

Geometrical Progression

Exercise 12A

Q. 1. Find the 6th and nth terms of the GP 2, 6, 18, 54....

Answer : Given: GP is 2, 6, 18, 54....

The given GP is of the form, a, ar, ar^2, ar^3, \dots

Where r is the common ratio.

First term in the given GP, $a_1 = a = 2$

Second term in GP, $a_2 = 6$

Now, the common ratio, $r = \frac{a_2}{a_1}$

$$r = \frac{6}{2} = 3$$

Now, n^{th} term of GP is, $a_n = ar^{n-1}$

So, the 6th term in the GP,

$$a_6 = ar^5$$

$$= 2 \times 3^5$$

$$= 486$$

n^{th} term in the GP,

$$a_n = ar^{n-1}$$

$$= 2 \cdot 3^{n-1}$$

Hence, 6th term = 486 and n^{th} term = $2 \cdot 3^{n-1}$

Q. 2. Find the 17th and nth terms of the GP 2, $2\sqrt{2}$, 4, $8\sqrt{2}$

Answer : Given GP is 2, $2\sqrt{2}$, 4, $8\sqrt{2}$

The given GP is of the form, a, ar, ar^2, ar^3, \dots

Where r is the common ratio.

First term in the given GP, $a_1 = a = 2$

Second term in GP, $a_2 = 2\sqrt{2}$

Now, the common ratio, $r = \frac{a_2}{a_1}$

$$r = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

Now, n^{th} term of GP is, $a_n = ar^{n-1}$

So, the 17^{th} term in the GP,

$$a_{17} = ar^{16}$$

$$= 2 \times (\sqrt{2})^{16}$$

$$= 512$$

n^{th} term in the GP,

$$a_n = ar^{n-1}$$

$$= 2(\sqrt{2})^{n-1}$$

$$= (\sqrt{2})^{n+1}$$

Hence, 17^{th} term = 512 and n^{th} term = $(\sqrt{2})^{n+1}$

Q. 3. Find the 7^{th} and n^{th} terms of the GP 0.4, 0.8, 1.6....

Answer : Given GP is 0.4, 0.8, 1.6....

The given GP is of the form, a, ar, ar^2, ar^3, \dots

Where r is the common ratio.

First term in the given GP, $a_1 = a = 0.4$

Second term in GP, $a_2 = 0.8$



Now, the common ratio, $r = \frac{a_2}{a_1}$

$$r = \frac{0.8}{0.4} = 2$$

Now, n^{th} term of GP is, $a_n = ar^{n-1}$

So, the 7th term in the GP,

$$a_7 = ar^6$$

$$= 0.4 \times 2^6$$

$$= 25.6$$

n^{th} term in the GP,

$$a_n = ar^{n-1}$$

$$= (0.4)(2)^{n-1}$$

$$= (0.2)2^n$$

Hence, 7th term = 25.6 and n^{th} term = $(0.2)2^n$



Q. 4. Find the 10th and n^{th} terms of the GP $-\frac{3}{4}, \frac{1}{2}, -\frac{1}{3}, \frac{2}{9}, \dots$

Answer : Given GP is $-\frac{3}{4}, \frac{1}{2}, -\frac{1}{3}, \frac{2}{9}, \dots$.

The given GP is of the form, a, ar, ar^2, ar^3, \dots

Where r is the common ratio.

The first term in the given GP, $a = a_1 = -\frac{3}{4}$

The second term in GP, $a_2 = \frac{1}{2}$

Now, the common ratio, $r = \frac{a_2}{a_1}$

$$r = -\frac{\frac{1}{2}}{\frac{3}{4}} = -\frac{2}{3}$$

Now, n^{th} term of GP is, $a_n = ar^{n-1}$

So, the 10^{th} term, $a_{10} = ar^9$

$$a_{10} = ar^9 = \left(-\frac{3}{4}\right)\left(-\frac{2}{3}\right)^9 = \frac{128}{6561}$$

Now, the required n^{th} term, $a_n = ar^{n-1}$

$$a_n = \left(-\frac{3}{4}\right)\left(-\frac{2}{3}\right)^{n-1} = \left(\frac{9}{8}\right)\left(-\frac{2}{3}\right)^n$$

Hence, the 10^{th} term, $a_{10} = \frac{128}{6561}$ and n^{th} term,

$$a_n = \left(\frac{9}{8}\right)\left(-\frac{2}{3}\right)^n.$$

Q. 5. Which term of the GP 3, 6, 12, 24.... Is 3072?

Answer : Given GP is 3, 6, 12, 24....

The given GP is of the form, a, ar, ar^2, ar^3, \dots

Where r is the common ratio.

First term in the given GP, $a_1 = a = 3$

Second term in GP, $a_2 = 6$

Now, the common ratio, $r = \frac{a_2}{a_1}$

$$r = \frac{6}{3} = 2$$

Let us consider 3072 as the n^{th} term of the GP.

Now, n^{th} term of GP is, $a_n = ar^{n-1}$

$$3072 = 3 \cdot 2^{n-1}$$

$$\frac{3072 \times 2}{3} = 2^n$$

$$2^n = 2^{11}$$

$$n = 11$$

So, 3072 is the 11th term in GP.

Q. 6. Which term of the GP $\frac{1}{4}, \frac{-1}{2}, 1, \dots$ is -128?

Answer : Given GP is $\frac{1}{4}, \frac{-1}{2}, 1, \dots$

The given GP is of the form, a, ar, ar^2, ar^3, \dots

Where r is the common ratio.



The first term in the given GP, $a = a_1 = \frac{1}{4}$

The second term in GP, $a_2 = -\frac{1}{2}$

Now, the common ratio, $r = \frac{a_2}{a_1}$

$$r = -\frac{4}{2} = -2$$

Let us consider -128 as the n^{th} term of the GP.

Now, n^{th} term of GP is, $a_n = ar^{n-1}$

$$-128 = \left(\frac{1}{4}\right)(-2)^{n-1}$$

$$(-2)^n = 1024 = (-2)^{10}$$

$$n = 10$$

So, -128 is the 10th term in GP.

