Hence, the required positive integers are 13 and 14.

5. The sum of the squares to two consecutive positive odd numbers is 514. Find the numbers.

Sol:

Let the two consecutive positive odd numbers be x and $(x+2)$.

According to the given condition,

$x^2+(x+2)^2=514$

$\Rightarrow x^2+x^2+4x+4=514$

$\Rightarrow 2x^2+4x-510=0$

$\Rightarrow x^2+2x-255=0$

$\Rightarrow x^2+17x-15x-255=0$

$\Rightarrow x(x+17)-15(x+17)=0$

$\Rightarrow (x+17)(x-15)=0$

$\Rightarrow x+17=0 \ or \ x-15=0$

$\Rightarrow x=-17 \ or \ x=15$

$\therefore x=15$ (x is a positive odd number)

When $x=15$,

$x+2=15+2=17$

Hence, the required positive integers are 15 and 17.

6. The sum of the squares of two consecutive positive even numbers is 452. Find the numbers.

Sol:

Let the two consecutive positive even numbers be x and $(x+2)$.

According to the given condition,

$x^2+(x+2)^2=452$

$\Rightarrow x^2+x^2+4x+4=452$

$\Rightarrow 2x^2+4x-448=0$

$\Rightarrow x^2+2x-224=0$

$\Rightarrow x^2+16x-14x-224=0$

$\Rightarrow x(x+16)-14(x+16)=0$

$\Rightarrow (x+16)(x-14)=0$

$\Rightarrow x+16=0 \ or \ x-14=0$

$\Rightarrow x=-16 \ or \ x=14$

$\therefore x=14$ (x is a positive even number)

When $x=14$,

$x+2=14+2=16$

Hence, the required numbers are 14 and 16.

7. The product of two consecutive positive integers is 306. Find the integers.

Sol:

Let the two consecutive positive integers be x and $(x+1)$.

According to the given condition,

$x(x+1)=306$

$\Rightarrow x^2+x-306=0$

$\Rightarrow x^2+18x-17x-306=0$

$\Rightarrow x(x+18)-17(x+18)=0$

$\Rightarrow (x+18)(x-17)=0$

$\Rightarrow x+18=0 \; or \; x-17=0$

$\Rightarrow x=-18 \; or \; x=17$

$\therefore x=17$ (x is a positive integers)

When $x=17$,

$x+1=17+1=18$

Hence, the required integers are 17 and 18.

8. Two natural number differ by 3 and their product is 504. Find the numbers.

Sol:

Let the required numbers be x and $(x+3)$.

According to the question:

$x(x+3)=504$

$\Rightarrow x^2+3x=504$

$\Rightarrow x^2+3x-504=0$

$\Rightarrow x^2+(24-21)x-504=0$

$\Rightarrow x^2+24x-21x-504=0$

$\Rightarrow x(x+24)-21(x+24)=0$

$\Rightarrow (x+24)(x-21)=0$

$\Rightarrow x+24=0 \; or \; x-21=0$

$\Rightarrow x=-24 \; or \; x=21$

If $x=-24$, the numbers are -24 and $\{(-24+3)=-21\}$.

If $x=21$, the numbers are 21 and $\{(21+3)=24\}$.

Hence, the numbers are $(-24,-21)$ and $(21,24)$.

9. Find two consecutive multiples of 3 whose product is 648.

Sol:

Le the required consecutive multiples of 3 be $3x$ and $3(x+1)$.

According to the given condition,

$3x \times 3(x+1) = 648$

$\Rightarrow 9(x^2 + x) = 648$

$\Rightarrow x^2 + x = 72$

$\Rightarrow x^2 + x - 72 = 0$

$\Rightarrow x^2 + 9x - 8x - 72 = 0$

$\Rightarrow x(x+9) - 8(x+9) = 0$

$\Rightarrow (x+9)(x-8) = 0$

$\Rightarrow x + 9 = 0 \text{ or } x - 8 = 0$

$\Rightarrow x = -9 \text{ or } x = 8$

$\therefore x = 8$ (Neglecting the negative value)

When $x = 8$,

$3x = 3 \times 8 = 24$

$3(x+1) = 3 \times (8+1) = 3 \times 9 = 27$

Hence, the required multiples are 24 and 27.

10. Find the tow consecutive positive odd integer whose product s 483.

Sol:

Let the two consecutive positive odd integers be x and $(x+2)$.

According to the given condition,

$x(x+2) = 483$

$\Rightarrow x^2 + 2x - 483 = 0$

$\Rightarrow x^2 + 23x - 21x - 483 = 0$

$\Rightarrow x(x+23) - 21(x+23) = 0$

$\Rightarrow (x+23)(x+21) = 0$

$\Rightarrow x + 23 = 0 \text{ or } x - 21 = 0$

$\Rightarrow x = -23 \text{ or } x = 21$

$\therefore x = 21$ (x is a positive odd integer)

When $x = 21$,

$x + 2 = 21 + 2 = 23$

Hence, the required integers are 21 and 23.

11. Find the two consecutive positive even integers whose product is 288.

Sol:

Let the two consecutive positive even integers be x and $(x+2)$.

According to the given condition,

$x(x+2)=288$

$\Rightarrow x^2+2x-288=0$

$\Rightarrow x^2+18x-16x-288=0$

$\Rightarrow x(x+18)-16(x+18)=0$

$\Rightarrow (x+18)(x-16)=0$

$\Rightarrow x+18=0 \text{ or } x-16=0$

$\Rightarrow x=-18 \text{ or } x=16$

$\therefore x=16$ (x is a positive even integer)

When $x=16$,

$x+2=16+2=18$

Hence, the required integers are 16 and 18.

12. The sum of two natural numbers is 9 and the sum of their reciprocals is $\frac{1}{2}$. Find the numbers.

Sol:

Let the required natural numbers be x and $(9,-x)$.

According to the given condition,

$$\frac{1}{x}+\frac{1}{9-x}=\frac{1}{2}$$

$$\Rightarrow \frac{9-x+x}{x(9-x)}=\frac{1}{2}$$

$$\Rightarrow \frac{9}{9x-x^2}=\frac{1}{2}$$

$\Rightarrow 9x-x^2=18$

$\Rightarrow x^2-9x+18=0$

$\Rightarrow x^2-6x-3x+18=0$

$\Rightarrow x(x-6)-3(x-6)=0$

$\Rightarrow x-3=0 \text{ or } x-6=0$

$\Rightarrow x=3 \text{ or } x=6$

When $x=3$,

$9-x=9-3=6$
When $x=6$,
$9-x=9-6=3$
Hence, the required natural numbers are 3 and 6.

13. The sum of two natural numbers is 15 and the sum of their reciprocals is $\frac{3}{10}$. Find the numbers.

Sol:

Let the required natural numbers be x and $(15-x)$.

According to the given condition,

$\frac{1}{x}+\frac{1}{15-x}=\frac{3}{10}$

$\Rightarrow \frac{15-x+x}{x(15-x)}=\frac{3}{10}$

$\Rightarrow \frac{15}{15x-x^2}=\frac{3}{10}$

$\Rightarrow 15x-x^2=50$

$\Rightarrow x^2-15x+50=0$

$\Rightarrow x^2-10x-5x+50=0$

$\Rightarrow x(x-10)-5(x-10)=0$

$\Rightarrow (x-5)(x-10)=0$

$\Rightarrow x-5=0 \text{ or } x-10=0$

$\Rightarrow x=5 \text{ or } x=10$

When $x=5$,
$15-x=15-5=10$
When $x=10$,
$15-x=15-10=5$
Hence, the required natural numbers are 5 and 10.

14. The difference of two natural number is 3 and the difference of their reciprocals is $\frac{3}{28}$. Find the numbers.

Sol:

Let the required natural numbers be x and $(x+3)$.

