac{1}{3}$

Hence, $\tan\frac{x}{2} = \frac{1}{3}$

Q. 6. Prove that

$\cot\frac{x}{2} - \tan\frac{x}{2} = 2\cot x$

Answer :

To Prove: $\cot\frac{x}{2} - \tan\frac{x}{2} = 2\cot x$

Proof: Consider L.H.S,

$\cot\frac{x}{2} - \tan\frac{x}{2} = \frac{\cos\frac{x}{2}}{\sin\frac{x}{2}} - \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}$

$= \frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{\sin\frac{x}{2}\cos\frac{x}{2}}$

$= \frac{\cos x}{\sin\frac{x}{2}\cos\frac{x}{2}}$ $(\because \cos^2 x - \sin^2 x = \cos 2x)$

$\Rightarrow (\cos^2\frac{x}{2} - \sin^2\frac{x}{2} = \cos x)$

Here multiply and divide L.H.S by 2

$= \frac{2\cos x}{2\sin\frac{x}{2}\cos\frac{x}{2}}$

$= \frac{2\cos x}{\sin x}$ $(\because 2\sin x\cos x = \sin 2x)$

$\Rightarrow (2\sin\frac{x}{2}\cos\frac{x}{2} = \sin x)$

$\cot \; - \tan\frac{x}{2} = 2\cot x$ = R.H.S

$\therefore$L.H.S = R.H.S, Hence proved

Q. 7. Prove that

$$\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)=\tan x+\sec x$$

Answer : To Prove: $\tan(\frac{\pi}{4}+\frac{\pi}{2})$ = tan x + sec x

Proof: Consider L.H.S,

$$\tan(\frac{\pi}{4}+\frac{x}{2})=\frac{\tan\frac{\pi}{4}+\tan\frac{x}{2}}{1-\tan\frac{\pi}{4}\tan\frac{x}{2}}\ (\because \text{this is of the form } \tan(x+y)=\frac{\tan x+\tan y}{1-\tan x\tan y})$$

$$=\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}=\frac{1+\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}{1-\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}$$

$$=\frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}$$

Multiply and divide L.H.S by $\cos\frac{x}{2}+\sin\frac{x}{2}$

$$=\frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}\times\frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}$$

$$=\frac{(\cos\frac{x}{2}+\sin\frac{x}{2})^2}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}$$

$$=\frac{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\cos\frac{x}{2}\sin\frac{x}{2}}{\cos x}\ (\because \cos^2\frac{x}{2}-\sin^2\frac{x}{2}=\cos x)$$

$$=\frac{1+2\cos\frac{x}{2}\sin\frac{x}{2}}{\cos x}$$

$= \frac{1+\sin x}{\cos x}$ ($\because 2\cos\frac{x}{2}\sin\frac{x}{2} = \sin x$)

$= \frac{1}{\cos x} + \frac{\sin x}{\cos x}$

$\tan(\frac{\pi}{4} + \frac{x}{2})$ = sec x + tan x = R.H.S

$\because$ L.H.S = R.H.S, Hence proved

Q. 8. Prove that

$$\sqrt{\frac{1+\sin x}{1-\sin x}} = \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$$

Answer :

To Prove: $\sqrt{\frac{1+\sin x}{1-\sin x}} = \tan(\frac{\pi}{4} + \frac{x}{2})$

Proof: Consider, L.H.S = $\sqrt{\frac{1+\sin x}{1-\sin x}}$

Multiply and divide L.H.S by $\sqrt{1+\sin x}$

$= \sqrt{\frac{1+\sin x}{1-\sin x}} \times \frac{\sqrt{1+\sin x}}{\sqrt{1+\sin x}} = \frac{1+\sin x}{\sqrt{1-\sin^2 x}}$

$= \frac{1+\sin x}{\cos x} = \frac{1+2\cos\frac{x}{2}\sin\frac{x}{2}}{\cos x}$ ($\because 2\cos\frac{x}{2}\sin\frac{x}{2} = \sin x$)