Q. 7. Which term of the GP $\sqrt{3}, 3, 3\sqrt{3}, \dots$ is 729?

Answer : Given GP is $\sqrt{3}, 3, 3\sqrt{3}, \dots$

The given GP is of the form, a, ar, ar^2, ar^3, \dots

Where r is the common ratio.

First term in the given GP, $a_1 = a = \sqrt{3}$

Second term in GP, $a_2 = 3$

Now, the common ratio, $r = \frac{a_2}{a_1}$



$$r = \frac{3}{\sqrt{3}} = \sqrt{3}$$

Let us consider 729 as the n^{th} term of the GP.

Now, n^{th} term of GP is, $a_n = ar^{n-1}$

$$729 = \sqrt{3} (\sqrt{3})^{n-1}$$

$$\sqrt{3}^n = \sqrt{3}^{12}$$

$$n = 12$$

So, 729 is the 12th term in GP.

Q. 8. Find the geometric series whose 5th and 8th terms are 80 and 640 respectively.

Answer : The n^{th} term of a GP is $a_n = ar^{n-1}$

It's given in the question that 5th term of the GP is 80 and 8th term of GP is 640.

$$\text{So, } a_5 = ar^4 = 80 \rightarrow (1)$$

$$a_8 = ar^7 = 640 \rightarrow (2)$$

$$\frac{(2)}{(1)} \rightarrow \frac{ar^7}{ar^4} = r^3 = \frac{640}{80} = 8$$

Common ratio, $r = 2$,

$$ar^4 = 80$$

$$16a = 80$$

$$a = 5$$

The required GP is of the form $a, ar, ar^2, ar^3, ar^4, \dots$

First term of GP, $a = 5$

Second term of GP, $ar = 5 \times 2 = 10$

Third term of GP, $ar^2 = 5 \times 2^2 = 20$

Fourth term of GP, $ar^3 = 5 \times 2^3 = 40$

Fifth term of GP, $ar^4 = 5 \times 2^4 = 80$

And so on...

The required GP is 5, 10, 20, 40, 80...

Q. 9. Find the GP whose 4th and 7th terms are $\frac{1}{18}$ and $-\frac{1}{486}$ respectively.

Answer : The nth term of a GP is $a_n = ar^{n-1}$

It's given in the question that 4th term of the GP is $\frac{1}{18}$ and 7th term of GP is $-\frac{1}{486}$.

$$\text{So, } a_4 = ar^3 = \frac{1}{18} \rightarrow (1)$$

$$a_7 = ar^6 = -\frac{1}{486} \rightarrow (2)$$

$$\frac{(2)}{(1)} \rightarrow \frac{ar^6}{ar^3} = r^3 = -\frac{1}{27}$$

$$\text{Common ratio, } r = -\frac{1}{3}$$

$$ar^3 = \frac{1}{18}$$

$$a = -\frac{3}{2}$$

The required GP is of form $a, ar, ar^2, ar^3, ar^4, \dots$

The first term of GP, $a = -\frac{3}{2}$



The second term of GP, $ar = -\frac{3}{2}x - \frac{1}{3} = \frac{1}{2}$

The third term of GP, $ar^2 = \frac{1}{2}x - \frac{1}{3} = -\frac{1}{6}$

The fourth term of GP, $ar^3 = -\frac{1}{6}x - \frac{1}{3} = \frac{1}{18}$

The fifth term of GP, $ar^4 = \frac{1}{18}x - \frac{1}{3} = -\frac{1}{54}$

And so on...

The required GP is $-\frac{3}{2}, \frac{1}{2}, -\frac{1}{6}, \frac{1}{18}, -\frac{1}{54}, \dots$

Q. 10. The 5th, 8th and 11th terms of a GP are a, b, c respectively. Show that $b^2 = ac$

Answer : It is given in the question that 5th, 8th and 11th terms of GP are a, b and c respectively.

Let us assume the GP is A, AR, AR², and AR³....

So, the nth term of this GP is $a_n = AR^{n-1}$

$$\text{Now, 5}^{\text{th}} \text{ term, } a_5 = AR^4 = a \rightarrow (1)$$

$$\text{8}^{\text{th}} \text{ term, } a_8 = AR^7 = b \rightarrow (2)$$

$$\text{11}^{\text{th}} \text{ term, } a_{11} = AR^{10} = c \rightarrow (3)$$

Dividing equation (3) by (2) and (2) by (1),

$$\frac{(3)}{(2)} \rightarrow \frac{AR^{10}}{AR^7} = R^3 = \frac{c}{b} \rightarrow (4)$$

$$\frac{(2)}{(1)} \rightarrow \frac{AR^7}{AR^4} = R^3 = \frac{b}{a} \rightarrow (5)$$

So, both equation (4) and (5) gives the value of R^3 . So we can equate them.

$$\frac{c}{b} = \frac{b}{a} = R^3,$$

$$\therefore b^2 = ac,$$

Hence proved.

Q. 11. The first term of a GP is -3 and the square of the second term is equal to its 4th term. Find its 7th term.

Answer : It is given that the first term of GP is -3.

$$\text{So, } a = -3$$

It is also given that the square of the second term is equal to its 4th term.

$$\therefore (a_2)^2 = a_4$$

$$n^{\text{th}} \text{ term of GP, } a_n = ar^{n-1}$$

$$\text{So, } a_2 = ar; a_4 = ar^3$$

$$(ar)^2 = ar^3 \rightarrow a = r = -3$$

$$\text{Now, the 7th term in the GP, } a_7 = ar^6$$

$$a_7 = (-3)^7 = -2187$$

Hence, the 7th term of GP is -2187.

Q. 12. Find the 6th term from the end of GP 8, 4, 2... $\frac{1}{1024}$.

Answer : The given GP is 8, 4, 2... $\frac{1}{1024}$ \rightarrow (1)

First term in the GP, $a_1 = a = 8$

Second term in the GP, $a_2 = ar = 4$

The common ratio, $r = \frac{4}{8} = \frac{1}{2}$

The last term in the given GP is $\frac{1}{1024}$.