Now, $x < x+3$

$\therefore \frac{1}{x} > \frac{1}{x+3}$

According to the given condition,

$\frac{1}{x} - \frac{1}{x+3} = \frac{3}{28}$

$\Rightarrow \frac{x+3-x}{x(x+3)} = \frac{3}{28}$

$\Rightarrow \frac{3}{x^2+3x} = \frac{3}{28}$

$\Rightarrow x^2+3x = 28$

$\Rightarrow x^2+3x-28 = 0$

$\Rightarrow x^2+7x-4x-28 = 0$

$\Rightarrow x(x+7)-4(x+7) = 0$

$\Rightarrow (x+7)(x-4) = 0$

$\Rightarrow x+7 = 0 \text{ or } x-4 = 0$

$\Rightarrow x = -7 \text{ or } x = 4$

$\therefore x = 4$ (−7 is not a natural number)

When $x = 4$,

$x+3 = 4+3 = 7$

Hence, the required natural numbers are 4 and 7.

15. The difference of two natural numbers is 5 and the difference of heir reciprocals is $\frac{5}{14}$. Find the numbers.

Sol:

Let the required natural numbers be x and $(x+5)$.

Now, $x < x+5$

$\therefore \frac{1}{x} > \frac{1}{x+5}$

According to the given condition,

$\frac{1}{x} - \frac{1}{x+5} = \frac{5}{14}$

$\Rightarrow \frac{x+5-x}{x(x+5)} = \frac{5}{14}$

$\Rightarrow \frac{5}{x^2+5x} = \frac{5}{14}$

$\Rightarrow x^2+5x = 14$

$\Rightarrow x^2+5x-14=0$

$\Rightarrow x^2+7x-2x-14=0$

$\Rightarrow x(x+7)-2(x+7)=0$

$\Rightarrow (x+7)(x-2)=0$

$\Rightarrow x+7=0 \text{ or } x-2=0$

$\Rightarrow x=-7 \text{ or } x=2$

$\therefore x=2$ (-7 is not a natural number)

When $x=2$,

$x+5=2+5=7$

Hence, the required natural numbers are 2 and 7.

16. The sum of the squares two consecutive multiples of 7 is 1225. Find the multiples.

Sol:

Let the required consecutive multiplies of 7 be $7x$ and $7(x+1)$.

According to the given condition,

$(7x)^2+[7(x+1)]^2=1225$

$\Rightarrow 49x^2+49(x^2+2x+1)=1225$

$\Rightarrow 49x^2+49x^2+98x+49=1225$

$\Rightarrow 98x^2+98x-1176=0$

$\Rightarrow x^2+x-12=0$

$\Rightarrow x^2+4x-3x-12=0$

$\Rightarrow x(x+4)-3(x+4)=0$

$\Rightarrow (x+4)(x-3)=0$

$\Rightarrow x+4=0 \text{ or } x-3=0$

$\Rightarrow x=-4 \text{ or } x=3$

$\therefore x=3$ (Neglecting the negative value)

When $x=3$,

$7x=7\times 3=21$

$7(x+1)=7(3+1)=7\times 4=28$

Hence, the required multiples are 21 and 28.

17. The sum of natural number and its reciprocal is $\frac{65}{8}$. Find the number.

Sol:

Let the natural number be x.
According to the given condition,

$$x+\frac{1}{x}=\frac{65}{8}$$
$$\Rightarrow \frac{x^2+1}{x}=\frac{65}{8}$$
$$\Rightarrow 8x^2+8=65x$$
$$\Rightarrow 8x^2-65x+8=0$$
$$\Rightarrow 8x^2-64x-x+8=0$$
$$\Rightarrow 8x(x-8)-1(x-8)=0$$
$$\Rightarrow (x-8)(8x-1)=0$$
$$\Rightarrow x-8=0 \text{ or } 8x-1=0$$
$$\Rightarrow x=8 \text{ or } x=\frac{1}{8}$$
$\therefore x=8$ (x is a natural number)

Hence, the required number is 8.

18. Divide 57 into two parts whose product is 680.

Sol:

Let the two parts be x and $(57-x)$.
According to the given condition,

$$x(57-x)=680$$
$$\Rightarrow 57x-x^2=680$$
$$\Rightarrow x^2-57x+680=0$$
$$\Rightarrow x^2-40x-17x+680=0$$
$$\Rightarrow x(x-40)-17(x-40)=0$$
$$\Rightarrow (x-40)(x-17)=0$$
$$\Rightarrow x-40=0 \text{ or } x-17=0$$
$$\Rightarrow x=40 \text{ or } x=17$$

When $x=40$,
$57-x=57-40=17$
When $x=17$,
$57-x=57-17=40$
Hence, the required parts are 17 and 40.

19. Divide 27 into two parts such that the sum of their reciprocal is $\frac{3}{20}$.

Sol:

Let the two parts be x and $(27-x)$.

According to the given condition,

$\frac{1}{x}+\frac{1}{27-x}=\frac{3}{20}$

$\Rightarrow \frac{27-x+x}{x(27-x)}=\frac{3}{20}$

$\Rightarrow \frac{27}{27x-x^2}=\frac{3}{20}$

$\Rightarrow 27x-x^2=180$

$\Rightarrow x^2-27x+180=0$

$\Rightarrow x^2-15x-12x+180=0$

$\Rightarrow x(x-15)-12(x-15)=0$

$\Rightarrow (x-12)(x-15)=0$

$\Rightarrow x-12=0$ or $x-15=0$

$\Rightarrow x=12$ or $x=15$

When $x=12$,

$27-x=27-12=15$

When $x=15$,

$27-x=27-15=12$

Hence, the required parts are 12 and 15.

20. Divide 16 into two parts such that twice the square of the larger part exceeds the square of the smaller part by 164.

Sol:

Let the larger and smaller parts be x and y, respectively.

According to the question:

$x+y=16$ ....(i)

$2x^2=y^2+164$ ....(ii)

From (i), we get:

$x=16-y$ ......(iii)

From (ii) and (iii), we get:

$2(16-y)^2 = y^2 + 164$

$\Rightarrow 2(256-32y+y^2) = y^2 + 164$

$\Rightarrow 512-64y+2y^2 = y^2 + 164$

$\Rightarrow y^2 - 64y + 348 = 0$

$\Rightarrow y^2 - (58+6)y + 348 = 0$

$\Rightarrow y^2 - 58y - 6y + 348 = 0$

$\Rightarrow y(y-58) - 6(y-58) = 0$

$\Rightarrow (y-58)(y-6) = 0$

$\Rightarrow y-58 = 0 \ or \ y-6 = 0$

$\Rightarrow y = 6 (\because y < 16)$

Putting the value of y in equation (iii), we get

$x = 16 - 6 = 10$

Hence, the two natural numbers are 6 and 10.

21. Divide two natural numbers, the sum of whose squares is 25 times their sum and also equal to 50 times their difference.

Sol:

Let the two natural numbers be x and y.

According to the question:

$x^2 + y^2 = 25(x+y)$ ......(i)

$x^2 + y^2 = 50(x-y)$ .....(ii)

From (i) and (ii), we get:

$25(x+y) = 50(x-y)$

$\Rightarrow x + y = 2(x-y)$

$\Rightarrow x + y = 2x - 2y$

$\Rightarrow y + 2y = 2x - x$

$\Rightarrow 3y = x$ ....(iii)

From (ii) and (iii), we get:

$(3y)^2 + y^2 = 50(3y - y)$

$\Rightarrow 9y^2 + y^2 = 100y$

$\Rightarrow 10y^2 = 100y$

$\Rightarrow y = 10$

From (iii), we have:

$3 \times 10 = x$

$\Rightarrow 30 = x$

Hence, the two natural numbers are 30 and 10.

22. The difference of the squares of two natural numbers is 45. The square of the smaller number is four times the larger number. Find the numbers.

Sol:

Let the greater number be x and the smaller number be y.

According to the question:

$x^2 - y^2 = 45$ ....(i)

$y^2 = 4x$ ....(ii)

From (i) and (ii), we get:

$x^2 - 4x = 45$

$\Rightarrow x^2 - 4x - 45 = 0$

$\Rightarrow x^2 - (9-5)x - 45 = 0$

$\Rightarrow x^2 - 9x + 5x - 45 = 0$

$\Rightarrow x(x-9) + 5(x-9) = 0$

$\Rightarrow (x-9)(x+5) = 0$

$\Rightarrow x - 9 = 0 \; or \; x + 5 = 0$

$\Rightarrow x = 9 \; or \; x = -5$

$\Rightarrow x = 9$ ($\because x$ is a natural number)

Putting the value of x in equation (ii), we get:

$y^2 = 4 \times 9$

$\Rightarrow y^2 = 36$

$\Rightarrow y = 6$

Hence, the two numbers are 9 and 6.

