$= -\binom{10}{r}(1)(x)^{2r}$

To get coefficient of x^{10} we must have,

$(x)^{2r} = x^{10}$

• 2r = 10

• r = 5

Therefore, coefficient of $x^{10} = -\binom{10}{5}$

For $\left(x - \frac{2}{x}\right)^{10}$,

Here, a=x, $b = \frac{-2}{x}$ and n=10

We have a formula,

$t_{r+1} = \binom{n}{r} a^{n-r} b^r$

$= \binom{10}{r}(x)^{10-r}\left(\frac{-2}{x}\right)^r$

$= \binom{10}{r}(x)^{10-r}(-2)^r(x)^{-r}$

$= \binom{10}{r}(x)^{10-r-r}(-2)^r$

$= \binom{10}{r}(-2)^r(x)^{10-2r}$

Now, to get coefficient of term independent of x that is coefficient of x^0 we must have,

$(x)^{10-2r} = x^0$

• 10 - 2r = 0

• 2r = 10

• r = 5

Therefore, coefficient of $x^0 = -\binom{10}{5}(2)^5$

Therefore,

$$\frac{\text{coefficient of } x^{10} \text{ in } (1-x^2)^{10}}{\text{coefficient of } x^0 \text{ in } \left(x-\frac{2}{x}\right)^{10}} = \frac{-\binom{15}{5}}{-\binom{15}{5}(2)^5}$$

$$= \frac{1}{(2)^5}$$

$$= \frac{1}{32}$$

Hence,

Coefficient of x^{10} in $(1-x^2)^{10}$: coefficient of x^0 in $\left(x-\frac{2}{x}\right)^{10}$ = 1:32

Q. 28. Find the term independent of x in the expansion of (91 + x + 2x³) $\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$.

Answer: To Find : term independent of x, i.e. coefficient of x^0

Formula: $t_{r+1} = \binom{n}{r} a^{n-r} b^r$

We have a formula,

$$t_{r+1} = \binom{n}{r} a^{n-r} b^r$$

Therefore, the expansion of $\left(x-\frac{2}{x}\right)^{10}$ is given by,

$$\left(x-\frac{2}{x}\right)^{10}=\sum_{r=0}^{10}\binom{10}{r}(x)^{10-r}\left(\frac{-2}{x}\right)^{r}$$

$$=\binom{10}{0}(x)^{10}\left(\frac{-2}{x}\right)^{0}+\binom{10}{1}(x)^{9}\left(\frac{-2}{x}\right)^{1}+\binom{10}{2}(x)^{8}\left(\frac{-2}{x}\right)^{2}+\cdots\ldots\ldots+\binom{10}{10}(x)^{0}\left(\frac{-2}{x}\right)^{10}$$

$$=x^{10}+\binom{10}{1}(x)^{9}(-2)\frac{1}{x}+\binom{10}{2}(x)^{8}(-2)^{2}\frac{1}{x^{2}}+\cdots+\binom{10}{10}(x)^{0}(-2)^{10}\frac{1}{x^{10}}$$

$$=x^{10}-(2)\binom{10}{1}(x)^{8}+(2)^{2}\binom{10}{2}(x)^{6}+\cdots\ldots\ldots+(2)^{10}\binom{10}{10}\frac{1}{x^{10}}$$

Now,

$$(91+x+2x^{3})\left(x-\frac{2}{x}\right)^{10}$$

$$=(91+x+2x^{3})\left(x^{10}-(2)\binom{10}{1}(x)^{8}+(2)^{2}\binom{10}{2}(x)^{6}+\cdots\ldots\ldots+(2)^{10}\binom{10}{10}\frac{1}{x^{10}}\right)$$

Multiplying the second bracket by 91, x and $2x^3$

$$=\left\{91x^{10}-91(2)\binom{10}{1}(x)^{8}+91(2)^{2}\binom{10}{2}(x)^{6}+\cdots+91(2)^{10}\binom{10}{10}\frac{1}{x^{10}}\right\}$$
$$+\left\{x.x^{10}-x.(2)\binom{10}{1}(x)^{8}+x.(2)^{2}\binom{10}{2}(x)^{6}+\cdots\ldots\ldots+x.(2)^{10}\binom{10}{10}\frac{1}{x^{10}}\right\}$$
$$+\left\{2x^{3}.x^{10}-2x^{3}.(2)\binom{10}{1}(x)^{8}+2x^{3}.(2)^{2}\binom{10}{2}(x)^{6}+\cdots\ldots\ldots+2x^{3}.(2)^{10}\binom{10}{10}\frac{1}{x^{10}}\right\}$$

In the first bracket, there will be a 6^{th} term of x^0 having coefficient $91(-2)^{5}\binom{10}{5}$

While in the second and third bracket, the constant term is absent.