Second last term in the GP = $a_{n-1} = ar^{n-2}$

Starting from the end, the series forms another GP in the form,

$ar^{n-1}, ar^{n-2}, ar^{n-3}, \dots, ar^3, ar^2, ar, a \rightarrow (2)$

Common ratio of this GP is $\frac{1}{r}$.

So, common ratio = 2

$$a = \frac{1}{1024}$$

So, 6th term of the GP (2),

$$a_6 = ar^5$$

$$= \frac{1}{1024} \times 2^5 = \frac{1}{32}$$

Hence, the 6th term from the end of the given GP is $\frac{1}{32}$.

Q. 13. Find the 4th term from the end of the GP $\frac{2}{27}, \frac{2}{9}, \frac{2}{3}, \dots, 162$.

Answer : The given GP is $\frac{2}{27}, \frac{2}{9}, \frac{2}{3}, \dots, 162 \rightarrow (1)$

The first term in the GP, $a_1 = a = \frac{2}{27}$

The second term in the GP, $a_2 = \frac{2}{9}$

The common ratio, $r = 3$

The last term in the given GP is $a_n = 162$.

Second last term in the GP = $a_{n-1} = ar^{n-2}$

Starting from the end, the series forms another GP in the form,

$ar^{n-1}, ar^{n-2}, ar^{n-3} \dots ar^3, ar^2, ar, a \rightarrow (2)$

Common ratio of this GP is $r' = \frac{1}{r}$.

So, $r' = \frac{1}{3}$

So, 4th term of the GP (2),

$a_4 = ar^3$

$$= 162 \times \frac{1}{3^3} = 6$$

Hence, the 4th term from the end of the given GP is 6.

Q. 14. If a, b, c are the $p^{\text{th}}, q^{\text{th}}$ and r^{th} terms of a GP, show that

$(q - r) \log a + (r - p) \log b + (p - q) \log c = 0$.

Answer : As per the question, a, b and c are the $p^{\text{th}}, q^{\text{th}}$ and r^{th} term of GP.

Let us assume the required GP as $A, AR, AR^2, AR^3 \dots$

Now, the n^{th} term in the GP, $a_n = AR^{n-1}$

p^{th} term, $a_p = AR^{p-1} = a \rightarrow (1)$

q^{th} term, $a_q = AR^{q-1} = b \rightarrow (2)$

r^{th} term, $a_r = AR^{r-1} = c \rightarrow (3)$

$$\frac{(1)}{(2)} \rightarrow \frac{R^{p-1}}{R^{q-1}} = R^{p-q} = \frac{a}{b} \rightarrow (i)$$

$$\frac{(2)}{(3)} \rightarrow \frac{R^{q-1}}{R^{r-1}} = R^{q-r} = \frac{b}{c} \rightarrow (ii)$$

$$\frac{(3)}{(1)} \rightarrow \frac{R^{r-1}}{R^{p-1}} = R^{r-p} = \frac{c}{a} \rightarrow (iii)$$

Taking logarithm on both sides of equation (i), (ii) and (iii).

$$(p - q) \log R = \log a - \log b,$$

$$\therefore (p - q) = \frac{\log a - \log b}{\log R} \rightarrow (4)$$

$$(q - r) \log R = \log b - \log c$$

$$\therefore (q - r) = \frac{\log b - \log c}{\log R} \rightarrow (5)$$

$$(r - p) \log R = \log c - \log a$$

$$\therefore (r - p) = \frac{\log c - \log a}{\log R} \rightarrow (6)$$

Now, multiply equation (4) with log c,



$$(p - q) \log c = \left(\frac{\log a - \log b}{\log R} \right) \log c \rightarrow (7)$$

Now, multiply equation (5) with log a,

$$(q - r) \log a = \left(\frac{\log b - \log c}{\log R} \right) \log a \rightarrow (8)$$

Now, multiply equation (6) with log b,

$$(r - p) \log b = \left(\frac{\log c - \log a}{\log R} \right) \log b \rightarrow (9)$$

Now, add equations (7), (8) and (9).

$$(p - q) \log c + (q - r) \log a + (r - p) \log b = \left(\frac{\log a - \log b}{\log R} \right) \log c + \left(\frac{\log b - \log c}{\log R} \right) \log a + \left(\frac{\log c - \log a}{\log R} \right) \log b$$

On solving the above equation, we will get,

$$(p - q) \log c + (q - r) \log a + (r - p) \log b = 0$$

Hence proved.

Q. 15. The third term of a GP is 4; Find the product of its five terms.

Answer : Given that the third term of the GP, $a_3 = 4$

Let us assume the GP mentioned in the question be,

$$\frac{A}{R^2}, \frac{A}{R}, A, AR, AR^2, \dots$$

With the first term $\frac{A}{R^2}$ and common ratio R.

Now, the third term in the assumed GP is A.

So, $A = 4$ (given data)

Now,

$$\text{Product of the first five terms of GP} = \frac{A}{R^2} \times \frac{A}{R} \times A \times AR \times AR^2 = A^5$$

$$\text{So, the required product} = A^5 = 4^5 = 1024$$

\therefore The product of first five terms of a GP with its third term 4 is 1024.

Q. 16. In a finite GP, prove that the product of the terms equidistant from the beginning and end is the product of first and last terms.

Answer : We need to prove that the product of the terms equidistant from the beginning and end is the product of first and last terms in a finite GP.

Let us first consider a finite GP.

$A, AR, AR^2, \dots, AR^{n-1}, AR^n$.

Where n is finite.

Product of first and last terms in the given GP = $A \cdot AR^n$

$$= A^2 R^n \rightarrow (a)$$

Now, n^{th} term of the GP from the beginning = $AR^{n-1} \rightarrow (1)$

Now, starting from the end,

First term = AR^n

Last term = A

$$\frac{1}{R} = \text{Common Ratio}$$

So, an n^{th} term from the end of GP, $A_n = (AR^n) \left(\frac{1}{R^{n-1}} \right) = AR \rightarrow (2)$

So, the product of n^{th} terms from the beginning and end of the considered GP from (1) and (2) = $(AR^{n-1})(AR)$

$$= A^2R^n \rightarrow (b)$$

So, from (a) and (b) its proved that the product of the terms equidistant from the beginning and end is the product of first and last terms in a finite GP.

