NCERT Solutions for Class 10 Maths Chapter 8 Introduction to Trigonometry EXERCISE 8.1

1. In Δ ABC, right-angled at B, AB = 24 cm, BC = 7 cm. Determine:

(i) Sin A, Cos A

(ii) Sin C, Cos C

Solution: Applying Pythagoras theorem for ΔABC, we obtain

$AC^2 = AB^2 + BC^2$

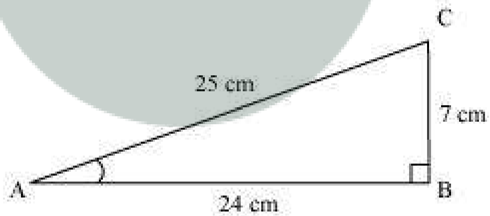
$= (24)^2 + (7)^2$

$= (576 + 49)$

$= 625\ cm^2$

$\therefore AC = \sqrt{625}$

$= 25$ cm



(i) Sin A = $\frac{BC}{AC}$

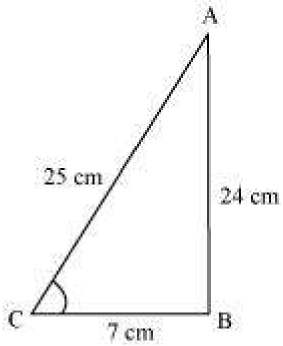
$= \frac{7}{25}$

Cos A = $\frac{AB}{AC}$

$= \frac{24}{25}$

(ii) Sin C = $\frac{AB}{AC}$

$= \frac{24}{25}$



Cos C = $\frac{BC}{AC}$

$= \frac{7}{25}$

2. In Fig. 8.13, find tan P – cot R.

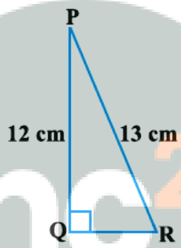
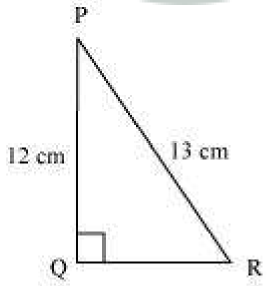


Fig. 8.13

Solution: Applying Pythagoras theorem for ΔPQR, we obtain



$PR^2 = PQ^2 + QR^2$

$(13\text{ cm})^2 = (12\text{ cm})^2 + QR^2$

$169\text{ cm}^2 = 144\text{ cm}^2 + QR^2$

$25\text{ cm}^2 = QR^2$

QR = 5 cm

tan P = $\frac{QR}{PQ}$

= $\frac{5}{12}$

cot R = $\frac{QR}{PQ}$

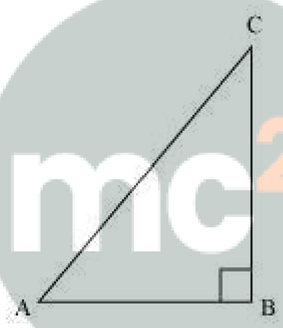
= $\frac{5}{12}$

tan P - cot R = $\frac{5}{12} - \frac{5}{12}$

= 0

3. If sin $A = \frac{3}{4}$, calculate Cos A and tan A

Solution: Let ΔABC be a right-angled triangle, right-angled at point B



Given that,

Sin A = $\frac{3}{4}$

$\frac{BC}{AC} = \frac{3}{4}$

Let BC be 3*k*. Therefore, AC will be 4*k*, where *k* is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$AC^2 = AB^2 + BC^2$

$(4k)^2 = AB^2 + (3k)^2$

$16k^2 - 9k^2 = AB^2$

$7k^2 = AB^2$

AB = $\sqrt{7}$k

Cos A = $\frac{AB}{AC}$

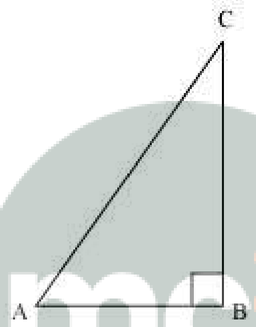
= $\frac{\sqrt{7}k}{4k}$ = $\frac{\sqrt{7}}{4}$

