Exercise 28

1. Find the distance between the following pairs of points:
(i) (-3, 6) and (2, -6)
(ii) (-a, -b) and (a, b)
(iii) (3/5, 2) and $\left(-\frac{1}{5}, 1\frac{2}{5}\right)$
(iv) (√3 + 1, 1) and (0, √3)
Solution:

(i) (-3, 6) and (2, -6)
Distance between the points is given by
$= \sqrt{[(2 - (-3))^2 + (-6 - 6)^2]}$
$= \sqrt{[5^2 + (-12)^2]}$
$= \sqrt{(25 + 144)}$
$= \sqrt{169}$
= 13 units

(ii) (-a, -b) and (a, b)
Distance between the points is given by
$= \sqrt{[(a - (-a))^2 + (b - (-b))^2]}$
$= \sqrt{[(2a)^2 + (2b)^2]}$
$= \sqrt{(4a^2 + 4b^2)}$
$= \sqrt{4(a^2 + b^2)}$
$= 2\sqrt{(a^2 + b^2)}$ units

(iii) (3/5, 2) and $\left(-\frac{1}{5}, 1\frac{2}{5}\right)$ i.e., (3/5, 2) and (-1/5, 7/5)
Distance between the points is given by
$= \sqrt{[(-1/5 - 3/5)^2 + (7/5 - 2)^2]}$
$= \sqrt{[(-4/5)^2 + ((7 - 10)/5)^2]}$
$= \sqrt{(16/25 + 9/25)}$
$= \sqrt{(25/25)}$
= 1 unit

(iv) (√3 + 1, 1) and (0, √3)
Distance between the points is given by
$= \sqrt{[(0 - (\sqrt{3} + 1))^2 + (\sqrt{3} - 1)^2]}$
$= \sqrt{[(\sqrt{3} + 1)^2 + (\sqrt{3} - 1)^2]}$
$= \sqrt{(3 + 1 + 2\sqrt{3} + 3 + 1 - 2\sqrt{3})}$
$= \sqrt{8}$
$= 2\sqrt{2}$ units

2. Find the distance between the origin and the point:
(i) (-8, 6) (ii) (-5, -12) (iii) (8, -15)

Solution:

Coordinates of the origin O are (0, 0)
Now, the distance between the origin and the points are
(i) A (-8, 6)
$AO = \sqrt{[(0 + 8)^2 + (0 - 6)^2]}$
$= \sqrt{[8^2 + (-6)^2]}$
$= \sqrt{(64 + 36)}$
$= \sqrt{100}$
= 10 units

(ii) B (-5, -12)
$BO = \sqrt{[(0 + 5)^2 + (0 + 12)^2]}$
$= \sqrt{[5^2 + 12^2]}$
$= \sqrt{(25 + 144)}$
$= \sqrt{169}$
= 13 units

(iii) C (8, -15)
$CO = \sqrt{[(0 - 8)^2 + (0 + 15)^2]}$
$= \sqrt{[8^2 + 15^2]}$
$= \sqrt{(64 + 225)}$
$= \sqrt{289}$
= 17 units

3. The distance between the points (3, 1) and (0, x) is 5. Find x.
Solution:

Given, the distance between the points A (3, 1) and B (0, x) is 5
AB = 5
On squaring on both sides, we get
$AB^2 = 5^2$
By distance formula, we have
$(0 - 3)^2 + (x - 1)^2 = 25$
$9 + x^2 - 2x + 1 = 25$
$x^2 - 2x + 10 = 25$
$x^2 - 2x - 15 = 0$
On factorization, we get
$x^2 + 3x – 5x - 15 = 0$
$x(x + 3) – 5(x + 3) = 0$
$(x - 5)(x + 3) = 0$
So, either $(x - 5) = 0$ or $(x + 3) = 0$
Hence,
x = 5 or -3

4. Find the co-ordinates of points on the x-axis which are at a distance of 17 units from

the point (11, -8).
Solution:

Let's assume the coordinates of the point on x-axis to be (x, 0)
Now, from the question, we have
$\sqrt{[(x - 11)^2 + (0 + 8)^2]} = 17$ [By using distance formula]
On squaring on both sides, we get
$(x - 11)^2 + (0 + 8)^2 = 289$
$x^2 + 121 – 22x + 64 = 289$
$x^2 – 22x – 104 = 0$
On factorization, we get
$x^2 – 26x + 4x – 104 = 0$
$x(x - 26) + 4(x - 26) = 0$
$(x - 26) (x + 4) = 0$
So, either $(x - 26) = 0$ or $(x + 4) = 0$
Hence,
x = 26 or -4

Therefore, the required co-ordinates of the points on x-axis are (26, 0) and (-4, 0)

5. Find the co-ordinates of the points on the y-axis, which are at a distance of 10 units from the point (-8, 4).
Solution:

Let's assume the coordinates of the point on y-axis to be (0, y)
Now, from the question, we have
$\sqrt{[(0 + 8)^2 + (y - 4)^2]} = 10$ [By using distance formula]
On squaring on both sides, we get
$8^2 + (y - 4)^2 = 100$
$64 + y^2 + 16 - 8y = 100$
$y^2 - 8y - 20 = 0$
On factorization, we get
$y^2 - 10y + 2y – 20 = 0$
$y(y - 10) + 2(y - 10) = 0$
$(y + 2) (y - 10) = 0$
So, either $(y + 2) = 0$ or $(y - 10) = 0$
Hence,
y = 10 or -2

Therefore, the required co-ordinates of the points on y-axis are (0, 10) and (0, -2)

6. A point A is at a distance of √10 unit from the point (4, 3). Find the co-ordinates of point A, if its ordinate is twice its abscissa.
Solution:

Given, the co-ordinates of point A are such that its ordinate is twice its abscissa.

Now, let's assume the co-ordinates of point A as (x, 2x)
And,
According to the question, we have
$\sqrt{[(x - 4)^2 + (2x - 3)^2]} = \sqrt{10}$ [By using distance formula]
On squaring on both sides, we get
$(x - 4)^2 + (2x - 3)^2 = 10$
$x^2 + 16 – 8x + 4x^2 + 9 – 12x = 10$
$5x^2 – 20x + 25 = 10$
$5x^2 – 20x + 15 = 0$
Dividing by 5, we get
$x^2 – 4x + 3 = 0$
On factorization, we get
$x^2 – 3x - x + 3 = 0$
$x(x - 3) - (x - 3) = 0$
$(x - 3) (x - 1) = 0$
So, either $(x - 3) = 0$ or $(x -1) = 0$
Hence,
$x = 3$ or 1

Thus, the co-ordinates of the point A are (1, 2) and (3, 6)

7. A point P (2, -1) is equidistant from the points (a, 7) and (-3, a). Find a.
Solution:

Given, the point P (2, -1) is equidistant from the points A (a, 7) and B (-3, a)
So, we have
PA = PB
$\Rightarrow PA^2 = PB^2$
By using the distance formula, we have
$(a - 2)^2 + (7 + 1)^2 = (-3 - 2)^2 + (a + 1)^2$
$a^2 + 4 – 4a + 64 = 25 + a^2 + 1 + 2a$
$68 – 4a = 26 + 2a$
$6a = 42$
$a = 7$

Hence, the value of a is 7

8. What point on the x-axis is equidistant from the points (7, 6) and (-3, 4)?
Solution:

Let's assume the co-ordinates of the required point on the x-axis to be P (x, 0)
The given points are A (7, 6) and B (-3, 4)
Given, PA = PB
So, on squaring on both sides, we get
$PA^2 = PB^2$
$(x - 7)^2 + (0 - 6)^2 = (x + 3)^2 + (0 - 4)^2$

$x^2 + 49 - 14x + 36 = x^2 + 9 + 6x + 16$
$85 - 14x = 6x + 25$
$20x = 60$
$x = 3$

Therefore, the required point is (3, 0)

9. Find a point on the y-axis which is equidistant from the points (5, 2) and (-4, 3).
Solution:

Let's assume the co-ordinates of the required point on the y-axis to be P (0, y)
And. the given points are A (5, 2) and B (-4, 3)
Given, PA = PB
So, on squaring on both sides, we get
$PA^2 = PB^2$
$(0 - 5)^2 + (y - 2)^2 = (0 + 4)^2 + (y - 3)^2$
$25 + y^2 + 4 - 4y = 16 + y^2 + 9 - 6y$
$29 - 4y = 25 - 6y$
$2y = -4$
$y = -2$

Thus, the required point is (0, -2).