Therefore, the coefficient of term independent of x, i.e. constant term in the above expansion

$$= 91(-2)^5 \binom{10}{5}$$

$$= -91.(2)^5 \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1}$$

$= -91(2)^5 (252)$

Conclusion: coefficient of term independent of x $= -91(2)^5 (252)$

Q. 29. Find the coefficient of x in the expansion of $(1 - 3x + 7x^2)(1 - x)^{16}$.

Answer : To Find : coefficient of x

Formula : $t_{r+1} = \binom{n}{r} a^{n-r} b^r$

We have a formula,

$$t_{r+1} = \binom{n}{r} a^{n-r} b^r$$

Therefore, expansion of $(1-x)^{16}$ is given by,

$$(1-x)^{16} = \sum_{r=0}^{16} \binom{16}{r} (1)^{16-r} (-x)^r$$

$$= \binom{16}{0} (1)^{16} (-x)^0 + \binom{16}{1} (1)^{15} (-x)^1 + \binom{16}{2} (1)^{14} (-x)^2 + \cdots \ldots \ldots + \binom{16}{16} (1)^0 (-x)^{16}$$

$$= 1 - \binom{16}{1} x + \binom{16}{2} x^2 + \cdots \ldots \ldots + \binom{16}{16} x^{16}$$

Now,

$(1-3x+7x^2)(1-x)^{16}$

$$= (1-3x+7x^2)\left(1-\binom{16}{1}x+\binom{16}{2}x^2+\cdots\ldots\ldots+\binom{16}{16}x^{16}\right)$$

Multiplying the second bracket by 1 , (-3x) and $7x^2$

$$=\left(1-\binom{16}{1}x+\binom{16}{2}x^2+\cdots\ldots\ldots+\binom{16}{16}x^{16}\right)$$
$$+\left(-3x+3x\binom{16}{1}x-3x\binom{16}{2}x^2+\cdots\ldots\ldots-3x\binom{16}{16}x^{16}\right)$$
$$+\left(7x^2-7x^2\binom{16}{1}x+7x^2\binom{16}{2}x^2+\cdots\ldots\ldots+7x^2\binom{16}{16}x^{16}\right)$$

In the above equation terms containing x are

$-\binom{16}{1}x$ and -3x

Therefore, the coefficient of x in the above expansion

$$=-\binom{16}{1}-3$$

=-16-3

=-19

Conclusion: coefficient of x = – 19

Q. 30. Find the coefficient of

(i)x^5 in the expansion of $(x + 3)^8$

(ii) x^6 in the expansion of $\left(3x^2-\frac{1}{3x}\right)^9$.

(iii) x^{-15} in the expansion of $\left(3x^2-\frac{a}{3x^3}\right)^{10}$.

(iv) a^7b^5 in the expansion of $(a – 2b)^{12}$.

Answer : (i) Here, a=x, b=3 and n=8

We have a formula,

$$t_{r+1} = \binom{n}{r} a^{n-r} b^r$$

$$= \binom{8}{r} (x)^{8-r}(3)^r$$

$$= \binom{8}{r} (3)^r (x)^{8-r}$$

To get coefficient of x^5 we must have,

$(x)^{8-r} = x^5$

• 8 - r = 5

• r = 3

Therefore, coefficient of $x^5 = \binom{8}{3}(3)^3$

$$= \frac{8 \times 7 \times 6}{3 \times 2 \times 1}.(27)$$

= 1512

(ii) Here, $a=3x^2$, $b = \frac{-1}{3x}$ and n=9

We have a formula,

$$t_{r+1} = \binom{n}{r} a^{n-r} b^r$$

$$= \binom{9}{r} (3x^2)^{9-r}\left(\frac{-1}{3x}\right)^r$$

$$= \binom{9}{r} (3)^{9-r} (x^2)^{9-r} \left(\frac{-1}{3}\right)^r (x)^{-r}$$

$$= \binom{9}{r} (3)^{9-r} (x)^{18-2r} \left(\frac{-1}{3}\right)^r (x)^{-r}$$

$= \binom{9}{r} (3)^{9-r} (x)^{18-2r-r} \left(\frac{-1}{3}\right)^r$

$= \binom{9}{r} (3)^{9-r} \left(\frac{-1}{3}\right)^r (x)^{18-3r}$

To get coefficient of x^6 we must have,

$(x)^{18-3r} = x^6$

- 18 - 3r = 6
- 3r = 12
- r = 4

Therefore, coefficient of $x^6 = \binom{9}{4} (3)^{9-4} \left(\frac{-1}{3}\right)^4$

$= \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} \cdot (3)^5 \left(\frac{1}{3}\right)^4$

= 126 × 3

= 378

(iii) Here, $a=3x^2$, $b = \frac{-a}{3x^3}$ and n=10

We have a formula,

$t_{r+1} = \binom{n}{r} a^{n-r} b^r$

$= \binom{10}{r} (3x^2)^{10-r} \left(\frac{-a}{3x^3}\right)^r$

$= \binom{10}{r} (3)^{10-r} (x^2)^{10-r} \left(\frac{-a}{3}\right)^r (x)^{-3r}$

$= \binom{10}{r} (3)^{10-r} (x)^{20-2r} \left(\frac{-a}{3}\right)^r (x)^{-3r}$

$$= \binom{10}{r} (3)^{10-r} (x)^{20-2r-3r} \left(\frac{-a}{3}\right)^r$$

$$= \binom{10}{r} (3)^{10-r} \left(\frac{-a}{3}\right)^r (x)^{20-5r}$$

To get coefficient of x^{-15} we must have,

$(x)^{20-5r} = x^{-15}$

• 20 - 5r = -15

• 5r = 35

• r = 7

Therefore, coefficient of $x^{-15} = \binom{10}{7} (3)^{10-7} \left(\frac{-a}{3}\right)^7$

But $\binom{10}{7} = \binom{10}{3}$ ……… $\left[\because \binom{n}{r} = \binom{n}{n-r}\right]$