Q. 17. If $\frac{a+bx}{a-bx} = \frac{b+cx}{b-cx} = \frac{c+dx}{c-dx}$ ($x \neq 0$) **then show that a, b, c, d are in GP.**

Answer : $\frac{a+bx}{a-bx} = \frac{b+cx}{b-cx}$ (Given data in the question) $\rightarrow (1)$

Cross multiplying (1) and expanding,

$$(a+bx)(b-cx) = (b+cx)(a-bx)$$

$$ab - acx + b^2x - bcx^2 = ba - b^2x + acx - bcx^2$$

$$2b^2x = 2acx$$

$$b^2 = ac \rightarrow (i)$$

If three terms are in GP, then the middle term is the Geometric Mean of first term and last term.

$$\rightarrow b^2 = ac$$

So, from (i) b, is the geometric mean of a and b.

So, a, b, c are in GP.

$$\frac{b+cx}{b-cx} = \frac{c+dx}{c-dx}$$
 (Given data in the question) $\rightarrow (2)$

Cross multiplying (2) and expanding,

$$(b+cx)(c-dx) = (c+dx)(b-cx)$$

$$bc - bdx + c^2x - cdx^2 = cb - c^2x + bdx - dcx^2$$

$$2c^2x = 2bdx$$

$$c^2 = bd \rightarrow (ii)$$



So, from (ii), c is the geometric mean of b and d .

So, b, c, d is in GP.

$\therefore a, b, c, d$ are in GP.

Q. 18. If a and b are the roots of $x^2 - 3x + p = 0$ and c and d are the roots of $x^2 - 12x + q = 0$, where a, b, c, d form a GP, prove that $(q + p) : (q - p) = 17 : 15$.

Answer : Given data is,

$$x^2 - 3x + p = 0 \rightarrow (1)$$

a and b are roots of (1)

$$\text{So, } (x + a)(x + b) = 0$$

$$x^2 - (a + b)x + ab = 0$$

$$\text{So, } a + b = 3 \text{ and } ab = p \rightarrow (2)$$

Given data is,

$$x^2 - 12x + q = 0 \rightarrow (3)$$

c and d are roots of (1)

$$\text{So, } (x + c)(x + d) = 0$$

$$x^2 - (c + d)x + cd = 0$$

$$\text{So, } c + d = 12 \text{ and } cd = q \rightarrow (4)$$

a, b, c, d are in GP. (Given data)

Similarly A, AR, AR^2, AR^3 also forms a GP, with common ratio R .

From (2),

$$a + b = 3$$

$$A + AR = 3$$

$$\frac{3}{A} = 1 + R \rightarrow (5)$$



From (4),

$$c + d = 12$$

$$AR^2 + AR^3 = 12$$

$$AR^2(1 + R) = 12 \rightarrow (6)$$

Substituting value of $(1 + R)$ in (6).

$$R = 2$$

Now, substitute value of R in (5) to get value of A ,

$$A = 1$$

Now, the GP required is A, AR, AR^2 , and AR^3

1, 2, 4, 8...is the required GP.

So,

$$a = 1, b = 2, c = 4, d = 8$$

From (2) and (4),

$$ab = p \text{ and } cd = q$$

So, $p = 2$, and $q = 32$.

$$\frac{q + p}{q - p} = \frac{cd + ab}{cd - ab} = \frac{34}{30} = \frac{17}{15}$$

So, $(q + p) : (q - p) = 17 : 15$.

Exercise 12C

Q. 1. A. Find the sum of the GP :

$$1 + 3 + 9 + 27 + \dots \text{ To 7 terms}$$



Answer : Sum of a G.P. series is represented by the formula, $S_n = a \frac{r^n - 1}{r - 1}$, when $r > 1$. 'S_n' represents the sum of the G.P. series upto nth terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

$$a = 1$$

$$r = (\text{ratio between the } n \text{ term and } n-1 \text{ term}) 3 \div 1 = 3$$

$$n = 7 \text{ terms}$$

$$\therefore S_n = 1 \frac{3^7 - 1}{3 - 1}$$

$$\Rightarrow S_n = \frac{2187 - 1}{3 - 1}$$

$$\Rightarrow S_n = \frac{2186}{2}$$

$$\Rightarrow S_n = 1093$$



Q. 1. B. Find the sum of the GP :

$$1 + \sqrt{3} + 3 + 3\sqrt{3} + \dots \text{ to 10 terms}$$

Answer : Sum of a G.P. series is represented by the formula, $S_n = a \frac{r^n - 1}{r - 1}$, when $r > 1$. 'S_n' represents the sum of the G.P. series upto nth terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

$$a = 1$$

$$r = (\text{ratio between the } n \text{ term and } n-1 \text{ term}) \sqrt{3} \div 1 = \sqrt{3} = 1.732$$

$$n = 10 \text{ terms}$$

$$\therefore S_n = 1 \cdot \frac{\sqrt{3}^{10} - 1}{\sqrt{3} - 1}$$

$$\Rightarrow S_n = \frac{1.732^{10} - 1}{1.732 - 1}$$

$$\Rightarrow S_n = \frac{242.929 - 1}{0.732}$$

$$\Rightarrow S_n = \frac{241.929}{0.732}$$

$$\Rightarrow S_n = 330.504$$

Q. 1. C. Find the sum of the GP :

0.15 + 0.015 + 0.0015 + To 6 terms

Answer : Sum of a G.P. series is represented by the formula, $S_n = a \frac{1-r^n}{1-r}$, when $|r| < 1$.
' S_n ' represents the sum of the G.P. series upto n^{th} terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

$$a = 0.15$$

$$r = (\text{ratio between the } n \text{ term and } n-1 \text{ term}) 0.015 \div 0.15 = 0.1$$