23. Three consecutive positive integers are such that the sum of the square of the first and product of the other two is 46. Find the integers.

Sol:

Let the three consecutive positive integers be $x, x+1$ and $x+2$.

According to the given condition,

$x^2 + (x+1)(x+2) = 46$

$\Rightarrow x^2 + x^2 + 3x + 2 = 46$

$\Rightarrow 2x^2 + 3x - 44 = 0$

$\Rightarrow 2x^2 + 11x - 8x - 44 = 0$

$\Rightarrow x(2x+11)-4(2x+11)=0$

$\Rightarrow (2x+11)(x-4)=0$

$\Rightarrow 2x+11=0 \text{ or } x-4=0$

$\Rightarrow x=-\frac{11}{2} \text{ or } x=4$

$\therefore x=4$ (x is a positive integer)

When $x=4$,

$x+1=4+1=5$

$x+2=4+2=6$

Hence, the required integers are 4, 5 and 6.

24. A two-digit number is 4 times the sum of its digits and twice the product of digits. Find the number.

Sol:

Let the digits at units and tens places be x and y, respectively.

Original number $=10y+x$

According to the question:

$10y+x=4(x+y)$

$\Rightarrow 10y+x=4x+4y$

$\Rightarrow 3x-6y=0$

$\Rightarrow 3x=6y$

$\Rightarrow x=2y$ ......(i)

Also,

$10y+x=2xy$

$\Rightarrow 10y+2y=2.2y.y$ $[From(i)]$

$\Rightarrow 12y=4y^2$

$\Rightarrow y=3$

From (i), we get:

$x=2\times 3=6$

$\therefore$ Original number $=10\times 3+6=36$

25. A two-digit number is such that the product of its digits is 14. If 45 is added to the number, the digit interchange their places. Find the number.

Sol:

Let the digits at units and tens places be x and y, respectively.

$\therefore xy=14$

$\Rightarrow y = \frac{14}{x}$ ....(i)

According to the question:

$(10y + x) + 45 = 10x + y$

$\Rightarrow 9y - 9x = -45$

$\Rightarrow y - x = -5$ ....(ii)

From (i) and (ii), we get:

$\frac{14}{x} - x = -5$

$\Rightarrow \frac{14 - x^2}{x} = -5$

$\Rightarrow 14 - x^2 = -5x$

$\Rightarrow x^2 - 5x - 14 = 0$

$\Rightarrow x^2 - (7 - 2)x - 14 = 0$

$\Rightarrow x^2 - 7x + 2x - 14 = 0$

$\Rightarrow x(x - 7) + 2(x - 7) = 0$

$\Rightarrow (x - 7)(x + 2) = 0$

$\Rightarrow x - 7 = 0$ *or* $x + 2 = 0$

$\Rightarrow x = 7$ *or* $x = -2$

$\Rightarrow x = 7$ ($\because$ the digit cannot be negative)

Putting $x = 7$ in equation (i), we get:

$y = 2$

$\therefore$ Required number $= 10 \times 2 + 7 = 27$

26. The denominator of a fraction is 3 more than its numerator. The sum of the fraction and its reciprocal is $2\frac{9}{10}$. Find the fraction.

Sol:

Let the numerator be x.

$\therefore$ Denominator $= x + 3$

$\therefore$ Original number $= \frac{x}{x+3}$

According to the question:

$\frac{x}{x+3} + \frac{1}{\left(\frac{x}{x+3}\right)} = 2\frac{9}{10}$

$\Rightarrow \frac{x}{x+3} + \frac{x+3}{x} = \frac{29}{10}$

$\Rightarrow \frac{x^2 + (x+3)^2}{x(x+3)} = \frac{29}{10}$

$\Rightarrow \frac{x^2 + x^2 + 6x + 9}{x^2 + 3x} = \frac{29}{10}$

$\Rightarrow \frac{2x^2 + 6x + 9}{x^2 + 3x} = \frac{29}{10}$

$\Rightarrow 29x^2 + 87x = 20x^2 + 60x + 90$

$\Rightarrow 9x^2 + 27x - 90 = 0$

$\Rightarrow 9(x^2 + 3x - 10) = 0$

$\Rightarrow x^2 + 3x - 10 = 0$

$\Rightarrow x^2 + 5x - 2x - 10 = 0$

$\Rightarrow x(x+5) - 2(x+5) = 0$

$\Rightarrow (x-2)(x+5) = 0$

$\Rightarrow x - 2 = 0$ *or* $x + 5 = 0$

$\Rightarrow x = 2$ *or* $x = -5$ (rejected)

So, number $= x = 2$

denominator $= x + 3 = 2 + 3 = 5$

So, required fraction $= \frac{2}{5}$

27. The numerator of a fraction is 3 less than its denominator. If 1 is added to the denominator, the fraction is decreased by $\frac{1}{15}$. Find the fraction.

Sol:

Let the denominator of the required fraction be x.

Numerator of the required fraction $= x - 3$

$\therefore$ Original fraction $= \frac{x-3}{x}$

If 1 is added to the denominator, then the new fraction obtained is $\frac{x-3}{x+1}$

According to the given condition,

$\frac{x-3}{x+1} = \frac{x-3}{x} - \frac{1}{15}$

$\Rightarrow \frac{x-3}{x} - \frac{x-3}{x+1} = \frac{1}{15}$

$\Rightarrow \frac{(x-3)(x+1) - x(x-3)}{x(x+1)} = \frac{1}{15}$

$\Rightarrow \frac{x^2 - 2x - 3 - x^2 + 3x}{x^2 + x} = \frac{1}{15}$

$\Rightarrow \frac{x-3}{x^2+x} = \frac{1}{15}$

$\Rightarrow x^2 + x = 15x - 45$

$\Rightarrow x^2 - 14x + 45 = 0$

$\Rightarrow x^2 - 9x - 5x + 45 = 0$

$\Rightarrow x(x-9) - 5(x-9) = 0$

$\Rightarrow (x-5)(x-9) = 0$

$\Rightarrow x - 5 = 0 \; or \; x - 9 = 0$

$\Rightarrow x = 5 \; or \; x = 9$

When $x = 5$,

$\frac{x-3}{x} = \frac{5-3}{5} = \frac{2}{5}$

When $x = 9$,

$\frac{x-3}{x} = \frac{9-3}{9} = \frac{6}{9} = \frac{2}{3}$ (This fraction is neglected because this does not satisfies the given condition.)

Hence, the required fraction is $\frac{2}{5}$.

28. The sum of a number and its reciprocal is $2\frac{1}{30}$. Find the number.

Sol:

Let the required number be x.

According to the given condition,

$x + \frac{1}{x} = 2\frac{1}{30}$

$\Rightarrow \frac{x^2+1}{x} = \frac{61}{30}$

$\Rightarrow 30x^2 + 30 = 61x$

$\Rightarrow 30x^2 - 61x + 30 = 0$

$\Rightarrow 30x^2 - 36x - 25x + 30 = 0$

$\Rightarrow 6x(5x-6) - 5(5x-6) = 0$

$\Rightarrow (5x-6)(6x-5) = 0$

$\Rightarrow 5x - 6 = 0 \ or \ 6x - 5 = 0$

$\Rightarrow x = \frac{6}{5} \ or \ x = \frac{5}{6}$

Hence, the required number is $\frac{5}{6}$ or $\frac{6}{5}$.

29. A teacher on attempting to arrange the students for mass drill in the form of solid square found that 24 students were left. When he increased the size of the square by one student, he found that he was short of 25 students. Find the number of students.

Sol:

Let there be x rows.

Then, the number of students in each row will also be x.

