EXERCISE 4.2

1. State whether the following quadratic equations have two distinct real roots. Justify your answer.

(i) $x^2 - 3x + 4 = 0$
(ii) $2x^2 + x - 1 = 0$
(iii) $2x^2 - 6x + 9/2 = 0$
(iv) $3x^2 - 4x + 1 = 0$
(v) $(x + 4)^2 - 8x = 0$
(vi) $(x - \sqrt{2})^2 - 2(x + 1) = 0$
(vii) $\sqrt{2}\, x^2 - (3/\sqrt{2})x + 1/\sqrt{2} = 0$
(viii) $x (1 - x) - 2 = 0$
(ix) $(x - 1) (x + 2) + 2 = 0$
(x) $(x + 1) (x - 2) + x = 0$

Solution:

(i)

The equation $x^2 - 3x + 4 = 0$ has no real roots.
$D = b^2 - 4ac$
$= (-3)^2 - 4(1)(4)$
$= 9 - 16 < 0$
Hence, the roots are imaginary.

(ii)

The equation $2x^2 + x - 1 = 0$ has two real and distinct roots.
$D = b^2 - 4ac$
$= 1^2 - 4(2) (-1)$
$= 1 + 8 > 0$
Hence, the roots are real and distinct.

(iii)

The equation $2x^2 - 6x + (9/2) = 0$ has real and equal roots.
$D = b^2 - 4ac$
$= (-6)^2 - 4(2) (9/2)$
$= 36 - 36 = 0$
Hence, the roots are real and equal.

(iv)

The equation $3x^2 - 4x + 1 = 0$ has two real and distinct roots.
$D = b^2 - 4ac$
$= (-4)^2 - 4(3)(1)$
$= 16 - 12 > 0$
Hence, the roots are real and distinct.

(v)

The equation $(x + 4)^2 - 8x = 0$ has no real roots.

Simplifying the above equation,
$x^2 + 8x + 16 - 8x = 0$
$x^2 + 16 = 0$
$D = b^2 - 4ac$
$= (0) - 4(1)(16) < 0$
Hence, the roots are imaginary.

(vi)

The equation $(x - \sqrt{2})^2 - \sqrt{2}(x+1)=0$ has two distinct and real roots.
Simplifying the above equation,
$x^2 - 2\sqrt{2}x + 2 - \sqrt{2}x - \sqrt{2} = 0$
$x^2 - \sqrt{2}(2+1)x + (2 - \sqrt{2}) = 0$
$x^2 - 3\sqrt{2}x + (2 - \sqrt{2}) = 0$
$D = b^2 - 4ac$
$= (-3\sqrt{2})^2 - 4(1)(2 - \sqrt{2})$
$= 18 - 8 + 4\sqrt{2} > 0$
Hence, the roots are real and distinct.

(vii)

The equation $\sqrt{2}x^2 - 3x/\sqrt{2} + ½ = 0$ has two real and distinct roots.
$D = b^2 - 4ac$
$= (-3/\sqrt{2})^2 - 4(\sqrt{2})(½)$
$= (9/2) - 2\sqrt{2} > 0$
Hence, the roots are real and distinct.

(viii)

The equation $x(1 - x) - 2 = 0$ has no real roots.
Simplifying the above equation,
$x^2 - x + 2 = 0$
$D = b^2 - 4ac$
$= (-1)^2 - 4(1)(2)$
$= 1 - 8 < 0$
Hence, the roots are imaginary.

(ix)

The equation $(x - 1)(x + 2) + 2 = 0$ has two real and distinct roots.
Simplifying the above equation,
$x^2 - x + 2x - 2 + 2 = 0$
$x^2 + x = 0$
$D = b^2 - 4ac$
$= 1^2 - 4(1)(0)$
$= 1 - 0 > 0$
Hence, the roots are real and distinct.

(x)

The equation $(x + 1)(x - 2) + x = 0$ has two real and distinct roots.

Simplifying the above equation,
$x^2 + x - 2x - 2 + x = 0$
$x^2 - 2 = 0$
$D = b^2 - 4ac$
$= (0)^2 - 4(1)(-2)$
$= 0 + 8 > 0$
Hence, the roots are real and distinct.

2. Write whether the following statements are true or false. Justify your answers.

- **(i) Every quadratic equation has exactly one root.**
- **(ii) Every quadratic equation has at least one real root.**
- **(iii) Every quadratic equation has at least two roots.**
- **(iv) Every quadratic equations has at most two roots.**
- **(v) If the coefficient of x2 and the constant term of a quadratic equation have opposite signs, then the quadratic equation has real roots.**
- **(vi) If the coefficient of x2 and the constant term have the same sign and if the coefficient of x term is zero, then the quadratic equation has no real roots.**

Solution:

(i) False. For example, a quadratic equation $x^2 - 9 = 0$ has two distinct roots – 3 and 3.

(ii) False. For example, equation $x^2 + 4 = 0$ has no real root.

(iii) False. For example, a quadratic equation $x^2 - 4x + 4 = 0$ has only one root which is 2.

(iv) True, because every quadratic polynomial has almost two roots.

(v) True, because in this case discriminant is always positive.
For example, in $ax^2 + bx + c = 0$, as a and c have opposite sign, $ac < 0$
$\Rightarrow$ Discriminant $= b^2 - 4ac > 0$.

(vi) True, because in this case discriminant is always negative.
For example, in $ax^2 + bx + c = 0$, as $b = 0$, and a and c have same sign then $ac > 0$
$\Rightarrow$ Discriminant $= b^2 - 4ac = -4ac < 0$

3. A quadratic equation with integral coefficient has integral roots. Justify your answer.

Solution:

No, a quadratic equation with integral coefficients may or may not have integral roots.

Justification

Consider the following equation,
$8x^2 - 2x - 1 = 0$
The roots of the given equation are ½ and - ¼ which are not integers.
Hence, a quadratic equation with integral coefficient might or might not have integral roots.