$$n = 6 \text{ terms}$$

$$\Rightarrow S_n = 0.15 \times \frac{1-0.1^6}{1-0.1}$$

$$\Rightarrow S_n = 0.15 \times \frac{1-0.000001}{0.9}$$

$$\Rightarrow S_n = 0.15 \times \frac{0.999999}{0.9}$$

$$\therefore S_n = 16.67$$

Q. 1. D. Find the sum of the GP :

$$1 - \frac{1}{2} + \frac{1}{4} = \frac{1}{8} + \dots \text{ to 9 terms}$$

Answer : Sum of a G.P. series is represented by the formula, $S_n = a \frac{1-r^n}{1-r}$, when $|r| < 1$. 'S_n' represents the sum of the G.P. series upto nth terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

$$a = 1$$

$$r = (\text{ratio between the } n \text{ term and } n-1 \text{ term}) = -\frac{1}{2} \div 1 = -\frac{1}{2}$$

$$n = 9 \text{ terms}$$

$$\therefore S_n = 1 \times \frac{1 - \left(\frac{-1}{2}\right)^9}{1 - \left(\frac{-1}{2}\right)}$$

$$\Rightarrow S_n = \frac{1 + \frac{1}{512}}{1 + \frac{1}{2}}$$

$$\Rightarrow S_n = \frac{\frac{513}{512}}{\frac{3}{2}}$$

$$\therefore S_n = 171$$

Q. 1. E. Find the sum of the GP :

$$\sqrt{2} + \frac{1}{\sqrt{2}} + \frac{1}{2\sqrt{2}} + \dots \text{to 8 terms}$$

Answer : Sum of a G.P. series is represented by the formula, $S_n = a \frac{1-r^n}{1-r}$, when $|r| < 1$. 'S_n' represents the sum of the G.P. series upto nth terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

$$a = \sqrt{2}$$

$$r = (\text{ratio between the } n \text{ term and } n-1 \text{ term}) \frac{1}{\sqrt{2}} \div \sqrt{2} = \frac{1}{2}$$

n = 8 terms

$$\therefore S_n = \sqrt{2} \times \frac{1 - \frac{1}{2^8}}{1 - \frac{1}{2}}$$

$$\Rightarrow S_n = \sqrt{2} \times \frac{1 - \frac{1}{256}}{\frac{1}{2}}$$

$$\Rightarrow S_n = \sqrt{2} \times \frac{255}{\frac{1}{2}}$$

$$\Rightarrow S_n = \sqrt{2} \times \frac{255}{128}$$

$$\therefore S_n = \frac{255\sqrt{2}}{128}$$

Q. 1. F. Find the sum of the GP :

$$\frac{2}{9} - \frac{1}{3} - \frac{1}{2} - \frac{3}{4} + \dots \text{ To 6 terms}$$

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Answer : Sum of a G.P. series is represented by the formula, $S_n = a \frac{r^n - 1}{r - 1}$, when $r > 1$. 'S_n' represents the sum of the G.P. series upto nth terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

$$a = \frac{2}{9}$$

$$r = (\text{ratio between the } n \text{ term and } n-1 \text{ term}) = -\frac{1}{3} \div \frac{2}{9} = -\frac{3}{2} = 1.5$$

n = 6 terms

$$\therefore S_n = \frac{2}{9} \times \frac{1.5^6 - 1}{1.5 - 1}$$

$$\Rightarrow S_n = \frac{2}{9} \times \frac{10.39}{0.5}$$

$$\therefore S_n = 4.62$$

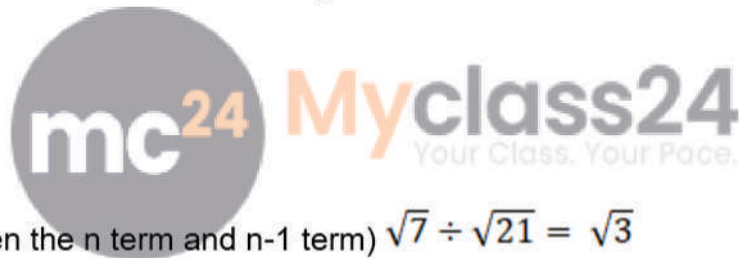
Q. 2. A. Find the sum of the GP :

$$\sqrt{7} + \sqrt{21} + 3\sqrt{7} + \dots \text{ to } n \text{ terms}$$

Answer : Sum of a G.P. series is represented by the formula, $S_n = a \frac{r^n - 1}{r - 1}$, when $r > 1$. 'S_n' represents the sum of the G.P. series upto nth terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

$$a = \sqrt{7}$$



$$r = (\text{ratio between the } n \text{ term and } n-1 \text{ term}) \sqrt{7} \div \sqrt{21} = \sqrt{3}$$

n terms

$$\therefore S_n = \sqrt{7} \times \frac{\sqrt{3}^n - 1}{\sqrt{3} - 1} \text{ [Rationalizing the denominator]}$$

$$\Rightarrow S_n = \sqrt{7} \times \frac{\sqrt{3}^n - 1}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

$$\Rightarrow S_n = \sqrt{7} \times \frac{(\sqrt{3}^n - 1)(\sqrt{3} + 1)}{3 - 1}$$

$$\therefore S_n = \frac{\sqrt{7}(\sqrt{3}^n - 1)(\sqrt{3} + 1)}{2}$$

Q. 2. B. Find the sum of the GP :

$$1 - \frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^3} + \dots \text{ to } n \text{ terms}$$