$\therefore$ Total number of students $= (x^2 + 24)$

According to the question:

$(x+1)^2 - 25 = x^2 + 24$

$\Rightarrow x^2 + 2x + 1 - 25 - x^2 - 24 = 0$

$\Rightarrow 2x - 48 = 0$

$\Rightarrow 2x = 48$

$\Rightarrow x = 24$

$\therefore$ Total number of students $= 24^2 + 24 = 576 + 24 = 600$

30. 300 apples are distributed equally among a certain number of students. Had there been 10 more students, each would have received one apple less. Find the number of students.

Sol:

Let the total number of students be x.

According to the question:

$\frac{300}{x} - \frac{300}{x+10} = 1$

$\Rightarrow \frac{300(x+10) - 300x}{x(x+10)} = 1$

$\Rightarrow \frac{300x + 3000 - 300x}{x^2 + 10x} = 1$

$\Rightarrow 3000 = x^2 + 10x$

$\Rightarrow x^2 + 10x - 3000 = 0$

$\Rightarrow x^2 + (60 - 50)x = 3000 = 0$

$\Rightarrow x^2 + 60x - 50x - 3000 = 0$

$\Rightarrow x(x + 60) - 50(x + 60) = 0$

$\Rightarrow (x + 60)(x - 50) = 0$

$\Rightarrow x = 50 \text{ or } x = -60$

x cannot be negative; therefore, the total number of students is 50.

31. In a class test, the sum of Kamal's marks in mathematics and English is 40. Had he got 3 marks more in mathematics and 4 marks less in English, the product of the marks would have been 360. Find his marks in two subjects separately.

Sol:

Let the marks of Kamal in mathematics and English be x and y, respectively.

According to the question:

$x + y = 40 \quad \text{......}(i)$

Also,

$(x + 3)(y - 4) = 360$

$\Rightarrow (x + 3)(40 - x - 4) = 360 \quad$ [From (i)]

$\Rightarrow (x + 3)(36 - x) = 360$

$\Rightarrow 36x - x^2 + 108 - 3x = 360$

$\Rightarrow 33x - x^2 - 252 = 0$

$\Rightarrow -x^2 + 33x - 252 = 0$

$\Rightarrow x^2 - 33x - 252 = 0$

$\Rightarrow x^2 - (21 + 12)x + 252 = 0$

$\Rightarrow x^2 - 21x - 12x + 252 = 0$

$\Rightarrow x(x - 21) - 12(x - 21) = 0$

$\Rightarrow (x - 21)(x - 12) = 0$

$\Rightarrow x = 21 \text{ or } x = 12$

If $x = 21$,

$y = 40 - 21 = 119$

Thus, Kamal scored 21 and 19 marks in mathematics and English, respectively.

If $x = 12$,

$y = 40 - 12 = 28$

Thus, Kamal scored 12 and 28 marks in mathematics and English, respectively.

32. Some students planned a picnic. The total budget for food was ₹ 2000. But, 5 students failed to attend the picnic and thus the cost for food for each member increased by ₹ 20. How many students attended the picnic and how much did each student pay for the food?

Sol:

Let x be the number of students who planned a picnic.

$\therefore$ Original cost of food for each member = ₹ $\frac{2000}{x}$

Five students failed to attend the picnic. So, $(x-5)$ students attended the picnic.

$\therefore$ New cost of food for each member = ₹ $\frac{2000}{(x-5)}$

According of the given condition,

$₹\frac{2000}{x-5} - ₹\frac{2000}{x} = ₹\,20$

$\Rightarrow \frac{2000x - 2000x + 10000}{x(x-5)} = 20$

$\Rightarrow \frac{10000}{x^2 - 5x} = 20$

$\Rightarrow x^2 - 5x = 500$

$\Rightarrow x^2 - 5x - 500 = 0$

$\Rightarrow x^2 - 25x + 20x - 500 = 0$

$\Rightarrow x(x-25) + 20(x-25) = 0$

$\Rightarrow (x-25)(x+20) = 0$

$\Rightarrow x - 25 = 0 \; or \; x + 20 = 0$

$\Rightarrow x = 25 \; or \; x = -20$

$\therefore x = 25$ (Number of students cannot be negative)

Number of students who attended the picnic $= x - 5 = 23 - 5 = 20$

Amount paid by each student for the food = ₹ $\frac{2000}{(25-5)}$ = ₹ $\frac{2000}{20}$ = ₹ 100

33. If the price of a book is reduced by ₹ 5, a person can buy 4 more books for ₹ 600. Find the original price of the book.

Sol:

Let the original price of the book be ₹ x.

$\therefore$ Number of books bought at original price for ₹ 600 $= \frac{600}{x}$

If the price of a book is reduced by ₹ 5, then the new price of the book is ₹ $(x-5)$.

$\therefore$ Number of books bought at reduced price for ₹ 600 $= \frac{600}{x-5}$

According to the given condition,

$\frac{600}{x-5} - \frac{600}{x} = 4$

$\Rightarrow \frac{600x - 600x + 3000}{x(x-5)} = 4$

$\Rightarrow \frac{3000}{x^2 - 5x} = 4$

$\Rightarrow x^2 - 5x = 750$

$\Rightarrow x^2 - 5x - 750 = 0$

$\Rightarrow x^2 - 30x + 25x - 750 = 0$

$\Rightarrow x(x-30) + 25(x-30) = 0$

$\Rightarrow x - 30 = 0$ *or* $x + 25 = 0$

$\Rightarrow x = 30$ *or* $x = -25$

$\therefore x = 30$ (Price cannot be negative)

Hence, the original price of the book is ₹30.

34. A person on tour has ₹ 10800 for his expenses. If he extends his tour by 4 days, he has to cut down his daily expenses by ₹ 90. Find the original duration of the tour.

Sol:

Let the original duration of the tour be x days.

$\therefore$ Original daily expenses = ₹ $\frac{10,800}{x}$

If he extends his tour by 4 days, then his new daily expenses =₹ $\frac{10,800}{x+4}$

According to the given condition,

₹ $\frac{10,800}{x}$ − ₹ $\frac{10,800}{x+4}$ = ₹ 90

$\Rightarrow \frac{10800x + 43200 - 10800x}{x(x+4)} = 90$

$\Rightarrow \frac{43200}{x^2 + 4x} = 90$

$\Rightarrow x^2 + 4x = 480$
$\Rightarrow x^2 + 4x - 480 = 0$
$\Rightarrow x^2 + 24x - 20x - 480 = 0$
$\Rightarrow x(x+24) - 20x - 480 = 0$
$\Rightarrow x(x+24) - 20(x+24) = 0$
$\Rightarrow (x+24)(x-20) = 0$
$\Rightarrow x + 24 = 0 \; or \; x - 20 = 0$
$\Rightarrow x = -24 \; or \; x = 20$

$\therefore x = 20$ (Number of days cannot be negative)

Hence, the original duration of the tour is 20 days.

35. In a class test, the sum of the marks obtained by P in mathematics and science is 28. Had he got 3 more marks in mathematics and 4 marks less in science, the product of marks obtained in the two subjects would have been 180. Find the marks obtained by him in the two subjects separately.

Sol:

Let the marks obtained by P in mathematics and science be x and $(28-x)$, respectively.

According to the given condition,

$(x+3)(28-x-4) = 180$
$\Rightarrow (x+3)(24-x) = 180$
$\Rightarrow -x^2 + 21x + 72 = 180$
$\Rightarrow x^2 - 21x + 108 = 0$
$\Rightarrow x^2 - 12x - 9x + 108 = 0$
$\Rightarrow x(x-12) - 9(x-12) = 0$
$\Rightarrow (x-12)(x-9) = 0$
$\Rightarrow x - 12 = 0 \; or \; x - 9 = 0$
$\Rightarrow x = 12 \; or \; x = 9$

When $x = 12$,

$28 - x = 28 - 12 = 16$

When $x = 9$,

$28 - x = 28 - 9 = 19$

Hence, he obtained 12 marks in mathematics and 16 marks in science or 9 marks in mathematics and 19 marks in science.

36. A man buys a number of pens for ₹ 180. If he had bought 3 more pens for the same amount, each pen would have cost him ₹ 3 less. How many pens did he buy?