10. A point P lies on the x-axis and another point Q lies on the y-axis.
(i) Write the ordinate of point P.
(ii) Write the abscissa of point Q.
(iii) If the abscissa of point P is -12 and the ordinate of point Q is -16; calculate the length of line segment PQ.
Solution:

(i) As the point P lies on the x-axis, its ordinate will be 0
(ii) As the point Q lies on the y-axis, its abscissa will be 0
(iii) The co-ordinates of P and Q are (-12, 0) and (0, -16) respectively
And,
$PQ = \sqrt{[(-12 - 0)^2 + (0 + 16)^2]}$
$= \sqrt{(144 + 256)}$
$= \sqrt{400}$
$= 20$

11. Show that the points P (0, 5), Q (5, 10) and R (6, 3) are the vertices of an isosceles triangle.
Solution:

Given points are P (0, 5), Q (5, 10) and R (6, 3)
Calculating:
$PQ = \sqrt{[(5 - 0)^2 + (10 - 5)^2]}$

$= \sqrt{(25 + 25)}$
$= \sqrt{50}$
$= 5\sqrt{2}$

$QR = \sqrt{[(6 - 5)^2 + (3 - 10)^2]}$
$= \sqrt{(1 + 49)}$
$= \sqrt{50}$
$= 5\sqrt{2}$

$RP = \sqrt{[(0 - 6)^2 + (5 - 3)^2]}$
$= \sqrt{(36 + 4)}$
$= \sqrt{40}$
$= 2\sqrt{10}$

As PQ = QR
Hence, ΔPQR is an isosceles triangle.

12. Prove that the points P (0, -4), Q (6, 2), R (3, 5) and S (-3, -1) are the vertices of a rectangle PQRS.
Solution:

Given points are P (0, -4), Q (6, 2), R (3, 5) and S (-3, -1)
Calculating:
$PQ = \sqrt{[(6 - 0)^2 + (2 + 4)^2]} = \sqrt{(36 + 36)} = \sqrt{72} = 6\sqrt{2}$ units

$QR = \sqrt{[(6 - 3)^2 + (2 - 5)^2]} = \sqrt{(9 + 9)} = \sqrt{18} = 3\sqrt{2}$ units

$RS = \sqrt{[(3 + 3)^2 + (5 + 1)^2]} = \sqrt{(36 + 36)} = \sqrt{72} = 6\sqrt{2}$ units

$PS = \sqrt{[(-3 - 0)^2 + (-1 + 4)^2]} = \sqrt{(9 + 9)} = \sqrt{18} = 3\sqrt{2}$ units

$PR = \sqrt{[(3 - 0)^2 + (5 + 4)^2]} = \sqrt{(9 + 81)} = \sqrt{90} = 3\sqrt{10}$ units

$QS = \sqrt{[(6 + 3)^2 + (2 - 5)^2]} = \sqrt{(9 + 9)} = \sqrt{18} = 3\sqrt{2}$ units
It's seen that,
PQ = RS and QR = PS
Also, PR = QS
Hence, PQRS is a rectangle

13. Prove that the points A (1, -3), B (-3, 0) and C (4, 1) are the vertices of an isosceles right-angled triangle. Find the area of the triangle.
Solution:

Given points are A (1, -3), B (-3, 0) and C (4, 1)
Calculating:
$AB = \sqrt{[(-3 - 1)^2 + (0 + 3)^2]} = \sqrt{(16 + 9)} = \sqrt{25} = 5$ units

BC = $\sqrt{[(4 + 3)^2 + (1 - 0)^2]} = \sqrt{(49 + 1)} = \sqrt{50} = 5\sqrt{2}$ units