Answer : Sum of a G.P. series is represented by the formula, $S_n = a \frac{1-r^n}{1-r}$, when $|r| < 1$. 'S_n' represents the sum of the G.P. series upto nth terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

$$a = 1$$

$$r = (\text{ratio between the } n \text{ term and } n-1 \text{ term}) = -\frac{1}{3} \div 1 = -\frac{1}{3}$$

n terms

$$\therefore S_n = 1 \times \frac{1 - \left(-\frac{1}{3}\right)^n}{1 - \left(-\frac{1}{3}\right)}$$

$$\Rightarrow S_n = \frac{1 - \left(-\frac{1}{3}\right)^n}{\frac{2}{3}}$$

$$\therefore S_n = \frac{3 - \left(-\frac{1}{3}\right)^{n-1}}{2}$$

Q. 2. C. Find the sum of the GP :

$$1 - a + a^2 - a^3 + \dots \text{ to } n \text{ terms (} a \neq 1 \text{)}$$

Answer : Sum of a G.P. series is represented by the formula, $S_n = a \frac{r^n - 1}{r - 1}$, when $r \neq 1$. 'S_n' represents the sum of the G.P. series upto nth terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

$$a = 1$$

$$r = (\text{ratio between the } n \text{ term and } n-1 \text{ term})^{-a \div 1} = -a$$

n terms

$$\therefore S_n = 1 \times \frac{(-a)^n - 1}{-a - 1}$$

[Multiplying both numerator and denominator by -1]

$$\Rightarrow S_n = \frac{1 - (-a)^n}{1 + a}$$

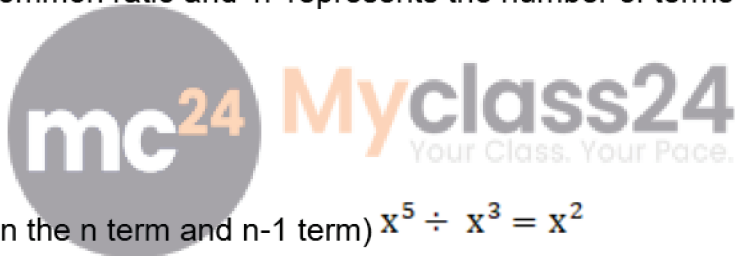
Q. 2. D. Find the sum of the GP :

$x^3 + x^5 + x^7 + \dots$ To n terms

Answer : Sum of a G.P. series is represented by the formula, $S_n = a \frac{r^n - 1}{r - 1}$, when $r \neq 1$. 'S_n' represents the sum of the G.P. series upto nth terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

$$a = x^3$$



$$r = (\text{ratio between the } n \text{ term and } n-1 \text{ term}) x^5 \div x^3 = x^2$$

n terms

$$\therefore S_n = x^3 \times \frac{x^{2n} - 1}{x^2 - 1}$$

$$\Rightarrow S_n = \frac{x^3 (x^n - 1) (x^n + 1)}{(x - 1)(x + 1)}$$

Q. 2. E. Find the sum of the GP :

$x(x + y) + x^2(x^2 + y^2) + x^3(x^3 + y^3) + \dots$ To n terms

Answer : The given expression can be written as

$$= (x^2 + xy) + (x^4 + x^2y^2) + (x^6 + x^3y^3) + \dots \text{ To n terms}$$

$$= (x^2 + x^4 + x^6 + \dots \text{ to n terms}) + (xy + x^2y^2 + x^3y^3 + \dots \text{ to n terms})$$

Sum of a G.P. series is represented by the formula, $S_n = a \frac{r^n - 1}{r - 1}$, when $r \neq 1$. 'S_n' represents the sum of the G.P. series upto nth terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

$a = x^2$ first part and xy for the second part

$r =$ (ratio between the n term and n-1 term) x^2 for the first part and xy for the second part

n terms

$$\therefore S_n = x^2 \times \frac{x^{2n} - 1}{x^2 - 1} + xy \times \frac{x^n y^n - 1}{xy - 1}$$

$$\Rightarrow S_n = \frac{x^2(x^{2n} - 1)(x^n + 1)}{(x + 1)(x - 1)} + \frac{x^{n+1}y^{n+1} - 1}{xy - 1}$$

Q. 3. Find the sum to n terms of the sequence :

(i) $\left(x + \frac{1}{x}\right)^2, \left(x^2 + \frac{1}{x^2}\right)^2, \left(x^3 + \frac{1}{x^3}\right)^2, \dots$ to n terms

(ii) $(x + y), 9x^2 + xy + y^2, (x^3 + x^2y + xy^2 + y^3), \dots$ to n terms

Answer : This can also be written as

$$= \left(x^2 + \frac{1}{x^2} + 2\right) + \left(x^4 + \frac{1}{x^4} + 2\right) + \left(x^6 + \frac{1}{x^6} + 2\right) + \dots \text{to n term}$$

$$= (x^2 + x^4 + x^6 + \dots \text{to n terms}) + \left(\frac{1}{x^2} + \frac{1}{x^4} + \frac{1}{x^6} + \dots \text{to n terms}\right) + (2 + 2 + \dots \text{to n terms})$$

$$= (x^2 + x^4 + x^6 + \dots \text{to n terms}) + \left(\frac{1}{x^2} + \frac{1}{x^4} + \frac{1}{x^6} + \dots \text{to n terms}\right) + 2n$$

Sum of a G.P. series is represented by the formula, $S_n = a \frac{r^n - 1}{r - 1}$, when $r \neq 1$. 'S_n' represents the sum of the G.P. series upto nth terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

$$a = x^2, \frac{1}{x^2}$$

$r =$ (ratio between the n term and $n-1$ term) $x^2, \frac{1}{x^2}$

n terms

$$\therefore S_n = x^2 \times \frac{x^{2n}-1}{x^2-1} + \frac{1}{x^2} \times \frac{\left(\frac{1}{x^2}\right)^n - 1}{\frac{1}{x^2}-1} + 2n$$

$$\Rightarrow S_n = \frac{x^2(x^n-1)(x^n+1)}{(x-1)(x+1)} + \frac{1}{x^2} \times \frac{\frac{1}{x^2}^n - 1}{\frac{1}{x^2}-1} + 2n$$

$$\Rightarrow S_n = \frac{x^2(x^n-1)(x^n+1)}{(x-1)(x+1)} + \frac{\frac{1}{x^2}^n - 1}{x^2-1} + 2n$$

$$\Rightarrow S_n = \frac{x^2(x^n-1)(x^n+1)}{(x-1)(x+1)} + \frac{\frac{1}{x^2}^n - 1}{(x-1)(x+1)} + 2n$$

$$\therefore S_n = \frac{x^2(x^n-1)(x^n+1) + \frac{1}{x^2}^n - 1}{(x-1)(x+1)} + 2n$$