Sol:

Let the total number of pens be x.

According to the question:

$$\frac{80}{x}-\frac{80}{x+4}=1$$

$$\Rightarrow \frac{80(x+4)-80x}{x(x+4)}=1$$

$$\Rightarrow \frac{80+320-80x}{x^2+4x}=1$$

$$\Rightarrow 320=x^2+4x$$

$$\Rightarrow x^2+4x-320=0$$

$$\Rightarrow x^2+(20-16)x-320=0$$

$$\Rightarrow x^2+20x-16x-320=0$$

$$\Rightarrow x(x+20)-16(x+20)=0$$

$$\Rightarrow (x+20)(x-16)=0$$

$$\Rightarrow x=-20 \text{ or } x=16$$

The total number of pens cannot be negative; therefore, the total number of pens is 16.

37. A dealer sells an article for ₹ 75 and gains as much per cent as the cost priced of the article. Find the cost price of the article.

Sol:

Let the cost price of the article be x

$\therefore$ Gain percent $=x\%$

According to the given condition,

$$₹\,x+₹\left(\frac{x}{100}\times x\right)= ₹75 \qquad \text{(Cost price + Gain = Selling price)}$$

$$\Rightarrow \frac{100x+x^2}{100}=75$$

$$\Rightarrow x^2+100x=7500$$

$$\Rightarrow x^2+100x-7500=0$$

$$\Rightarrow x^2+150x-50x-7500=0$$

$$\Rightarrow x(x+150)-50(x+150)=0$$

$$\Rightarrow (x-50)(x+150)=0$$

$$\Rightarrow x-50=0 \text{ or } x+150=0$$

$\Rightarrow x=50 \text{ or } x=-150$

$\therefore x=50$ (Cost price cannot be negative)

Hence, the cost price of the article is ₹50.

38. One year ago, man was 8 times as old as his son. Now, his age is equal to the square of his son's age. Find their present ages.

Sol:

Let the present age of the son be x years.

$\therefore$ Present age of the man $=x^2$ years

One year ago,

Age of the son $=(x-1)$ years

Age of the man $=(x^2-1)$ years

According to the given condition,

Age of the man $=8\times$ Age of the son

$\therefore x^2-1=8(x-1)$

$\Rightarrow x^2-1=8x-8$

$\Rightarrow x^2-8x+7=0$

$\Rightarrow x^2-7x-x+7=0$

$\Rightarrow x(x-7)-1(x-7)=0$

$\Rightarrow (x-1)(x-7)=0$

$\Rightarrow x-1=0 \text{ or } x-7=0$

$\Rightarrow x=1 \text{ or } x=7$

$\therefore x=7$ (Man's age cannot be 1 year)

Present age of the son $=7$ years

Present age of the man $=7^2$ years $=49$ years.

39. The sum of reciprocals of Meena's ages (in years) 3 years ago and 5 years hence $\frac{1}{3}$. Find her present ages.

Sol:

Let the present age of Meena be x years

Meena's age 3 years ago $=(x-3)$ years

Meena's age 5 years hence $=(x+5)$ years

According to the given condition,

$$\frac{1}{x-3}+\frac{1}{x+5}=\frac{1}{3}$$

$\Rightarrow \frac{x+5+x-3}{(x-3)(x+5)} = \frac{1}{3}$

$\Rightarrow \frac{2x+2}{x^2+2x-15} = \frac{1}{3}$

$\Rightarrow x^2+2x-15 = 6x+6$

$\Rightarrow x^2-4x-21 = 0$

$\Rightarrow x^2-7x+3x-21 = 0$

$\Rightarrow x(x-7)+3(x-7) = 0$

$\Rightarrow (x-7)(x+3) = 0$

$\Rightarrow x-7 = 0 \text{ or } x+3 = 0$

$\Rightarrow x = 7 \text{ or } x = -3$

$\therefore x = 7$ (Age cannot be negative)

Hence, the present age of Meena is 7 years.

40. The sum of the ages of a boy and his brother is 25 years, and the product of their ages in years is 126. Find their ages.

Sol:

Let the present ages of the boy and his brother be x years and $(25-x)$ years.

According to the question:

$x(25-x) = 126$

$\Rightarrow 25x - x^2 = 126$

$\Rightarrow x^2-(18-7)x+126 = 0$

$\Rightarrow x^2-18x-7x+126 = 0$

$\Rightarrow x(x-18)-7(x-18) = 0$

$\Rightarrow (x-18)(x-7) = 0$

$\Rightarrow x-18 = 0 \text{ or } x-7 = 0$

$\Rightarrow x = 18 \text{ or } x = 7$

$\Rightarrow x = 18$ ($\because$ Present age of the boy cannot be less than his brother)

If $x = 18$, we have

Present ages of the boy = 18 years

Present age of his brother $= (25-18)$ years = 7 years

Thus, the present ages of the boy and his brother are 18 years and 7 years, respectively.

41. The product of Tanvy's age (in years) 5 years ago and her age is 8 years later is 30. Find her present age.

Sol:

Let the present age of Meena be x years.

According to the question:

$(x-5)(x+8)=30$

$\Rightarrow x^2+3x-40=30$

$\Rightarrow x^2+3x-70=0$

$\Rightarrow x^2+(10-7)x-70=0$

$\Rightarrow x^2+10x-7x-70=0$

$\Rightarrow x(x+10)-7(x+10)=0$

$\Rightarrow (x+10)(x-7)=0$

$\Rightarrow x+10=0 \; or \; x-7=0$

$\Rightarrow x=-10 \; or \; x=7$

$\Rightarrow x=7$ ($\because$ Age cannot be negative)

Thus, the present age of Meena is 7 years.

42. Two years ago, man's age was three times the square of his son's age. In three years' time, his age will be four time his son's age. Find their present ages.

Sol:

Let son's age 2 years ago be x years. Then,

Man's age 2 years ago $=3x^2$ years

$\therefore$ Sons present age $=(x+2)$ years

Man's present age $=(3x^2+2)$ years

In three years time,

Son's age $=(x+2+3)$ years $=(x+5)$ years

Man's age $=(3x^2+2+3)$ years $=(3x^2+5)$ years

According to the given condition,

Man's age = $4\times$ Son's age

$\therefore 3x^2+5=4(x+5)$

$\Rightarrow 3x^2+5=4x+20$

$\Rightarrow 3x^2-4x-15=0$

$\Rightarrow 3x^2-9x+5x-15=0$

$\Rightarrow 3x(x-3)+5(x-3)=0$

$\Rightarrow (x-3)(3x+5)=0$

$\Rightarrow x-3=0 \text{ or } 3x+5=0$

$\Rightarrow x=3 \text{ or } x=-\frac{5}{3}$

$\therefore x=3$ (Age cannot be negative)

Son's present age $=(x+2)$ years $=(3+2)$ years $=5$ years

Man's present age $=(3x^2+2)$ years $=(3\times 9+2)$ years $=29$ years

43. A truck covers a distance of 150 km at a certain average speed and then covers another 200 km at an average speed which is 20 km per hour more than the first speed. If the truck covers the total distance in 5 hours, find the first speed of the truck.

Sol:

Let the first speed of the truck be $x\ km/h$.

$\therefore$ Time taken to cover 150 $km=\frac{150}{x}h$ $\left(Time=\frac{Distance}{Speed}\right)$

New speed of the truck $=(x+20)\,km/h$

$\therefore$ Time taken to cover 200 km $=\frac{200}{x+20}h$

According to the given condition,

Time taken to cover 150 km + Time taken to cover 200 km = 5 h

$\therefore \frac{150}{x}+\frac{200}{x+20}=5$

$\Rightarrow \frac{150x+3000+200x}{x(x+20)}=5$

$\Rightarrow 350x+3000=5(x^2+20x)$

$\Rightarrow 350x+3000=5x^2+100x$

$\Rightarrow 5x^2-250x-3000=0$

$\Rightarrow x^2-50x-600=0$

$\Rightarrow x^2-60x+10x-600=0$

$\Rightarrow x(x-60)+10(x-60)=0$

$\Rightarrow (x-60)(x+10)=0$

$\Rightarrow x-60=0 \text{ or } x+10=0$

$\Rightarrow x=60 \text{ or } x=-10$

$\therefore x=60$ (Speed cannot be negative)

Hence, the first speed of the truck is 60 km/h.