Tan A = $\frac{BC}{AB}$

= $\frac{3k}{\sqrt{7}k}$ = $\frac{3}{\sqrt{7}}$

4. Given 15 cot A = 8, find sin A and sec A

Solution: Consider a right-angled triangle, right-angled at B



Cot A = $\frac{AB}{BC}$

It is given that,

Cot A = $\frac{8}{15}$

$\frac{AB}{BC} = \frac{8}{15}$

Let AB be 8k.Therefore, BC will be 15k, where k is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$AC^2 = AB^2 + BC^2$

$= (8k)^2 + (15k)^2$

$= 64k^2 + 225k^2$

$= 289k^2$

AC = 17k

Sin A = $\frac{BC}{AC}$

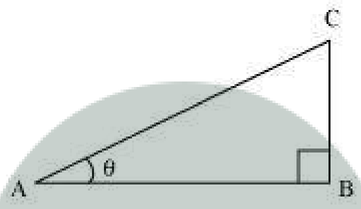
= $\frac{15k}{17k} = \frac{15}{17}$

Sec A = $\frac{AC}{AB}$

= $\frac{17}{8}$

5. Given sec $\theta = \frac{13}{12}$, calculate all other trigonometric ratios

Solution: Consider a right-angle triangle ΔABC, right-angled at point B



sec θ = $\frac{AC}{AB}$

$\frac{13}{12} = \frac{AC}{AB}$

If AC is 13k, AB will be 12k, where k is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$(AC)^2 = (AB)^2 + (BC)^2$

$(13k)^2 = (12k)^2 + (BC)^2$

$169k^2 = 144k^2 + BC^2$

$25k^2 = BC^2$

BC = 5k

Sin θ = $\frac{BC}{AC}$

= $\frac{5k}{13k} = \frac{5}{13}$

Cos θ = $\frac{AB}{AC}$

= $\frac{12k}{13k}$ = $\frac{12}{13}$

Tan θ = $\frac{BC}{AB}$

= $\frac{5k}{12k}$ = $\frac{5}{12}$

Cot θ = $\frac{AB}{BC}$

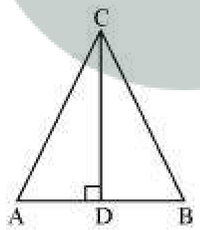
= $\frac{12k}{5k}$ = $\frac{12}{5}$

Cosec θ = $\frac{AC}{BC}$

= $\frac{13k}{5k}$ = $\frac{13}{5}$

6. If ∠ A and ∠ B are acute angles such that Cos A = Cos B, then show that ∠ A = ∠ B

Solution: Let us consider a triangle ABC in which CD ⊥ AB



It is given that

Cos A = Cos B

$\frac{AD}{AC} = \frac{BD}{BC}$ (i)

We have to prove ∠A = ∠B. To prove this, let us extend AC to P such that BC = CP

From equation (i), we obtain

$\frac{AD}{BD} = \frac{AC}{BC}$

$\frac{AD}{BD} = \frac{AC}{CP}$ (By construction BC = CP) (ii)

By using the converse of B.P.T,

CD||BP

∠ACD = ∠CPB (Corresponding angles) (iii)

And,

∠BCD = ∠CBP (Alternate interior angles) (iv)

By construction, we have BC = CP

∠CBP **=** ∠CPB (Angle opposite to equal sides of a triangle) (v)

From equations (iii), (iv), and (v), we obtain

∠ACD = ∠BCD (vi)

In ΔCAD and ΔCBD,

∠ACD = ∠BCD [Using equation (vi)]

∠CDA = ∠CDB (Both 90°)

Therefore, the remaining angles should be equal

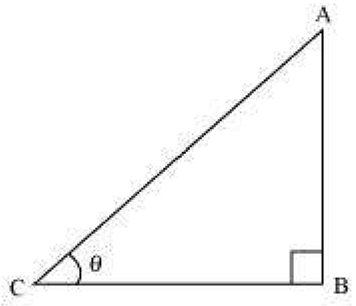
∠CAD = ∠CBD

∠A = ∠B

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$,

(ii) $\cot^2\theta$

Solution: Let us consider a right triangle ABC, right-angled at point B



Cot θ = $\frac{BC}{AB}$

= $\frac{7}{8}$ If BC is 7k, then AB will be 8k, where k is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$AC^2 = AB^2 + BC^2$