Therefore, the coefficient of $x^{-15} = \frac{10\times9\times8}{3\times2\times1} . (3)^3 \left(\frac{-a}{3}\right)^7$

$$= 120 . (-a)^7 \left(\frac{1}{3}\right)^4$$

$$= (-a)^7 \frac{120}{3^4}$$

$$= (-a)^7 \frac{40}{27}$$

(iv) Here, a=a, b=-2b and n=12

We have formula,

$$t_{r+1} = \binom{n}{r} a^{n-r} b^r$$

$$= \binom{12}{r} (a)^{12-r} (-2b)^r$$

$= \binom{12}{r} (-2)^r (a)^{12-r} (b)^r$

To get coefficient of a^7b^5 we must have,

$(a)^{12-r} (b)^r = a^7b^5$

• r = 5

Therefore, coefficient of $a^7b^5 = \binom{12}{5} (-2)^5$

$= \frac{12 \times 11 \times 10 \times 9 \times 8}{5 \times 4 \times 3 \times 2 \times 1} . (-32)$

= 792. (-32)

= -25344

Q. 31. Show that the term containing x^3 does not exist in the expansion of $\left(3x - \frac{1}{2x}\right)^8$.

Answer : For $\left(3x - \frac{1}{2x}\right)^8$,

a=3x, $b = \frac{-1}{2x}$ and n=8

We have a formula,

$t_{r+1} = \binom{n}{r} a^{n-r} b^r$

$= \binom{8}{r} (3x)^{8-r} \left(\frac{-1}{2x}\right)^r$

$= \binom{8}{r} (3)^{8-r} (x)^{8-r} \left(\frac{-1}{2}\right)^r (x)^{-r}$

$= \binom{8}{r} (3)^{8-r} (x)^{8-r-r} \left(\frac{-1}{2}\right)^{r}$

$= \binom{8}{r} (3)^{8-r} \left(\frac{-1}{2}\right)^{r} (x)^{8-2r}$

To get coefficient of x^3 we must have,

$(x)^{8-2r} = (x)^3$

• 8 - 2r = 3

• 2r = 5

• r = 2.5

As $\binom{8}{r} = \binom{8}{2.5}$ is not possible

Therefore, the term containing x^3 does not exist in the expansion of $\left(3x - \frac{1}{2x}\right)^8$

Q. 32. Show that the expansion of $\left(2x^2 - \frac{1}{x}\right)^{20}$ does not contain any term involving x^9.

Answer : For $\left(2x^2 - \frac{1}{x}\right)^{20}$,

a=$2x^2$, $b = \frac{-1}{x}$ and n=20

We have a formula,

$t_{r+1} = \binom{n}{r} a^{n-r} b^r$

$= \binom{20}{r} (3x^2)^{20-r} \left(\frac{-1}{x}\right)^{r}$

$= \binom{20}{r} (3)^{20-r} (x^2)^{20-r} (-1)^r (x)^{-r}$

$= \binom{20}{r} (3)^{20-r} (x)^{40-2r} (-1)^r (x)^{-r}$

$= \binom{20}{r} (3)^{20-r} (x)^{40-2r-r} (-1)^r$

$= \binom{20}{r} (3)^{20-r} (-1)^r (x)^{40-3r}$

To get coefficient of x^9 we must have,

$(x)^{40-3r} = (x)^9$

• 40 - 3r = 9

• 3r = 31

• r = 10.3333

As $\binom{20}{r} = \binom{20}{10.3333}$ is not possible

Therefore, the term containing x^9 does not exist in the expansion of $\left(2x^2 - \frac{1}{x}\right)^{20}$

Q. 33. Show that the expansion of $\left(x^2 + \frac{1}{x}\right)^{12}$ does not contain any term involving x^{-1}.

Answer : For $\left(x^2 + \frac{1}{x}\right)^{12}$,

$a=x^2$, $b = \frac{1}{x}$ and n=12

We have a formula,

$t_{r+1} = \binom{n}{r} a^{n-r} b^r$

$= \binom{12}{r} (x^2)^{12-r} \left(\frac{1}{x}\right)^r$

$= \binom{12}{r} (x)^{24-2r} (x)^{-r}$

$= \binom{12}{r} (x)^{24-2r-r}$

$= \binom{12}{r} (x)^{24-3r}$

To get coefficient of x^{-1} we must have,

$(x)^{24-3r} = (x)^{-1}$

• 24 - 3r = -1

• 3r = 25

• r = 8.3333

As $\binom{20}{r} = \binom{20}{8.3333}$ is not possible

Therefore, the term containing x^{-1} does not exist in the expansion of $\left(x^2 + \frac{1}{x}\right)^{12}$

Q. 34. Write the general term in the expansion of

$(x^2 – y)^6$

Answer : <u>To Find</u> : General term, i.e. t_{r+1}

For $(x^2 - y)^6$

$a=x^2$, $b=-y$ and $n=6$

General term t_{r+1} is given by,

$t_{r+1} = \binom{n}{r} a^{n-r} b^r$

$= \binom{6}{r} (x^2)^{6-r}(-y)^r$

<u>Conclusion</u> : General term $= \binom{6}{r} (x^2)^{6-r}(-y)^r$

Q. 35. Find the 5th term from the end in the expansion of $\left(x-\frac{1}{x}\right)^{12}$.

Answer : To Find : 5th term from the end

Formulae :

- $t_{r+1} = \binom{n}{r} a^{n-r} b^{r}$
- $\binom{n}{r} = \binom{n}{n-r}$

For $\left(x-\frac{1}{x}\right)^{12}$,

a=x, $b = \frac{-1}{x}$ and n=12

As n=12 , therefore there will be total (12+1)=13 terms in the expansion

Therefore,

5th term from the end = (13-5+1)th i.e. 9th term from the starting.