44. While boarding an aeroplane, a passengers got hurt. The pilot showing promptness and concern, made arrangements to hospitalize the injured and so the plane started late by 30 minutes. To reach the destination, 1500 km away, in time, the pilot increased the speed by 100 km/hour. Find the original speed of the plane.
Do you appreciate the values shown by the pilot, namely promptness in providing help to the injured and his efforts to reach in time?

Sol:

Let the original speed of the plane be x km/h.

$\therefore$ Actual speed of the plane $=(x+100)km/h$

Distance of the journey $=1500\ km$

Time taken to reach the destination at original speed $=\frac{1500}{x}h$ $\left(Time=\frac{Distance}{Speed}\right)$

Time taken to reach the destination at actual speed $=\frac{1500}{x+100}h$

According to the given condition,

Time taken to reach the destination at original speed = Time taken to reach the destination at actual speed + 30 min

$\therefore \frac{1500}{x}=\frac{1500}{x+100}+\frac{1}{2}$ $\left(30\ \text{min}=\frac{30}{60}h=\frac{1}{2}h\right)$

$\Rightarrow \frac{1500}{x}-\frac{1500}{x+100}=\frac{1}{2}$

$\Rightarrow \frac{1500x+150000-1500x}{x(x+100)}=\frac{1}{2}$

$\Rightarrow \frac{150000}{x^2+100x}=\frac{1}{2}$

$\Rightarrow x^2+100x=300000$

$\Rightarrow x^2+100x=300000=0$

$\Rightarrow x^2+600x-500x-300000=0$

$\Rightarrow x(x+600)-500(x+600)=0$

$\Rightarrow (x+600)(x-500)=0$

$\Rightarrow x+600=0\ or\ x-500=0$

$\Rightarrow x=-600\ or\ x=500$

$\therefore x=500$ (Speed cannot be negative)

Hence, the original speed of the plane is 500 km/h.

Yes, we appreciate the values shown by the pilot, namely promptness in providing help to the injured and his efforts to reach in time. This reflects the caring nature of the pilot and his dedication to the work.

45. A train covers a distance of 480 km at a uniform speed. If the speed had been 8 km/hr less then it would have taken 3 hours more to cover the same distance. Find the usual speed of the train.

Sol:

Let the usual speed of the train be x km/h.

$\therefore$ Reduced speed of the train $=(x-8)\,km/h$

Total distance to be covered = 480 km

Time taken by the train to cover the distance at usual speed $=\frac{480}{x}h$ $\left(Time=\frac{Distance}{Speed}\right)$

Time taken by the train to cover the distance at reduced speed $=\frac{480}{x-8}h$

According to the given condition,

Time taken by the train to cover the distance at reduced speed = Time taken by the train to cover the distance at usual speed + 3 h

$\therefore \frac{480}{x-8}=\frac{480}{x}+3$

$\Rightarrow \frac{480}{x-8}-\frac{480}{x}=3$

$\Rightarrow \frac{480x-480x+3840}{x(x-8)}=3$

$\Rightarrow \frac{3840}{x^2-8x}=3$

$\Rightarrow x^2-8x=1280$

$\Rightarrow x^2-8x-1280=0$

$\Rightarrow x(x-40)+32(x-40)=0$

$\Rightarrow (x-40)(x+32)=0$

$\Rightarrow x-40=0\ or\ x+32=0$

$\Rightarrow x=40\ or\ x=-32$

$\therefore x=40$ (Speed cannot be negative)

Hence, the usual speed of the train is 40 km/h.

46. A train travels at a certain average speed for a distanced of 54 km and then travels a distance of 63 km at an average speed of 6 km/hr more than the first speed. If it takes 3 hours to complete the total journey, what is its first speed?

Sol:

Let the first speed of the train be x km/h.

Time taken to cover $54\ km = \frac{54}{x} h$ $\left(Time = \frac{Distance}{Speed}\right)$

New speed of the train $= (x+6)\ km/h$

$\therefore$ Time taken to cover $63\ km = \frac{63}{x+6} h$

According to the given condition,

Time taken to cover 54 km + Time taken to cover 63 km = 3 h

$\therefore \frac{54}{x} + \frac{63}{x+6} = 3$

$\Rightarrow \frac{54x+324+63x}{x(x+6)} = 3$

$\Rightarrow 117x+324 = 3(x^2+6x)$

$\Rightarrow 117x+324 = 3x^2+18x$

$\Rightarrow 3x^2-99x-324 = 0$

$\Rightarrow x^2-33x-324 = 0$

$\Rightarrow x^2-33x-108 = 0$

$\Rightarrow x^2-36x+3x-108 = 0$

$\Rightarrow x(x-36)+3(x-36) = 0$

$\Rightarrow (x-36)(x+3) = 0$

$\Rightarrow x-36 = 0$ or $x+3 = 0$

$\Rightarrow x = 36$ or $x = -3$

$\therefore x = 36$ (Speed cannot be negative)

Hence, the first speed of the train is 36 km/h.

47. A train travels 180 km at a uniform speed. If the speed had been 9 km/hr more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Sol: 36km/hr

48. A train covers a distance of 90 km at a uniform speed. Had the speed been 15 km/hr more, it would have taken 30 minutes less for the journey. Find the original speed of the train.

Sol:

Let the original speed of the train be x km/hr.

According to the question:

$\frac{90}{x} - \frac{90}{(x+15)} = \frac{1}{2}$

$$\Rightarrow \frac{90(x+15)-90x}{x(x+15)}=\frac{1}{2}$$
$$\Rightarrow \frac{90x+1350-90x}{x^2+15x}=\frac{1}{2}$$
$$\Rightarrow \frac{1350}{x^2+15x}=\frac{1}{2}$$
$$\Rightarrow 2700=x^2+15x$$
$$\Rightarrow x^2+(60-45)x-2700=0$$
$$\Rightarrow x^2+60x-45x-2700=0$$
$$\Rightarrow x(x+60)-45x(x+60)=0$$
$$\Rightarrow (x+60)(x-45)=0$$
$$\Rightarrow x=-60 \ or \ x=45$$

x cannot be negative; therefore, the original speed of train is 45 km/hr.

49. A passenger train takes 2 hours less for a journey of 300 km if its speed is increased by 5 km/hr from its usual speed. Find its usual speed.

Sol:

Let the usual speed x km/hr.

According to the question:

$$\frac{300}{x}-\frac{300}{(x+5)}=2$$
$$\Rightarrow \frac{300(x+5)-300x}{x(x+5)}=2$$
$$\Rightarrow \frac{300x+1500-300x}{x^2+5x}=2$$
$$\Rightarrow 1500=2(x^2+5x)$$
$$\Rightarrow 1500=2x^2+10x$$
$$\Rightarrow x^2+5x-750=0$$
$$\Rightarrow x^2+(30-25)x-750=0$$
$$\Rightarrow x^2+30x-25x-750=0$$
$$\Rightarrow x(x+30)-25(x+30)=0$$
$$\Rightarrow (x+30)(x-25)=0$$
$$\Rightarrow x=-30 \ or \ x=25$$

The usual speed cannot be negative; therefore, the speed is 25 km/hr.

50. The distance between Mumbai and Pune is 192 km. Travelling by the Deccan Queen, it takes 48 minutes less than another train. Calculate the speed of the Deccan Queen if the speeds of the two train differ by 20km/hr.

Sol:

Let the speed of the Deccan Queen be x km/hr.