$= (8k)^2 + (7k)^2$

$= 64k^2 + 49k^2$

$= 113k^2$

AC = $\sqrt{113}$k

Sin θ = $\frac{AB}{AC}$

= $\frac{8k}{\sqrt{113}k}$

= $\frac{8}{\sqrt{113}}$

Cos θ = $\frac{BC}{AC}$

= $\frac{7k}{\sqrt{113}k}$

= $\frac{7}{\sqrt{113}}$

(i) $\frac{(1+sin\theta)(1-\sin\theta)}{(1+\cos\theta)\,(1-\cos\theta)}$ = $(1 - \sin^2 \theta)/(1 - \cos^2 \theta)$

$= \frac{1-\frac{8}{\sqrt{113}}*\frac{8}{\sqrt{113}}}{1-\frac{7}{\sqrt{113}}*\frac{7}{\sqrt{113}}}$

$= \frac{1-\frac{64}{113}}{1-\frac{49}{113}}$

$= \frac{\frac{49}{113}}{\frac{64}{113}} = \frac{49}{64}$

(ii) $\cot^2 \theta = (\cot \theta)^2$

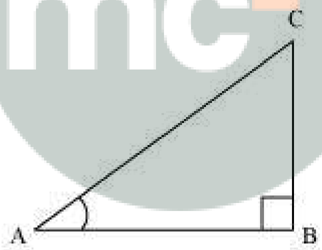
$= (\frac{7}{8})^2 = \frac{49}{64}$

8. If 3 cot A = 4, check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not

Solution: It is given that 3cot A = 4

Or, cot A = $\frac{4}{3}$

Consider a right triangle ABC, right-angled at point B



Cot A = $\frac{AB}{BC}$

$= \frac{4}{3}$

If AB is $4k$, then BC will be $3k$, where k is a positive integer

In ΔABC,

$(AC)^2 = (AB)^2 + (BC)^2$

$= (4k)^2 + (3k)^2$

$= 16k^2 + 9k^2$

$= 25k^2$

$AC = 5k$

Cos A = $\frac{AB}{AC}$

$= \frac{4k}{5k}$

$= \frac{4}{5}$

Sin A = $\frac{BC}{AC}$

$= \frac{3k}{5k}$

$= \frac{3}{5}$

Tan A = $\frac{BC}{AB}$

$= \frac{3k}{4k}$

$= \frac{3}{4}$

$1 - \tan^2 A/1 + \tan^2 A = \frac{1-\frac{3}{4}*\frac{3}{4}}{1+\frac{3}{4}*\frac{3}{4}}$