We have a formula,

$$t_{r+1} = \binom{n}{r} a^{n-r} b^{r}$$

For t_9 , r=8

$$\therefore t_9 = t_{8+1}$$

$$= \binom{12}{8} (x)^{12-8} \left(\frac{-1}{x}\right)^{8}$$

$$= \binom{12}{4} (x)^{4} (x)^{-8} \quad \ldots\ldots\ldots. \left[\because \binom{n}{r} = \binom{n}{n-r}\right]$$

$$= \frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1} (x)^{4-8}$$

$= 495\ (x)^{-4}$

Therefore, a 5th term from the end $= 495\ (x)^{-4}$

Conclusion : 5th term from the end $= 495\ (x)^{-4}$

Q. 36. Find the 4th term from the end in the expansion of $\left(\frac{4x}{5} - \frac{5}{2x}\right)^9$.

Answer : To Find : 4th term from the end

Formulae :

- $t_{r+1} = \binom{n}{r} a^{n-r} b^r$
- $\binom{n}{r} = \binom{n}{n-r}$

For $\left(\frac{4x}{5} - \frac{5}{2x}\right)^9$,

$a = \frac{4x}{5}$, $b = \frac{-5}{2x}$ and n=9

As n=9, therefore there will be total (9+1)=10 terms in the expansion

Therefore,

4th term from the end = (10-4+1)th, i.e. 7th term from the starting.

We have a formula,

$t_{r+1} = \binom{n}{r} a^{n-r} b^r$

For t_7 , r=6

$\therefore t_7 = t_{6+1}$

$= \binom{10}{6} \left(\frac{4x}{5}\right)^{10-6} \left(\frac{-5}{2x}\right)^6$

$= \binom{10}{4}\left(\frac{4x}{5}\right)^4\left(\frac{-5}{2x}\right)^6$ .... $\left[\because \binom{n}{r} = \binom{n}{n-r}\right]$

$= \binom{10}{4}\frac{(4)^4}{(5)^4}(x)^4\frac{(-5)^6}{(2)^6}(x)^{-6}$

$= \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1}(100)(x)^{-2}$

$= 21000\,(x)^{-2}$

Therefore, a 4th term from the end $= 21000\,(x)^{-2}$

Conclusion : 4th term from the end $= 21000\,(x)^{-2}$

Q. 37. Find the 4th term from the beginning and end in the expansion of $\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{3}}\right)^n$.

Answer : To Find :

I. 4th term from the beginning

II. 4th term from the end

Formulae :

- $t_{r+1} = \binom{n}{r} a^{n-r} b^r$
- $\binom{n}{r} = \binom{n}{n-r}$

For $\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{3}}\right)^n$,

$a = \sqrt[3]{2}$, $b = \frac{1}{\sqrt[3]{3}}$ and n=9

As n=n , therefore there will be total (n+1) terms in the expansion

Therefore,

I. For the 4^{th} term from the starting.

We have a formula,

$$t_{r+1} = \binom{n}{r} a^{n-r} b^r$$

For t_4 , r=3

$$\therefore t_4 = t_{3+1}$$

$$= \binom{n}{3} (\sqrt[3]{2})^{n-3} \left(\frac{1}{\sqrt[3]{3}}\right)^3$$

$$= \binom{n}{3} (2)^{\frac{n-3}{3}} \frac{1}{3}$$

$$= \binom{n}{3} . \frac{(2)^{\frac{n-3}{3}}}{3}$$

$$= \frac{n!}{(n-3)! \times 3!} . \frac{(2)^{\frac{n-3}{3}}}{3}$$

Therefore, a 4^{th} term from the starting $= \frac{n!}{(n-3)! \times 3!} . \frac{(2)^{\frac{n-3}{3}}}{3}$

Now,

II. For the 4^{th} term from the end

We have a formula,

$$t_{r+1} = \binom{n}{r} a^{n-r} b^r$$

For $t_{(n-2)}$, r = (n-2)-1 = (n-3)

$$\therefore t_{(n-2)} = t_{(n-3)+1}$$

$$= \binom{n}{n-3} \left(\sqrt[3]{2}\right)^{n-(n-3)} \left(\frac{1}{\sqrt[3]{3}}\right)^{(n-3)}$$

$$= \binom{n}{3} \left(\sqrt[3]{2}\right)^{3} (3)^{\frac{-(n-3)}{3}} \quad .... \left[\because \binom{n}{r} = \binom{n}{n-r}\right]$$

$$= \binom{n}{4} (2)\, (3)^{\frac{3-n}{3}}$$

$$= \frac{n!}{(n-4)! \times 4!} (2)\, (3)^{\frac{3-n}{3}}$$

Therefore, a 4th term from the end $= \frac{n!}{(n-4)! \times 4!} (2)\, (3)^{\frac{3-n}{3}}$

<u>Conclusion</u> :