(ii) If we divide and multiply the terms by $(x-y)$

$$= \frac{(x-y)(x+y) + (x-y)(x^2+xy+y^2) + (x-y)(x^3+x^2y+xy^2+y^3) + \dots \text{to } n \text{ terms}}{(x-y)}$$

$$= \frac{(x^2-y^2) + (x^3-y^3) + (x^4-y^4) + \dots \text{to } n \text{ terms}}{(x-y)}$$

$$= \frac{(x^2+x^3+x^4+\dots \text{to } n \text{ terms}) + (y^2+y^3+y^4+\dots \text{to } n \text{ terms})}{(x-y)}$$

Sum of a G.P. series is represented by the formula, $S_n = a \frac{r^n-1}{r-1}$, when $r \neq 1$. 'S_n' represents the sum of the G.P. series upto n^{th} terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

$$a = x^2, y^2$$

$r =$ (ratio between the n term and $n-1$ term) x, y

n terms

$$\therefore S_n = \frac{x^2 \times \frac{x^{n-1}}{x-1} + y^2 \times \frac{y^{n-1}}{y-1}}{(x-y)}$$

$$\Rightarrow S_n = \frac{\frac{x^2(x^{n-1})}{x-1} + \frac{y^2(y^{n-1})}{y-1}}{(x-y)}$$

Q. 4. Find the sum :

$$\frac{3}{5} + \frac{4}{5^2} + \frac{3}{5^3} + \frac{4}{5^4} + \dots \text{ To } 2n \text{ terms}$$

Answer : We can split the above expression into 2 parts. We will split 2n terms into 2 parts also which will leave it as n terms and another n terms.

$$= \left(\frac{3}{5} + \frac{3}{5^3} + \dots \text{ to } n \text{ terms} \right) + \left(\frac{4}{5} + \frac{4}{5^2} + \dots \text{ to } n \text{ terms} \right)$$

Sum of a G.P. series is represented by the formula, $S_n = a \frac{1-r^n}{1-r}$, when $|r| < 1$. 'S_n' represents the sum of the G.P. series upto nth terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

$$a = \frac{3}{5}, \frac{4}{5}$$

$$r = (\text{ratio between the } n \text{ term and } n-1 \text{ term}) \frac{3}{5^3} \div \frac{3}{5}, \frac{4}{5^2} \div \frac{4}{5} = \frac{1}{5^2}, \frac{1}{5}$$

n terms

$$\therefore S_n = \frac{3}{5} \times \frac{1 - \frac{1}{5^2}^n}{1 - \frac{1}{5^2}} + \frac{4}{5} \times \frac{1 - \frac{1}{5}^n}{1 - \frac{1}{5}}$$

$$\Rightarrow S_n = \frac{3}{5} \times \frac{1 - \frac{1}{5^2}^n}{\frac{24}{5^2}} + \frac{4}{5} \times \frac{1 - \frac{1}{5}^n}{\frac{4}{5}}$$

$$\Rightarrow S_n = \frac{5\left(1 - \frac{1}{5^2}\right)}{8} + \left(1 - \frac{1}{5}\right)$$

$$\Rightarrow S_n = \frac{\left(5 - \frac{5}{5^{2n}}\right)}{8} + \left(1 - \frac{1}{5}\right)$$

$$\therefore S_n = \frac{\left(5 - \frac{1}{5^{2n-1}}\right)}{8} + \left(1 - \frac{1}{5}\right)$$

Q. 5. Evaluate :

NOTE: In an expression like this $\Rightarrow \sum_{i=1}^n x$, n represents the upper limit, 1 represents the lower limit, x is the variable expression which we are finding out the sum of and i represents the index of summarization.

$$\sum_{n=1}^{10} (2 + 3^n)$$

(i)

$$\sum_{k=1}^n [2^k + 3^{(k-1)}]$$

(ii)

$$\sum_{n=1}^8 5^n$$

(iii)



Answer : We can write this as $(2 + 3^1) + (2 + 3^2) + (2 + 3^3) + \dots$ to 10 terms

$$= (2 + 2 + 2 + \dots \text{ to 10 terms}) + (3 + 3^2 + 3^3 + \dots \text{ to 10 terms})$$

$$= 2 \times 10 + (3 + 3^2 + 3^3 + \dots \text{ to 10 terms})$$

$$= 20 + (3 + 3^2 + 3^3 + \dots \text{ to 10 terms})$$

Sum of a G.P. series is represented by the formula, $S_n = a \frac{r^n - 1}{r - 1}$, when $r \neq 1$. 'S_n' represents the sum of the G.P. series upto nth terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

$$a = 3$$

$$r = (\text{ratio between the } n \text{ term and } n-1 \text{ term}) = 3$$

n = 10 terms

$$S_n = 3 \times \frac{3^{10} - 1}{3 - 1}$$

$$\Rightarrow S_n = 3 \times \frac{59049-1}{2}$$

$$\Rightarrow S_n = 3 \times \frac{59048}{2}$$

$$\Rightarrow S_n = 88572$$

Thus, sum of the given expression is

$$= 20 + (3+3^2+3^3+\dots \text{ to } 10 \text{ terms})$$

$$= 20 + 88572$$

$$= 88592$$

(ii) The given expression can be written as,

$$(2^1 + 3^{1-1}) + (2^2 + 3^{2-1}) + \dots \text{ to } n \text{ terms}$$

$$= (2 + 3^0) + (2^2 + 3^1) + \dots \text{ to } n \text{ terms}$$

$$= (2 + 1) + (2^2 + 3) + \dots \text{ to } n \text{ terms}$$

$$= (2 + 2^2 + \dots \text{ to } \frac{n}{2} \text{ terms}) + (1 + 3 + \dots \text{ to } \frac{n}{2} \text{ terms})$$