$= \frac{1-\frac{9}{16}}{1+\frac{9}{16}}$

$= \frac{\frac{7}{16}}{\frac{25}{16}} = \frac{7}{25}$

$\text{Cos}^2 A - \text{Sin}^2 A = (\frac{4}{5})^2 - (\frac{3}{5})^2$

$= \frac{16}{25} - \frac{9}{25}$

$= \frac{7}{25}$

Therefore,

$1 - \tan^2 A/1 + \tan^2 A = \text{Cos}^2 A - \text{Sin}^2 A$

9. In triangle ABC, right-angled at B, if $\tan A = \frac{1}{\sqrt{3}}$, find the value of:

(i) Sin A Cos C + Cos A Sin C

(ii) Cos A Cos C – Sin A Sin C

Solution: tan A = $\frac{BC}{AB}$ = $\frac{1}{\sqrt{3}}$

If BC is *k*, then AB will be, where *k* is a positive integer

In ΔABC,

$AC^2 = AB^2 + BC^2$

$= (\sqrt{3}k)^2 + (k)^2$

$= 3k^2 + k^2 = 4k^2$

AC = $2k$

Sin A = $\frac{BC}{AC}$

$= \frac{k}{2k} = \frac{1}{2}$

Cos A = $\frac{AB}{AC}$

$= \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$

Sin C = $\frac{AB}{AC}$

$= \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$

Cos C = $\frac{BC}{AC}$

$= \frac{k}{2k} = \frac{1}{2}$

(i) sin A cos C + cos A sin C

$= (\frac{1}{2})\ (\frac{1}{2}) + (\frac{\sqrt{3}}{2})\ (\frac{\sqrt{3}}{2})$

$= \frac{1}{4} + \frac{3}{4}$

$= \frac{4}{4} = 1$

(ii) cos A cos C - sin A sin C

$= (\frac{\sqrt{3}}{2})\ (\frac{1}{2}) - (\frac{1}{2})\ (\frac{\sqrt{3}}{2})$

$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$

= 0

10. In Δ PQR, right-angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of Sin P, Cos P and tan P

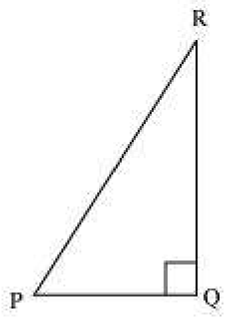
Solution: Given that, PR + QR = 25

PQ = 5

Let PR be x

Therefore,

QR = 25 - x



Applying Pythagoras theorem in ΔPQR, we obtain

$PR^2 = PQ^2 + QR^2$

$x^2 = (5)^2 + (25 - x)^2$

$x^2 = 25 + 625 + x^2 - 50x$

$50x = 650$

$x = 13$

Therefore,

PR = 13 cm

QR = (25 - 13) cm = 12 cm

Sin P = $\frac{QR}{PR} = \frac{12}{13}$

Cos P = $\frac{PQ}{PR} = \frac{5}{13}$

Tan P = $\frac{QR}{PQ} = \frac{12}{5}$

11. State whether the following are true or false. Justify your answer.

(i) The value of tan A is always less than 1

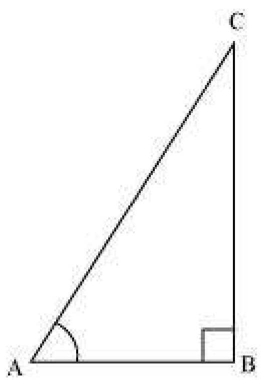
(ii) Sec $A = \frac{12}{5}$ for some value of angle A

(iii) Cos A is the abbreviation used for the cosecant of angle A

(iv) Cot A is the product of cot and A.

(v) $\sin\theta = \frac{4}{3}$ for some angle θ

Solution: (i) Consider a ΔABC, right-angled at B



Tan A = $\frac{12}{5}$

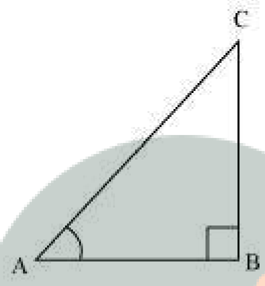
But $\frac{12}{5} > 1$

Tan A > 1

So, Tan A < 1 is not always true

Hence, the given statement is false

(ii) Sec A = $\frac{12}{5}$



$\frac{AC}{AB} = \frac{12}{5}$

Let AC be 12k, AB will be 5k, where k is a positive integer

Applying Pythagoras theorem in ΔABC, we obtain

$AC^2 = AB^2 + BC^2$

$(12k)^2 = (5k)^2 + BC^2$

$144k^2 = 25k^2 + BC^2$

$BC^2 = 119k^2$

BC = 10.9k

It can be observed that for given two sides AC = 12k and AB = 5k,

BC should be such that,

AC - AB < BC < AC + AB

$12k - 5k < BC < 12k + 5k$

$7k < BC < 17\ k$

However, BC = 10.9k. Clearly, such a triangle is possible and hence, such value of sec A is possible

Hence, the given statement is true

(iii) Abbreviation used for cosecant of angle A is cosec A. And Cos A is the abbreviation used for cosine of angle A

Hence, the given statement is false

(iv) Cot A is not the product of cot and A. It is the cotangent of ∠A

Hence, the given statement is false

(v) $\sin\theta = \frac{4}{3}$

In a right-angled triangle, hypotenuse is always greater than the remaining two sides. Therefore, such value of sin θ is not possible
Hence, the given statement is false