CA = $\sqrt{[(1 - 4)^2 + (-3 - 1)^2]} = \sqrt{(9 + 16)} = \sqrt{25} = 5$ units
Hence, its seen that AB = CA
So, A, B and C are the vertices of an isosceles triangle
And,
$AB^2 + CA^2 = 25 + 25 = 50$
$BC^2 = (5\sqrt{2})^2 = 25 \times 2 = 50$
Thus, $AB^2 + CA^2 = BC^2$
Therefore, we can conclude that A, B and C are the vertices of a right-angle triangle
i.e., ∆ABC is a right-angle isosceles triangle
Now,
Area of ∆ABC = ½ x AB x CA
= ½ x 5 x 5
= 25/2
= 12.5 sq. units

14. Show that the points A (5, 6), B (1, 5), C (2, 1) and D (6, 2) are the vertices of a square ABCD.
Solution:

Given points are A (5, 6), B (1, 5), C (2, 1) and D (6, 2)
Calculating the sides:
AB = $\sqrt{[(1 - 5)^2 + (5 - 6)^2]} = \sqrt{(16 + 1)} = \sqrt{17}$ units
BC = $\sqrt{[(2 - 1)^2 + (1 - 5)^2]} = \sqrt{(1 + 16)} = \sqrt{17}$ units
CD = $\sqrt{[(6 - 2)^2 + (2 - 1)^2]} = \sqrt{(16 + 1)} = \sqrt{17}$ units
DA = $\sqrt{[(5 - 6)^2 + (6 - 2)^2]} = \sqrt{(1+ 16)} = \sqrt{17}$ units

Now, calculating the diagonals:
AC = $\sqrt{[(2 - 5)^2 + (1 - 6)^2]} = \sqrt{(9 + 25)} = \sqrt{34}$ units
BD = $\sqrt{[(6 - 1)^2 + (2 - 5)^2]} = \sqrt{(25 + 9)} = \sqrt{34}$ units

As, AB = BC = CD = DA and AC = BD
Hence, we can conclude that A, B, C and D are the vertices of a square

15. Show that (-3, 2), (-5, -5), (2, -3) and (4, 4) are the vertices of a rhombus.
Solution:

Let the given points be taken as A (-3, 2), B (-5, -5), C (2, -3) and D (4, 4)
Calculating the sides:
AB = $\sqrt{[(-5 + 3)^2 + (-5 - 2)^2]} = \sqrt{(4 + 49)} = \sqrt{53}$ units
BC = $\sqrt{[(2 + 5)^2 + (-3 + 5)^2]} = \sqrt{(49 + 4)} = \sqrt{53}$ units
CD = $\sqrt{[(4 - 2)^2 + (4 + 3)^2]} = \sqrt{(4 + 49)} = \sqrt{53}$ units
DA = $\sqrt{[(-3 - 4)^2 + (2 - 4)^2]} = \sqrt{(49 + 4)} = \sqrt{53}$ units

Now, calculating the diagonals:
AC = $\sqrt{[(2 + 3)^2 + (-3 - 2)^2]} = \sqrt{(25 + 25)} = \sqrt{50} = 5\sqrt{2}$ units
BD = $\sqrt{[(4 - 5)^2 + (4 + 5)^2]} = \sqrt{(81 + 81)} = \sqrt{162} = 9\sqrt{2}$ units

As, AB = BC = CD = DA and AC ≠ BD
Hence, we can conclude that the given vertices are of a rhombus

16. Points A (-3, -2), B (-6, a), C (-3, -4) and D (0, -1) are the vertices of quadrilateral ABCD; find a if 'a' is negative and AB = CD.
Solution:

Given, points A (-3, -2), B (-6, a), C (-3, -4) and D (0, -1) are the vertices of quadrilateral ABCD
Also given that AB = CD
On squaring on both sides, we get
$AB^2 = CD^2$
$(-6 + 3)^2 + (a + 2)^2 = (0 + 3)^2 + (-1 + 4)^2$ [By distance formula]
$9 + a^2 + 4 + 4a = 9 + 9$
$a^2 + 4a - 5 = 0$
$a^2 - a + 5a - 5 = 0$
$a(a - 1) + 5 (a - 1) = 0$
$(a - 1) (a + 5) = 0$
So, either (a - 1) = 0 or (a + 5) = 0
⇒ a = 1 or -5

It is given that 'a' is negative,
Hence, the value of a is -5

17. The vertices of a triangle are (5, 1), (11, 1) and (11, 9). Find the co-ordinates of the circumcentre of the triangle.
Solution:

Let's assume the circumcentre to be P (x, y)
Given, vertices of a triangle are (5, 1), (11, 1) and (11, 9)
If P is the circumcentre,
Then, PA = PB
Squaring on both sides, we get
$PA^2 = PB^2$
$(x - 5)^2 + (y - 1)^2 = (x - 11)^2 + (y - 1)^2$ [By distance formula]
$x^2 + 25 - 10x = x^2 + 121 - 22x$
$12x = 96$
$x = 8$

Also, PA = PC
Squaring on both sides, we get
$PA^2 = PC^2$
$(x - 5)^2 + (y - 1)^2 = (x - 11)^2 + (y - 9)^2$

$x^2 + 25 - 10x + y^2 + 1 - 2y = x^2 + 121 - 22x + y^2 + 81 - 18y$
$12x + 16y = 176$
$3x + 4y = 44$
$24 + 4y = 44$
$4y = 20$
$y = 5$

Therefore, the co-ordinates of the circumcentre of the triangle are (8, 5)

18. Given A = (3, 1) and B = (0, y - 1). Find y if AB = 5.
Solution:

Given, points A = (3, 1) and B = (0, y - 1)
According to the question, we have
AB = 5
On squaring it on both sides, we have
$AB^2 = 25$
$(0 - 3)^2 + (y - 1 - 1)^2 = 25$
$9 + y^2 + 4 - 4y = 25$
$y^2 - 4y - 12 = 0$
On factorization, we get
$y^2 - 6y + 2y - 12 = 0$
$y(y - 6) + 2(y - 6) = 0$
$(y - 6)(y + 2) = 0$
So, $(y - 6) = 0$ or $(y + 2) = 0$
Hence,
y = 6 or -2

19. Given A = (x + 2, -2) and B (11, 6). Find x if AB = 17.
Solution:

Given, points A = (x + 2, -2) and B (11, 6)
According to the question, we have
AB = 17
On squaring it on both sides, we have
$AB^2 = 289$
$(11 - x - 2)^2 + (6 + 2)^2 = 289$
$x^2 + 81 - 18x + 64 = 289$
$x^2 - 18x - 144 = 0$
$x^2 - 24x + 6x - 144 = 0$
$x(x - 24) + 6(x - 24) = 0$
$(x - 24)(x + 6) = 0$
So, $(x – 24) = 0$ or $(x + 6) = 0$
Hence,
x = 24, -6

20. The centre of a circle is (2x - 1, 3x + 1). Find x if the circle passes through (-3, -1) and the length of its diameter is 20 unit.
Solution:

Given, the centre of the circle O is (2x - 1, 3x + 1)
Form the question, we have
Distance between the centre O (2x - 1, 3x + 1) and point A (-3, -1) should be equal to the radius of the circle
OA = 10 units (As given, diameter = 20 units)
On squaring on both sides, we get
$OA^2 = 100$
$(-3 - 2x + 1)^2 + (-1 - 3x - 1)^2 = 100$ [By distance formula]
$(-2 - 2x)^2 + (-2 - 3x)^2 = 100$
$4 + 4x^2 + 8x + 4 + 9x^2 + 12x = 100$
$13x^2 + 20x - 92 = 0$
By using the quadratic formula, we have
$x = -20 \pm \sqrt{[(20)^2 - 4(13)(-92)]} / 26$
$= (-20 \pm 72)/26$
$= -92/26$ or $52/26$
$= -46/13$ or 2

Therefore, the value of x is 2 or -46/13