Sum of a G.P. series is represented by the formula, $S_n = a \frac{r^n - 1}{r - 1}$, when $r \neq 1$. 'S_n' represents the sum of the G.P. series upto nth terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

$$a = 2, 1$$

$$r = (\text{ratio between the } n \text{ term and } n-1 \text{ term}) 2, 3$$

$$\frac{n}{2} \text{ terms}$$

$$S_n = 2 \times \frac{2^{\frac{n}{2}} - 1}{2 - 1} + 1 \times \frac{3^{\frac{n}{2}} - 1}{3 - 1}$$

$$\Rightarrow S_n = 2 \times \frac{2^{\frac{n}{2}} - 1}{1} + 1 \times \frac{3^{\frac{n}{2}} - 1}{2}$$

$$\Rightarrow S_n = 2^{\frac{n}{2}+1} - 2 + \frac{3^{\frac{n}{2}} - 1}{2}$$

(iii) We can rewrite the given expression as

$(5^1 + 5^2 + 5^3 + \dots \text{to } 8 \text{ terms})$

Sum of a G.P. series is represented by the formula, $S_n = a \frac{r^n - 1}{r - 1}$, when $r > 1$. 'S_n' represents the sum of the G.P. series upto nth terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

$$a = 5$$

$$r = (\text{ratio between the } n \text{ term and } n-1 \text{ term}) = 5$$

$$n = 8 \text{ terms}$$

$$S_n = 5 \times \frac{5^8 - 1}{5 - 1}$$

$$\Rightarrow S_n = 5 \times \frac{390625 - 1}{4}$$

$$\Rightarrow S_n = 5 \times \frac{390624}{4}$$

$$\Rightarrow S_n = 488280$$

Q. 6. Find the sum of the series :

NOTE: The following terms are not G.P. series, but we can convert them to form one.

(i) $8 + 88 + 888 + \dots$ To n terms

(ii) $3 + 33 + 333 + \dots$ To n terms

(iii) $0.7 + 0.77 + 0.777 + \dots$ To n terms

Answer : The expression can be rewritten as

[Taking 8 as a common factor]

$$8(1+ 11 + 111+ \dots \text{ to } n \text{ terms})$$

[Multiplying and dividing the expression by 9]

$$= \frac{8}{9} (9 + 99+ 999 + \dots \text{ to } n \text{ terms})$$

$$= \frac{8}{9} ((10-1) + (100-1) + (1000-1) + \dots \text{ to } n \text{ terms})$$

$$= \frac{8}{9} ((10 + 100 + 1000 + \dots \text{ to } n \text{ terms}) - (1+1+1+ \dots \text{ to } n \text{ terms}))$$

$$= \frac{8}{9} ((10 + 100 + 1000 + \dots \text{ to } n \text{ terms}) - n)$$

Sum of a G.P. series is represented by the formula, $S_n = a \frac{r^n - 1}{r - 1}$, when $r > 1$. 'S_n' represents the sum of the G.P. series upto nth terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

$$a = 10$$

$$r = (\text{ratio between the } n \text{ term and } n-1 \text{ term}) = 10$$

n terms

$$S_n = 10 \times \frac{10^n - 1}{10 - 1}$$

$$\Rightarrow S_n = 10 \times \frac{10^n - 1}{9}$$

$$\Rightarrow S_n = \frac{10^{n+1} - 10}{9}$$

∴ The sum of the given expression is

$$= \frac{8}{9} ((10 + 100 + 1000 + \dots \text{ to } n \text{ terms}) - n)$$

$$= \frac{8}{9} \left(\frac{10^{n+1} - 10}{9} - n \right)$$

(ii) The given expression can be rewritten as

[taking 3 common]

$$= 3(1 + 11 + 111 + \dots \text{to } n \text{ terms})$$

[Multiplying and dividing the expression by 9]

$$= \frac{3}{9} (9 + 99 + 999 + \dots \text{to } n \text{ terms})$$

$$= \frac{3}{9} ((10-1) + (100-1) + (1000-1) + \dots \text{to } n \text{ terms})$$

$$= \frac{3}{9} ((10+100+1000+ \dots \text{to } n \text{ terms}) - (1+1+1+ \dots \text{to } n \text{ terms}))$$

$$= \frac{3}{9} ((10+100+1000+ \dots \text{to } n \text{ terms}) - n)$$

Sum of a G.P. series is represented by the formula, $S_n = a \frac{r^n - 1}{r - 1}$, when $r > 1$. 'S_n' represents the sum of the G.P. series upto nth terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

$$a = 10$$

$$r = (\text{ratio between the } n \text{ term and } n-1 \text{ term}) = 10$$

n terms

$$S_n = 10 \times \frac{10^n - 1}{10 - 1}$$

$$\Rightarrow S_n = 10 \times \frac{10^n - 1}{9}$$

$$\Rightarrow S_n = \frac{10^{n+1} - 10}{9}$$

∴ The sum of the given expression is

$$= \frac{3}{9} ((10+100+1000+ \dots \text{ to } n \text{ terms}) - n)$$

$$= \frac{3}{9} (\frac{10^{n+1} - 10}{9} - n)$$

(iii) We can rewrite the expression as

[taking 7 as a common factor]

$$= 7(0.1+0.11+0.111+ \dots \text{ to } n \text{ terms})$$

[multiplying and dividing by 9]

$$= \frac{7}{9} (0.9+0.99+0.999+ \dots \text{ to } n \text{ terms})$$

$$= \frac{7}{9} ((1-0.1)+(1-0.01)+(1-0.001)+ \dots \text{ to } n \text{ terms})$$

$$= \frac{7}{9} ((1+1+1+ \dots \text{ to } n \text{ terms}) - (0.1+0.01+0.001+ \dots \text{ to } n \text{ terms}))$$

$$= \frac{7}{9} (n - (0.1+0.01+0.001+ \dots \text{ to } n \text{ terms}))$$

Sum of a G.P. series is represented by the formula, $S_n = a \frac{1-r^n}{1-r}$, when $|r| < 1$. 'S_n' represents the sum of the G.P. series upto nth terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

$$a = 0.1$$

$$r = (\text{ratio between the } n \text{ term and } n-1 \text{ term}) 0.1$$

n terms

$$S_n = 0.1 \times \frac{1 - 0.1^n}{1 - 0.1}$$

$$\Rightarrow S_n = 0.1 \times \frac{1 - 0.1^n}{0.9}$$

[multiplying both numerator and denominator by 10]