I. 4th term from the beginning $= \frac{n!}{(n-3)! \times 3!} \cdot \frac{(2)^{\frac{n-3}{3}}}{3}$

II. 4th term from the end $= \frac{n!}{(n-4)! \times 4!} (2)\, (3)^{\frac{3-n}{3}}$

Q. 38. Find the middle term in the expansion of :

(i) $(3 + x)^6$

(ii) $\left(\frac{x}{3} + 3y\right)^8$

(iii) $\left(\frac{x}{a} - \frac{a}{x}\right)^{10}$

(iv) $\left(x^2 - \frac{2}{x}\right)^{10}$

Answer : (i) For $(3 + x)^6$,

a=3, b=x and n=6

As n is even, $\left(\frac{n+2}{2}\right)^{th}$ is the middle term

Therefore, the middle term $= \left(\frac{6+2}{2}\right)^{th} = \left(\frac{8}{2}\right)^{th} = (4)^{th}$

General term t_{r+1} is given by,

$$t_{r+1} = \binom{n}{r} a^{n-r} b^r$$

Therefore, for 4^{th} , r=3

Therefore, the middle term is

$$t_4 = t_{3+1}$$

$$= \binom{6}{3} (3)^{6-3} (x)^3$$

$$= \frac{6 \times 5 \times 4}{3 \times 2 \times 1} . (3)^3 (x)^3$$

= (20). (27) x^3

= 540 x^3

(ii) For $\left(\frac{x}{3} + 3y\right)^8$,

$a = \frac{x}{3}$, b=3y and n=8

As n is even, $\left(\frac{n+2}{2}\right)^{th}$ is the middle term

Therefore, the middle term $= \left(\frac{8+2}{2}\right)^{th} = \left(\frac{10}{2}\right)^{th} = (5)^{th}$

General term t_{r+1} is given by,

$$t_{r+1} = \binom{n}{r} a^{n-r} b^r$$

Therefore, for 5^{th} , r=4

Therefore, the middle term is

$$t_5 = t_{4+1}$$

$$= \binom{8}{4} \left(\frac{x}{3}\right)^{8-4} (3y)^4$$

$$= \binom{8}{4} \left(\frac{x}{3}\right)^{4} (3)^4 \ (y)^4$$

$$= \binom{8}{4} \frac{(x)^4}{(3)^4} (3)^4 \ (y)^4$$

$$= \frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1} . (x)^4 \ (y)^4$$

= (70). x^4 y^4

(iii) For $\left(\frac{x}{a} - \frac{a}{x}\right)^{10}$,

$a = \frac{x}{a}$, $b = \frac{-a}{x}$ and n=10

As n is even, $\left(\frac{n+2}{2}\right)^{th}$ is the middle term

Therefore, the middle term $= \left(\frac{10+2}{2}\right)^{th} = \left(\frac{12}{2}\right)^{th} = (6)^{th}$

General term t_{r+1} is given by,

$$t_{r+1} = \binom{n}{r} a^{n-r} b^r$$

Therefore, for 6^{th} , r=5

Therefore, the middle term is

$$t_6 = t_{5+1}$$

$= \binom{10}{5} \left(\frac{x}{a}\right)^{10-5} \left(\frac{-a}{x}\right)^{5}$

$= \binom{10}{5} \left(\frac{x}{a}\right)^{5} (-a)^5 \left(\frac{1}{x}\right)^{5}$

$= \binom{10}{5} \frac{(x)^5}{(a)^5} (-a)^5 \left(\frac{1}{x}\right)^{5}$

$= \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1} . (-1)$

= -252

(iv) For $\left(x^2 - \frac{2}{x}\right)^{10}$,

$a=x^2$, $b = \frac{-2}{x}$ and n=10

As n is even, $\left(\frac{n+2}{2}\right)^{th}$ is the middle term

Therefore, the middle term $= \left(\frac{10+2}{2}\right)^{th} = \left(\frac{12}{2}\right)^{th} = (6)^{th}$

General term t_{r+1} is given by,

$t_{r+1} = \binom{n}{r} a^{n-r} b^r$

Therefore, for the 6^{th} middle term, r=5

Therefore, the middle term is

$t_6 = t_{5+1}$

$= \binom{10}{5} (x^2)^{10-5} \left(\frac{-2}{x}\right)^{5}$

$= \binom{10}{5} (x^2)^5 (-2)^5 \left(\frac{1}{x}\right)^5$

$= \binom{10}{5} \frac{(x)^{10}}{(x)^5} (-32)$

$= \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1} . (-32)\ (x)^5$

= -252 (32) x^5

= -8064 x^5

Q. 39. A. Find the two middle terms in the expansion of :

$(x^2 + a^2)^5$

Answer : For $(x^2 + a^2)^5$,

a= x^2, b= a^2 and n=5

As n is odd, there are two middle terms i.e.

I. $\left(\frac{n+1}{2}\right)^{th}$ and II. $\left(\frac{n+3}{2}\right)^{th}$

General term t_{r+1} is given by,

$t_{r+1} = \binom{n}{r} a^{n-r} b^r$

I. The first, middle term is $\left(\frac{n+1}{2}\right)^{th} = \left(\frac{5+1}{2}\right)^{th} = \left(\frac{6}{2}\right)^{th} = (3)^{rd}$

Therefore, for the 3^{rd} middle term, r=2

Therefore, the first middle term is

$t_3 = t_{2+1}$

$= \binom{5}{2} (x^2)^{5-2} (a^2)^2$

$= \binom{5}{2} (x^2)^3 (a)^4$

$= \binom{5}{2} (x)^6 (a)^4$

$= \frac{5 \times 4}{2 \times 1}. (x)^6 (a)^4$

= 10. a^4. x^6

II. The second middle term is $\left(\frac{n+3}{2}\right)^{th} = \left(\frac{5+3}{2}\right)^{th} = \left(\frac{8}{2}\right)^{th} = (4)^{th}$