According to the question:

Speed of another train $=(x-20)\,km/hr$

$$\therefore \frac{192}{x-20}-\frac{192}{x}=\frac{48}{60}$$

$$\Rightarrow \frac{4}{x-20}-\frac{4}{x}=\frac{1}{60}$$

$$\Rightarrow \frac{4x-4(x-20)}{(x-20)x}=\frac{1}{60}$$

$$\Rightarrow \frac{4x-4x+80}{x^2-20x}=\frac{1}{60}$$

$$\Rightarrow \frac{80}{x^2-20x}=\frac{1}{60}$$

$$\Rightarrow x^2-20x=4800$$

$$\Rightarrow x^2-20x-4800=0$$

$$\Rightarrow x^2-(80-60)x-4800=0$$

$$\Rightarrow x^2-80x+60x-4800=0$$

$$\Rightarrow x(x-80)+60(x-80)=0$$

$$\Rightarrow (x-80)(x+60)=0$$

$$\Rightarrow x=80 \; or \; x=-60$$

The value of

x cannot be negative; therefore, the original speed of Deccan Queen 180 km/hr.

51. A motor boat whose speed in still water is 178 km/hr, takes 1 hour more to go 24 km upstream than to return to the same spot. Find the speed of the stream

Sol:

Let the speed of the stream be $x\ km/hr$.

Given:

Speed of the boat $=18\ km/hr$

$\therefore$ Speed downstream $=(18+x)\ km/hr$

Speed upstream $=(18-x)\ km/hr$

$$\therefore \frac{24}{(18-x)} - \frac{24}{(18-x)} = 1$$

$$\Rightarrow \frac{1}{(18-x)} - \frac{1}{(18+x)} = \frac{1}{24}$$

$$\Rightarrow \frac{18+x-18+x}{(18-x)(18+x)} = \frac{1}{24}$$

$$\Rightarrow \frac{2x}{18^2-x^2} = \frac{1}{24}$$

$$\Rightarrow 324 - x^2 = 48x$$

$$\Rightarrow 324 - x^2 - 48x = 0$$

$$\Rightarrow x^2 + 48x - 324 = 0$$

$$\Rightarrow x^2 + (54-6)x - 324 = 0$$

$$\Rightarrow x^2 + 54x - 6x - 324 = 0$$

$$\Rightarrow x(x+54) - 6(x+54) = 0$$

$$\Rightarrow (x+54)(x-6) = 0$$

$$\Rightarrow x = -54 \text{ or } x = 6$$

The value of x cannot be negative; therefore, the speed of the stream is 6 km/hr.

52. The speed of a boat in still water is 8 km/hr. It can go 15 km upstream and 22 km downstream is 5 hours. Fid the speed of the stream

Sol:

Speed of the boat in still water $= 8\ km/hr$.

Let the speed of the stream be x km/hr.

$\therefore$ Speed upstream $= (8-x)\,km/hr$.

Speed downstream $= (8+x)\,km/hr$.

Time taken to go 22 km downstream $= \frac{22}{(8+x)}\,hr$

Time taken to go 15 km upstream $= \frac{15}{(8-x)}\,hr$

According to the question:

$$\Rightarrow \frac{22}{(8+x)} + \frac{15}{(8-x)} = 5$$

$$\Rightarrow \frac{22}{(8+x)} + \frac{15}{(8-x)} - 5 = 0$$

$\Rightarrow \frac{22(8-x)+15(8+x)-5(8-x)(8+x)}{(8-x)(8+x)}=0$

$\Rightarrow 176-22x+120+15x-320+5x^2=0$

$\Rightarrow 5x^2-7x-24=0$

$\Rightarrow 5x^2-(15-8)x-24=0$

$\Rightarrow 5x^2-15x+8x-24=0$

$\Rightarrow 5x(x-3)-8(x-3)=0$

$\Rightarrow (x-3)(5x-8)=0$

$\Rightarrow x-3=0 \text{ or } 5x-8=0$

$\Rightarrow x=3 \text{ or } x=\frac{8}{5}$

$\Rightarrow x=3$ ($\because$ Speed cannot be a fraction)

$\therefore$ Speed of the stream $=3\ km/hr$

53. A motorboat whose speed is 9 km/hr in still water, goes 15 km downstream and comes back in a total time of 3 hours 45 minutes. Find the speed of the stream.

Sol:

Let the speed of the stream be x km/hr.

$\therefore$ Downstream speed $=(9+x)\ km/hr$.

Upstream speed $=(9-x)\ km/hr$

Distance covered downstream = Distance covered upstream = 15 km

Total time taken = 3hours 45 minutes $=\left(3+\frac{45}{60}\right)$ minutes $=\frac{225}{60}$ minutes $=\frac{15}{4}$ minutes

$\therefore \frac{15}{(9+x)}+\frac{15}{(9-x)}=\frac{15}{4}$

$\Rightarrow \frac{1}{(9+x)}+\frac{1}{(9-x)}=\frac{1}{4}$

$\Rightarrow \frac{9-x+9+x}{(9+x)(9-x)}=\frac{1}{4}$

$\Rightarrow \frac{18}{9^2-x^2}=\frac{1}{4}$

$\Rightarrow \frac{18}{81-x^2}=\frac{1}{4}$

$\Rightarrow 81-x^2=72$

$\Rightarrow 81-x^2-72=0$

$\Rightarrow -x^2+9=0$

$\Rightarrow x^2=9$

$\Rightarrow x=3 \text{ or } x=-3$

The value of x cannot be negative; therefore, the speed of the stream is 3 km/hr.

54. A takes 10 days less than the time taken by B to finish a piece of work. If both A and B together can finish the work in 12 days, find the time taken by B to finish the work.

Sol:

Let B takes x days to complete the work.

Therefore, A will take $(x-10)$ days.

$\therefore \frac{1}{x}+\frac{1}{(x-10)}=\frac{1}{12}$

$\Rightarrow \frac{(x-10)+x}{x(x-10)}=\frac{1}{12}$

$\Rightarrow \frac{2x-10}{x^2-10x}=\frac{1}{12}$

$\Rightarrow x^2-10x=12(2x-10)$

$\Rightarrow x^2-10x=24x-120$

$\Rightarrow x^2-34x+120=0$

$\Rightarrow x^2-(30+4)x+120=0$

$\Rightarrow x^2-30x-4x+120=0$

$\Rightarrow x(x-30)-4(x-30)=0$

$\Rightarrow (x-30)(x-4)=0$

$\Rightarrow x=30 \text{ or } x=4$

Number of days to complete the work by

B cannot be less than that by A; therefore, we get: $x=30$

Thus, B completes the work in 30 days.

55. Two pipes running together can fill a cistern in $3\frac{1}{13}$ minutes. If one pipe takes 3 minutes more than the other to fill it, find the time in which each pipe would fill the cistern.

Sol:

Let one pipe fills the cistern in x mins.

Therefore, the other pipe will fill the cistern in $(x+3)$ mins.

Time taken by both, running together, to fill the cistern $=3\frac{1}{13} \min s=\frac{40}{13} \min s$

Part filled by one pipe in 1 min $= \frac{1}{x}$

Part filled by the other pipe in 1 min $= \frac{1}{x+3}$

Part filled by both pipes, running together, in 1 min $= \frac{1}{x} + \frac{1}{x+3}$

$$\therefore \frac{1}{x} + \frac{1}{x+3} = \frac{1}{\frac{40}{13}}$$

$$\Rightarrow \frac{(x+3)+x}{x(x+3)} = \frac{13}{40}$$

$$\Rightarrow \frac{2x+3}{x^2+3a} = \frac{13}{40}$$

$$\Rightarrow 13x^2 + 39x = 80x + 120$$

$$\Rightarrow 13x^2 - 41x - 120 = 0$$

$$\Rightarrow 13x^2 - (65-24)x - 120 = 0$$

$$\Rightarrow 13x^2 - 65x + 24x - 120 = 0$$

$$\Rightarrow 13x(x-5) + 24(x-5) = 0$$

$$\Rightarrow (x-5)(13x+24) = 0$$

$$\Rightarrow x - 5 = 0 \ or \ 13x + 24 = 0$$

$$\Rightarrow x = 5 \ or \ x = \frac{-24}{13}$$

$\Rightarrow x = 5$ ($\because$ Speed cannot be a negative fraction)

Thus, one pipe will take 5 mins and other will take $\{(5+3)=8\}$ mins to fill the cistern.