$= \frac{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\cos\frac{x}{2}\sin\frac{x}{2}}{\cos x}$ ($\because \cos^2 x + \sin^2 x = 1$)

$$= \frac{(\cos\frac{x}{2}+\sin\frac{x}{2})^2}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}$$

$$= \frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}} \times \frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}} \; (\because x^2+y^2 = (x+y)(x-y))$$

$$= \frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}$$

Multiply and divide the above with cos $\frac{x}{2}$

$$= \frac{1+\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}{1-\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}$$

Here, since $\tan\frac{\pi}{4} = 1$

Here, since $\tan\frac{\pi}{4} = 1$

$$\sqrt{\frac{1+\sin x}{1-\sin x}} = \frac{\tan\frac{\pi}{4}+\tan\frac{x}{2}}{1-\tan\frac{\pi}{4}\tan\frac{x}{2}} = \tan(\frac{\pi}{4}+\frac{x}{2}) = \text{R.H.S}$$

Since, L.H.S = R.H.S, Hence proved.

Q. 9. Prove that

$$\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)+\tan\left(\frac{\pi}{4}-\frac{x}{2}\right) = 2\sec x$$

Answer :

To prove: $\tan(\frac{\pi}{4}+\frac{x}{2})+\tan(\frac{\pi}{4}-\frac{x}{2}) = 2\sec x$

Proof: Consider, L.H.S $= \tan(\frac{\pi}{4}+\frac{x}{2})+\tan(\frac{\pi}{4}-\frac{x}{2})$

$$\tan(\frac{\pi}{4}+\frac{x}{2})+\tan(\frac{\pi}{4}-\frac{x}{2}) = \frac{\tan\frac{\pi}{4}+\tan\frac{x}{2}}{1-\tan\frac{\pi}{4}\tan\frac{x}{2}}+\frac{\tan\frac{\pi}{4}-\tan\frac{x}{2}}{1+\tan\frac{\pi}{4}\tan\frac{x}{2}}$$

$$(\because \tan(x+y) = \frac{\tan x+\tan y}{1-\tan x\tan y} \text{ and } \tan(x-y) = \frac{\tan x-\tan y}{1+\tan x\tan y})$$

$$= \frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}+\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}$$

$$= \frac{1+\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}{1-\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}+\frac{1-\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}{1+\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}$$

$$= \frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}} + \frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}$$

$$= \frac{(\cos\frac{x}{2}+\sin\frac{x}{2})^2+(\cos\frac{x}{2}-\sin\frac{x}{2})^2}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}$$

By Expanding the numerator we get,

$$= \frac{2}{\cos x} \left(\because \cos^2\frac{x}{2} - \sin^2\frac{x}{2} = \cos x\right)$$

$\tan(\frac{\pi}{4} + \frac{x}{2}) + \tan(\frac{\pi}{4} - \frac{x}{2}) = 2\sec x = \text{R.H.S}$

since L.H.S = R.H.S, Hence proved.

Q. 10. Prove that

$$\frac{\sin x}{1+\cos x} = \tan\frac{x}{2}$$

Answer :

To Prove: $\frac{\sin x}{1+\cos x} = \tan\frac{x}{2}$

Proof: consider, L.H.S = $\frac{\sin x}{1+\cos x}$

$$\frac{\sin x}{1+\cos x} = \frac{2\cos\frac{x}{2}\sin\frac{x}{2}}{1+\cos^2\frac{x}{2}-\sin^2\frac{x}{2}} \left(\because \cos^2\frac{x}{2} - \sin^2\frac{x}{2} = \cos x \text{ and } 2\cos\frac{x}{2}\sin\frac{x}{2} = \sin x\right)$$

$$= \frac{2\cos\frac{x}{2}\sin\frac{x}{2}}{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+\cos^2\frac{x}{2}-\sin^2\frac{x}{2}} \left(\because \cos^2\frac{x}{2} - \sin^2\frac{x}{2} = 1\right)$$

$$= \frac{2\cos\frac{x}{2}\sin\frac{x}{2}}{2\cos^2\frac{x}{2}} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \tan\frac{x}{2}$$

$$\frac{\sin x}{1+\cos x} = \tan\frac{x}{2} = \text{R.H.S}$$

Since L.H.S = R.H.S, Hence proved.