Therefore, for the 4^{th} middle term, r=3

Therefore, the second middle term is

$t_4 = t_{3+1}$

$= \binom{5}{3} (x^2)^{5-3} (a^2)^3$

$= \binom{5}{3} (x^2)^2 (a)^6$

$= \binom{5}{2} (x)^4 (a)^6$ .... $[\because \binom{n}{r} = \binom{n}{n-r}]$

$= \frac{5 \times 4}{2 \times 1}. (x)^4 (a)^6$

= 10. a^6. x^4

Q. 39. B. Find the two middle terms in the expansion of:

$\left(x^4 - \frac{1}{x^3}\right)^{11}$

Answer : For $\left(x^4 - \frac{1}{x^3}\right)^{11}$,

$a = x^4$, $b = \frac{-1}{x^3}$ and n=11

As n is odd, there are two middle terms i.e.

II. $\left(\frac{n+1}{2}\right)^{th}$ and II. $\left(\frac{n+3}{2}\right)^{th}$

General term t_{r+1} is given by,

$$t_{r+1} = \binom{n}{r} a^{n-r} b^r$$

I. The first middle term is $\left(\frac{n+1}{2}\right)^{th} = \left(\frac{11+1}{2}\right)^{th} = \left(\frac{12}{2}\right)^{th} = (6)^{th}$

Therefore, for the 6^{th} middle term, r=5

Therefore, the first middle term is

$$t_6 = t_{5+1}$$

$$= \binom{11}{5} (x^4)^{11-5} \left(\frac{-1}{x^3}\right)^5$$

$$= \binom{11}{5} (x^4)^6 (-1)^5 \left(\frac{1}{x^3}\right)^5$$

$$= \binom{11}{5} (x)^{24} (-1) \frac{1}{x^{15}}$$

$$= \frac{11 \times 10 \times 9 \times 8 \times 7}{5 \times 4 \times 3 \times 2 \times 1} . (x)^9 (-1)$$

$= - 462. x^9$

II. The second middle term is $\left(\frac{n+3}{2}\right)^{th} = \left(\frac{11+3}{2}\right)^{th} = \left(\frac{14}{2}\right)^{th} = (7)^{th}$

Therefore, for the 7^{th} middle term, r=6

Therefore, the second middle term is

$t_7 = t_{6+1}$

$= \binom{11}{6} (x^4)^{11-6} \left(\frac{-1}{x^3}\right)^6$

$= \binom{11}{5} (x^4)^5 (-1)^6 \left(\frac{1}{x^3}\right)^6$ ………$[\because \binom{n}{r} = \binom{n}{n-r}]$

$= \binom{11}{5} (x)^{20} (1) \frac{1}{x^{18}}$

$= \frac{11 \times 10 \times 9 \times 8 \times 7}{5 \times 4 \times 3 \times 2 \times 1} . (x)^2$

= 462. x^2

Q. 39. C. Find the two middle terms in the expansion of :

$\left(\frac{p}{x} + \frac{x}{p}\right)^9$

Answer : For $\left(\frac{p}{x} + \frac{x}{p}\right)^9$,

$a = \frac{p}{x}$, $b = \frac{x}{p}$ and n=9

As n is odd, there are two middle terms i.e.

I. $\left(\frac{n+1}{2}\right)^{th}$ and II. $\left(\frac{n+3}{2}\right)^{th}$

General term t_{r+1} is given by,

$t_{r+1} = \binom{n}{r} a^{n-r} b^r$

I. The first middle term is $\left(\frac{n+1}{2}\right)^{th} = \left(\frac{9+1}{2}\right)^{th} = \left(\frac{10}{2}\right)^{th} = (5)^{th}$

Therefore, for 5^{th} middle term, r=4

Therefore, the first middle term is

$t_5 = t_{4+1}$

$= \binom{9}{4}\left(\frac{p}{x}\right)^{9-4}\left(\frac{x}{p}\right)^4$

$= \binom{9}{4}\left(\frac{p}{x}\right)^5 (x)^4 \left(\frac{1}{p}\right)^4$

$= \binom{9}{4}\left(\frac{p}{x}\right)$

$= \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1}.(p).(x)^{-1}$

= $126p.\ x^{-1}$

II. The second middle term is $\left(\frac{n+3}{2}\right)^{th} = \left(\frac{9+3}{2}\right)^{th} = \left(\frac{12}{2}\right)^{th} = (6)^{th}$

Therefore, for the 6^{th} middle term, r=5

Therefore, the second middle term is

$t_6 = t_{5+1}$

$= \binom{9}{5}\left(\frac{p}{x}\right)^{9-5}\left(\frac{x}{p}\right)^5$

$= \binom{9}{4}\left(\frac{p}{x}\right)^4 (x)^5 \left(\frac{1}{p}\right)^5$ .... $\left[\because \binom{n}{r} = \binom{n}{n-r}\right]$

$= \binom{9}{4}\left(\frac{x}{p}\right)$

$= \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1}.\left(\frac{1}{p}\right).(x)$

$= 126\left(\frac{1}{p}\right).(x)$

Q. 39. D. Find the two middle terms in the expansion of :

$$\left(3x - \frac{x^3}{6}\right)^9$$

Answer : For $\left(3x - \frac{x^3}{6}\right)^9$,

a=3x, $b = \frac{-x^3}{6}$ and n=9

As n is odd, there are two middle terms i.e.

