NCERT Solutions for Class-XII Maths

Chapter-7.8

NCERT Maths Class 12

1. $\int_a^b x\,dx$

1. It is known that,

$$\int_0^b f(x)dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)+...+f(a+(n-1)h)\right], \text{where } h=\frac{b-a}{n}$$

Here, a = a, b = b, and f(x) = x

$$\therefore \int_a^b f(x)dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[a+(a+h)...(a+2h)...a+(n-1)h\right]$$

$$=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[\left(\underbrace{a+a+a+...+a}_{n\text{ times}}\right)+\left(h+2h+3h+...+(n-1)h\right)\right]$$

$$=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[na+h(1+2+3+..+(n-1))\right]$$

$$=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[na+h\left\{\frac{(n-1)(2)}{2}\right\}\right]$$

$$=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[a+\frac{(n-1)h}{2}\right]$$

$$=(b-a)\lim_{n\to\infty}\left[a+\frac{(n-1)h}{2}\right]$$

$$=(b-a)\lim_{n\to\infty}\left[a+\frac{(n-1)(b-a)}{2n}\right]$$

$$=(b-a)\lim_{n\to\infty}\left[a+\frac{\left(1-\frac{1}{n}\right)(b-a)}{2}\right]$$

$$=(b-a)\left[a+\frac{(b-a)}{2}\right]$$

$$=(b-a)\left[\frac{2a+b-a}{2}\right]$$

$$=\frac{1}{2}\left(b^2-a^2\right)$$

2. $\int_0^b (x+1)dx$

2. Given: $\int_0^5 (x+1)dx$

$$\text{Let} \int (x+1)\,dx = F(x) = \frac{(x+1)^{1+1}}{1+1} \quad (\because \int x^n dx = \frac{x^{n+1}}{n+1})$$

$$\therefore \int (x+1)\,dx = \frac{(x+1)^2}{2}$$

$$\text{Weknowthat} \int_a^b f(x)\,dx = F(b) - F(a)$$

$$\therefore \int_0^5 (x+1)\,dx = F(5) - F(0)$$

$$\Rightarrow \int_0^5 (x+1)\,dx = \frac{(5+1)^2}{2} - (0+1)22$$

$$\Rightarrow \int_0^5 (x+1)\,dx = \frac{(6)^2}{2} - (1)22$$

$$\Rightarrow \int_0^5 (x+1)\,dx = \frac{36}{2} - 12$$

$$\Rightarrow \int_0^5 (x+1)\,dx = \frac{36-1}{2} = \frac{35}{2}$$

$$\therefore \int_0^5 (x+1)\,dx = \frac{35}{2}$$

3. $\int_2^b x^2 dx$

3. It is known that,

$$\int_a^b f(x)dx = (b-a)\lim_{x\to\infty}\frac{1}{n}\left[f(a)+f(a+h)+f(a+2h)..f\{a+(n-1)h\}\right], \text{where } h = \frac{b-a}{n}$$

Here, a = 2, b = 3, and f(x) = x^2

$$\Rightarrow h = \frac{3-2}{n} = \frac{1}{n}$$

$$\therefore \int_2^3 x^2 dx = (3-2)\lim_{x\to\infty}\frac{1}{4}\left[f(2)+f\left(2+\frac{1}{n}\right)+f\left(2+\frac{2}{n}\right)...f\left\{2+(n-1)\frac{1}{n}\right\}\right]$$

$$= \lim_{n\to\infty}\frac{1}{n}\left[(2)^2+\left(2+\frac{1}{n}\right)^2+\left(2+\frac{2}{n}\right)^2+...\left(2+\frac{(n-1)}{n}\right)^2\right]$$

$$= \lim_{n\to\infty}\frac{1}{n}\left[2^2+\left\{2^2+\left(\frac{1}{n}\right)^2+2.2.\frac{1}{n}\right\}+...+\left\{(2)^2+\frac{(n-1)^2}{n^2}+2.2.\frac{(n-1)}{n}\right\}\right]$$

$$= \lim_{n\to\infty}\frac{1}{n}\left[\left(2^2+\underset{\text{a times}}{...}+2^2\right)+\left\{\left(\frac{1}{n}\right)^2+\left(\frac{2}{n}\right)^2+...+\left(\frac{n-1}{n}\right)^2\right\}+2.2.\left\{\frac{1}{n}+\frac{2}{n}+\frac{3}{n}+...+\frac{(n-1)}{n}\right\}\right]$$

$$= \lim_{n\to\infty} \frac{1}{n}\left[4n + \frac{1}{n^2}\{1^2+2^2+3^2...+(n-1)^2\} + \frac{4}{n}\{1+2+...+(n-1)\}\right]$$

$$= \lim_{n\to\infty} \frac{1}{n}\left[4n \frac{1}{n^2}\left\{\frac{n(n-1)(2n-1)}{6}\right\} + \frac{4}{n}\left\{\frac{n(n-1)}{2}\right\}\right]$$

$$= \lim_{n\to\infty} \frac{1}{n}\left[4n + \frac{n\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)}{6} + \frac{4n-4}{2}\right]$$

$$= \lim_{n\to\infty}\left[4 + \frac{1}{6}\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right) + 2 - \frac{2}{n}\right]$$

$$= 4 + \frac{2}{6} + 2 = \frac{19}{3}$$

4. $\int_1^4 (x^2 - x)dx$

4. Explanation:

Given: $\int_1^4 (x^2 - x)dx$

Here,

$\int_1^4 (x^2 - x)\,dx = \int_1^4 x^2\,dx - 14xdx (\because \int_a^b (x_1 \pm _2)\,dx = \int_a^b x_1 dx \pm \int_a^b x_2 dx)$