I. $\left(\frac{n+1}{2}\right)^{th}$ and II. $\left(\frac{n+3}{2}\right)^{th}$

General term t_{r+1} is given by,

$$t_{r+1} = \binom{n}{r} a^{n-r} b^r$$

I. The first middle term is $\left(\frac{n+1}{2}\right)^{th} = \left(\frac{9+1}{2}\right)^{th} = \left(\frac{10}{2}\right)^{th} = (5)^{th}$

Therefore, for 5^{th} middle term, r=4

Therefore, the first middle term is

$$t_5 = t_{4+1}$$

$$= \binom{9}{4} (3x)^{9-4} \left(\frac{-x^3}{6}\right)^4$$

$$= \binom{9}{4} (3x)^5 (x^3)^4 \left(\frac{1}{6}\right)^4$$

$= \binom{9}{4} (3)^5 (x)^5 (x)^{12} \left(\frac{1}{6}\right)^4$

$= \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} . \frac{243}{1296} (x)^{17}$

$= \frac{189}{8} (x)^{17}$

II. The second middle term is $\left(\frac{n+3}{2}\right)^{th} = \left(\frac{9+3}{2}\right)^{th} = \left(\frac{12}{2}\right)^{th} = (6)^{th}$

Therefore, for the 6^{th} middle term, r=5

Therefore, the second middle term is

$t_6 = t_{5+1}$

$= \binom{9}{5} (3x)^{9-5} \left(\frac{-x^3}{6}\right)^5$

$= \binom{9}{4} (3x)^4 (-x^3)^5 \left(\frac{1}{6}\right)^5$ ……… $[\because \binom{n}{r} = \binom{n}{n-r}]$

$= \binom{9}{4} (3)^4 (x)^4 (-x)^{15} \left(\frac{1}{6}\right)^5$

$= \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} . \frac{81}{7776} (-x)^{19}$

$= -\frac{21}{16} (x)^{19}$

Q. 40. A. Find the term independent of x in the expansion of :

$\left(2x + \frac{1}{3x^2}\right)^9$

Answer : To Find : term independent of x, i.e. x^0

For $\left(2x + \frac{1}{3x^2}\right)^9$

$a = 2x$, $b = \frac{1}{3x^2}$ and $n = 9$

We have a formula,

$$t_{r+1} = \binom{n}{r} a^{n-r} b^r$$

$$= \binom{9}{r} (2x)^{9-r} \left(\frac{1}{3x^2}\right)^r$$

$$= \binom{9}{r} (x)^{9-r} (2)^{9-r} \left(\frac{1}{3}\right)^r \left(\frac{1}{x^2}\right)^r$$

$$= \binom{9}{r} (x)^{9-r} \frac{(2)^{9-r}}{(3)^r} (x)^{-2r}$$

$$= \binom{9}{r} \frac{(2)^{9-r}}{(3)^r} (x)^{9-r-2r}$$

$$= \binom{9}{r} \frac{(2)^{9-r}}{(3)^r} (x)^{9-3r}$$

Now, to get coefficient of term independent of x that is coefficient of x^0 we must have,

$(x)^{9-3r} = x^0$

- $9 - 3r = 0$
- $3r = 9$
- $r = 3$

Therefore, coefficient of $x^0 = \binom{9}{3} \frac{(2)^{9-3}}{(3)^3}$

$= \frac{9 \times 8 \times 7}{3 \times 2 \times 1} \frac{(2)^6}{(3)^3}$

$= \frac{1792}{3}$

Conclusion : coefficient of $x^0 = \frac{1792}{3}$

Q. 40. B. Find the term independent of x in the expansion of :

$$\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^6$$

Answer : To Find : term independent of x, i.e. x^0

For $\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^6$

$a = \frac{3x^2}{2}$, $b = -\frac{1}{3x}$ and n=6

We have a formula,

$$t_{r+1} = \binom{n}{r} a^{n-r} b^r$$

$$= \binom{6}{r} \left(\frac{3x^2}{2}\right)^{6-r} \left(-\frac{1}{3x}\right)^r$$

$$= \binom{6}{r} \left(\frac{3}{2}\right)^{6-r} (x^2)^{6-r} \left(\frac{-1}{3}\right)^r \left(\frac{1}{x}\right)^r$$

$$= \binom{6}{r} \left(\frac{3}{2}\right)^{6-r} \left(\frac{-1}{3}\right)^r (x)^{12-2r} (x)^{-r}$$

$$= \binom{6}{r} \left(\frac{3}{2}\right)^{6-r} \left(\frac{-1}{3}\right)^r (x)^{12-2r-r}$$

$$= \binom{6}{r} \left(\frac{3}{2}\right)^{6-r} \left(\frac{-1}{3}\right)^{r} (x)^{12-3r}$$

Now, to get coefficient of term independent of x that is coefficient of x^0 we must have,

$(x)^{12-3r} = x^0$

• 12 - 3r = 0

• 3r = 12

• r = 4

Therefore, coefficient of $x^0 = \binom{6}{4} \left(\frac{3}{2}\right)^{6-4} \left(\frac{-1}{3}\right)^{4}$

$= \binom{6}{2} \left(\frac{3}{2}\right)^{2} \frac{1}{81}$ .... $[\because \binom{n}{r} = \binom{n}{n-r}]$

$$= \frac{6 \times 5}{2 \times 1} . \frac{9}{4} . \frac{1}{81}$$

$$= \frac{15}{36}$$

Conclusion : coefficient of $x^0 = \frac{15}{36}$

Q. 40. C. Find the term independent of x in the expansion of :

$$\left(x - \frac{1}{x^2}\right)^{3n}$$

Answer : To Find : term independent of x, i.e. x^0

For $\left(x - \frac{1}{x^2}\right)^{3n}$

$a=x$, $b = -\frac{1}{x^2}$ and $N=3n$

We have a formula,

$t_{r+1} = \binom{N}{r} a^{N-r} b^r$

$= \binom{3n}{r} (x)^{3n-r} \left(-\frac{1}{x^2}\right)^r$

$= \binom{3n}{r} (x)^{3n-r} (-1)^r \left(\frac{1}{x^2}\right)^r$

$= \binom{3n}{r} (x)^{3n-r} (-1)^r (x)^{-2r}$

$= \binom{3n}{r} (-1)^r (x)^{3n-r-2r}$

$= \binom{3n}{r} (-1)^r (x)^{3n-3r}$

Now, to get coefficient of term independent of x that is coefficient of x^0 we must have,

$(x)^{3n-3r} = x^0$

• 3n - 3r = 0

• 3r = 3n

• r = n

Therefore, coefficient of $x^0 = \binom{3n}{n} (-1)^n$

<u>Conclusion</u> : coefficient of $x^0 = \binom{3n}{n} (-1)^n$

Q. 40. D. Find the term independent of x in the expansion of :

$\left(3x - \frac{2}{x^2}\right)^{15}$

Answer : <u>To Find</u> : term independent of x, i.e. x^0

For $\left(3x - \frac{2}{x^2}\right)^{15}$

a=3x, $b = \frac{-2}{x^2}$ and n=15

We have a formula,

$t_{r+1} = \binom{n}{r} a^{n-r} b^r$

$= \binom{15}{r} (3x)^{15-r} \left(\frac{-2}{x^2}\right)^r$

$= \binom{15}{r} (3)^{15-r} (x)^{15-r} (-2)^r \left(\frac{1}{x^2}\right)^r$

$= \binom{15}{r} (3)^{15-r} (x)^{15-r} (-2)^r (x)^{-2r}$

$= \binom{15}{r} (3)^{15-r} (-2)^r (x)^{15-r-2r}$

$= \binom{15}{r} (3)^{15-r} (-2)^r (x)^{15-3r}$

Now, to get coefficient of term independent of x that is coefficient of x^0 we must have,

$(x)^{15-3r} = x^0$

• 15 - 3r = 0

• 3r = 15

• r = 5

Therefore, coefficient of $x^0 = \binom{15}{5} (3)^{15-5} (-2)^5$

$= \frac{15 \times 14 \times 13 \times 12 \times 11}{5 \times 4 \times 3 \times 2 \times 1}.(3)^{10}.(-32)$

$= -3003.(3)^{10}.(32)$

Conclusion : coefficient of $x^0 = -3003.(3)^{10}.(32)$

Q. 41. Find the coefficient of x^5 in the expansion of $(1 + x)^3 (1 - x)^6$.

Answer : To Find : coefficient of x^5

For $(1+x)^3$

a=1, b=x and n=3

We have a formula,

$$(1+x)^3 = \sum_{r=0}^{3} \binom{3}{r} (1)^{3-r} x^r$$

$$= \binom{3}{0} (1)^3 x^0 + \binom{3}{1} (1)^2 x^1 + \binom{3}{2} (1)^1 x^2 + \binom{3}{3} (1)^0 x^3$$

$$= 1 + 3x + 3x^2 + x^3$$

For $(1-x)^6$

a=1, b=-x and n=6

We have formula,

$$(1-x)^6 = \sum_{r=0}^{6} \binom{6}{r} (1)^{6-r} (-x)^r$$

$$= \binom{6}{0} (1)^6 (-x)^0 + \binom{6}{1} (1)^5 (-x)^1 + \binom{6}{2} (1)^4 (-x)^2 + \binom{6}{3} (1)^3 (-x)^3 + \binom{6}{4} (1)^2 (-x)^4 + \binom{6}{5} (1)^1 (-x)^5 + \binom{6}{6} (1)^0 (-x)^6$$

We have a formula,

$$\binom{n}{r} = \frac{n!}{(n-r)! \times r!}$$

By using this formula, we get, ×

$(1-x)^6 = 1 - 6x + 15x^2 - 20x^3 + 15x^4 - 6x^5 + x^6$

$\therefore (1+x)^3(1-x)^6$

$= (1 + 3x + 3x^2 + x^3)(1 - 6x + 15x^2 - 20x^3 + 15x^4 - 6x^5 + x^6)$

Coefficients of x^5 are

$x^0.x^5 = 1\times(-6) = -6$

$x^1.x^4 = 3\times15 = 45$

$x^2.x^3 = 3\times(-20) = -60$

$x^3.x^2 = 1\times15 = 15$

Therefore, Coefficients of $x^5 = -6+45-60+15 = -6$

Conclusion : Coefficients of $x^5 = -6$

Q. 42. Find numerically the greatest term in the expansion of $(2 + 3x)^9$, where $x = \frac{3}{2}$.

Answer : To Find : numerically greatest term

For $(2+3x)^9$,

a=2, b=3x and n=9

We have relation,

$t_{r+1} \geq t_r$ or $\frac{t_{r+1}}{t_r} \geq 1$

We have a formula,

$t_{r+1} = \binom{n}{r} a^{n-r} b^r$

$= \binom{9}{r} 2^{9-r} (3x)^r$