Here,

$\int x^2 dx = F(x) = \frac{(x)^{2+1}}{2+1} (\because \int x^n dx = \frac{x^{n+1}}{n+1})$

$\therefore \int x^2 dx = \frac{(x)^3}{3}$

We know that $\int_a^b f(x)\,dx = F(b) - F(a)$

$\therefore \int_1^4 x^2\,dx = F(4) - F(1)$

$\therefore \int_1^4 x^2\,dx = \frac{(4)^3}{3} - (1)33$

$\Rightarrow \int_1^4 x^2\,dx = \frac{64}{3} - 13$

$\Rightarrow \int_1^4 x^2\,dx = \frac{64-1}{3} = \frac{63}{3} = 21$

Now,

$\int xdx = F(x) = \frac{x^{1+1}}{1+1} (\because \int x^n dx = \frac{x^{n+1}}{n+1})$

$\therefore \int dx = \frac{x^2}{2}$

We know that $\int_a^b f(x)\,dx = F(b) - F(a)$

$\therefore \int_1^4 x\,dx = F(4) - F(1)$

$\Rightarrow \int_1^4 x\,dx = \frac{4^2}{2} - 122$

$\Rightarrow \int_1^4 x\,dx = \frac{16}{2} - 12$

$\Rightarrow \int_1^4 x\,dx = \frac{16-1}{2} = \frac{15}{2}$

Here,

$\int_1^4 (x^2 - x)\,dx = \int_1^4 x^2\,dx - 14xdx$

$\Rightarrow \int_1^4 (x^2 - x)\,dx = 21 - 152 = 42 - 152 = 272$

5. $\int_{-1}^{1} e^x dx$

5. Given : $\int_{-1}^{1} e^x dx$

Let $\int e^x dx = F(x) = e^x$ $(\because \int e^x dx = e^x)$

$\therefore \int e^x dx = e^x$

We know that $\int_a^b f(x)\,dx = F(b) - F(a)$

$\therefore \int_{-1}^{1} e^x\,dx = F(1) - F(-1)$

$\Rightarrow \int_{-1}^{1} e^x\,dx = e^1 - e - 1$

$\Rightarrow \int_{-1}^{1} e^x\,dx = e - 1e$

$\therefore \int_{-1}^{1} e^x\,dx = e - 1e$

6. $\int_0^4 \left(x + e^{2x}\right) dx$

6. Given: $\int_0^4 (x + e^{2x}) dx$

Here,

$\int_0^4 (x + e^{2x})\,dx = \int_0^4 x\,dx + \int_0^4 e^{2x}\,dx (\because \int_a^b (x_1 \pm_2)\,dx = \int_a^b x_1 dx \pm \int_a^b x_2 dx)$

Here,

$\int dx = F(x) = \frac{x^{1+1}}{1+1} (\because \int xndx = \frac{x^{n+1}}{n+1})$

$\therefore \int dx = \frac{x^2}{2}$

Weknowthat $\int_a^b f(x)\,dx = F(b) - F(a)$

$\therefore \int_0^4 x\,dx = F(4) - F(0)$

$\Rightarrow \int_1^4 x\,dx = \frac{4^2}{2} - 02$

$\Rightarrow \int_1^4 x\,dx = \frac{16}{2} = 8$

Now,

$\int_0^4 e^{2x}\,dx = F(x)$

Let u = 2x

so that

du = 2 dx,

⇒ ½ × du = dx.

Now substituting this into F(x), we get

$\frac{1}{2}\int e^u\,du = \frac{e^u}{2} (\because \int e^x\,dx = e^x)$

$\Rightarrow F(x) = \frac{e^{2x}}{2} (\because u = 2x)$

We know that $\int_a^b x\,dx = F(b) - F(a)$

$\therefore \int_0^4 e^{2x}\,dx = F(4) - F(0)$

$\Rightarrow \int_1^4 e^{2x}\,dx = \frac{e^{2(4)}}{2} - e2(0)2$

$\Rightarrow \int_1^4 e^{2x}\,dx = \frac{e^8}{2} - 12$

Here,

$\int_0^4 (x + e^{2x})\,dx = \int_0^4 x\,dx + \int_0^4 e^{2x}\,dx$

$\Rightarrow \int_0^4 (x + e^{2x})\,dx = 8 + \frac{e^8}{2} - 12 = e82 + 152$

$$\therefore \int_0^4 \left(x + e^{2x}\right)dx = \frac{e^8}{2} + \frac{15}{2}$$