56. Two pipes running together can fill a tank in $11\frac{1}{9}$ minutes. If on pipe takes 5 minutes more than the other to fill the tank separately, find the time in which each pipe would fill the tank separately.

Sol:

Let the time taken by one pipe to fill the tank be x minutes.

$\therefore$ Time taken by the other pipe to fill the tank $= (x+5)$ min

Suppose the volume of the tank be V.

Volume of the tank filled by one pipe in x minutes = V

$\therefore$ Volume of the tank filled by one pipe in 1 minute $= \frac{V}{x}$

$\Rightarrow$ Volume of the tank filled by one pipe in $11\frac{1}{9}$ minutes $= \frac{V}{x} \times 11\frac{1}{9} = \frac{V}{x} \times \frac{100}{9}$

Similarly,

Volume of the tank filled by the other pipe in $11\frac{1}{9}$ minutes $= \frac{V}{(x+5)} \times 11\frac{1}{9} = \frac{V}{(x+5)} \times \frac{100}{9}$

Now,

Volume of the tank filled by one pipe in $11\frac{1}{9}$ minutes + Volume of the tank filled by the other pipe in $11\frac{1}{9}$ minutes $= V$

$\therefore V\left(\frac{1}{x} + \frac{1}{x+5}\right) \times 100 = V$

$\Rightarrow \frac{1}{x} + \frac{1}{x+5} = \frac{9}{100}$

$\Rightarrow \frac{x+5+x}{x(x+5)} = \frac{9}{100}$

$\Rightarrow \frac{2x+5}{x^2+5x} = \frac{9}{100}$

$\Rightarrow 200x + 500 = 9x^2 + 45x$

$\Rightarrow 9x^2 - 155x - 500 = 0$

$\Rightarrow 9x^2 - 180x + 25x - 500 = 0$

$\Rightarrow 9x(x-20) + 25(x-20) = 0$

$\Rightarrow (x-20)(9x+25) = 0$

$\Rightarrow x - 20 = 0 \text{ or } 9x + 25 = 0$

$\Rightarrow x = 20 \text{ or } x = -\frac{25}{9}$

$\therefore x = 20$ (Time cannot be negative)

Time taken by one pipe to fill the tank = 20 min

Time taken by other pipe to fill the tank = (20 + 5) 25 min

57. Two water taps together can fill a tank in 6 hours. The tap of larger diameter takes 9 hours less than the smaller one to fill the tank separately. Find the time which each tap can separately fill the tank.

Sol:

Let the tap of smaller diameter fill the tank in x hours.

$\therefore$ Time taken by the tap of larger diameter to fill the tank $= (x-9)h$

Suppose the volume of the tank be V.

Volume of the tank filled by the tap of smaller diameter in x hours = V

$\therefore$ Volume of the tank filled by the tap of smaller diameter in 1 hour $= \dfrac{V}{x}$

$\Rightarrow$ Volume of the tank filled by the tap of smaller diameter in 6 hours $= \dfrac{V}{x} \times 6$

Similarly,

Volume of the tank filled by the tap of larger diameter in 6 hours $= \dfrac{V}{(x-9)} \times 6$

Now,

Volume of the tank filled by the tap of smaller diameter in 6 hours + Volume of the tank filled by the tap of larger diameter in 6 hours = V

$$\therefore V\left(\frac{1}{x}+\frac{1}{x-9}\right)\times 6 = V$$

$$\Rightarrow \frac{1}{x}+\frac{1}{x-9}=\frac{1}{6}$$

$$\Rightarrow \frac{x-9+x}{x(x-9)}=\frac{1}{6}$$

$$\Rightarrow \frac{2x-9}{x^2-9x}=\frac{1}{6}$$

$$\Rightarrow 12x-54=x^2-9x$$

$$\Rightarrow x^2-21x+54=0$$

$$\Rightarrow x^2-81x-3x+54=0$$

$$\Rightarrow x(x-18)-3(x-18)=0$$

$$\Rightarrow (x-18)(x-3)=0$$

$$\Rightarrow x-18=0 \ or \ x-3=0$$

$$\Rightarrow x=18 \ or \ x=3$$

For $x=3$, time taken by the tap of larger diameter to fill the tank is negative which is not possible.

$\therefore x=18$

Time taken by the tap of smaller diameter to fill the tank = 18 h

Time taken by the tap of larger diameter to fill the tank $=(18-9)=9h$

Hence, the time taken by the taps of smaller and larger diameter to fill the tank is 18 hours and 9 hours, respectively.

58. The length of rectangle is twice its breadth and its areas is 288 cm $288cm^2$. Find the dimension of the rectangle.

Sol:

Let the length and breadth of the rectangle be $2x$ m and x m, respectively.

According to the question:

$2x \times x = 288$

$\Rightarrow 2x^2 = 288$

$\Rightarrow x^2 = 144$

$\Rightarrow x = 12 \text{ or } x = -12$

$\Rightarrow x = 12$ ($\because x$ cannot be negative)

$\therefore \text{Length} = 2 \times 12 = 24\,m$

Breath $= 12\,m$

59. The length of a rectangular field is three times its breadth. If the area of the field be 147 sq meters, find the length of the field.

Sol:

Let the length and breadth of the rectangle be $3x$ m and x m, respectively.

According to the question:

$3x \times x = 147$

$\Rightarrow 3x^2 = 147$

$\Rightarrow x^2 = 49$

$\Rightarrow x = 7 \text{ or } x = -7$

$\Rightarrow x = 7$ ($\because x$ cannot be negative)

$\therefore \text{Length} = 3 \times 7 = 21\,m$

Breath $= 7\,m$

60. The length of a hall is 3 meter more than its breadth. If the area of the hall is 238 sq meters, calculate its length and breadth.

Sol:

Let the breath of the rectangular hall be x meter.

Therefore, the length of the rectangular hall will be $(x+3)$ meter.

According to the question:

$x(x+3) = 238$

$\Rightarrow x^2 + 3x = 238$

$\Rightarrow x^2 + 3x - 238 = 0$

$\Rightarrow x^2 + (17-14)x - 238 = 0$

$\Rightarrow x^2 + 17x - 14x - 238 = 0$

$\Rightarrow x(x+17) - 14(x+17) = 0$

$\Rightarrow (x+17)(x-14) = 0$

$\Rightarrow x = -17 \text{ or } x = 14$

But the value x cannot be negative.
Therefore, the breath of the hall is 14 meter and the length is 17 meter.

61. The perimeter of a rectangular plot is 62 m and its area is 288 sq meters. Find the dimension of the plot

Sol:
Let the length and breadth of the rectangular plot be x and y meter, respectively.
Therefore, we have:

Perimeter $=2(x+y)=62$ .....(i) and

Area $=xy=228$

$\Rightarrow y=\frac{228}{x}$

Putting the value of y in (i), we get

$\Rightarrow 2\left(x+\frac{228}{x}\right)=62$

$\Rightarrow x+\frac{228}{x}=31$

$\Rightarrow \frac{x^2+228}{x}=31$

$\Rightarrow x^2+228=31x$

$\Rightarrow x^2-31x+228=0$

$\Rightarrow x^2-(19+12)x+228=0$

$\Rightarrow x^2-19x-12x+228=0$

$\Rightarrow x(x-19)-12(x-19)=0$

$\Rightarrow (x-19)(x-12)=0$

$\Rightarrow x=19 \; or \; x=12$

If $x=19m, y=\frac{228}{19}=12m$

Therefore, the length and breadth of the plot are 19 m and 12 m, respectively.

62. A rectangular filed in 16 m long and 10 m wide. There is a path of uniform width all around it, having an area of $120m^2$. Find the width of the path

Sol:
Let the width of the path be x m.
$\therefore$ Length of the field including the path $=16+x+x=16+2x$
Breadth of the field including the path $=10+x+x=10+2x$
Now,

(Area of the field including path) - (Area of the field excluding path) = Area of the path

$\Rightarrow (16+2x)(10+2x)-(16\times 10)=120$

$\Rightarrow 160+32x+20x+4x^2-160=120$

$\Rightarrow 4x^2+52x-120=0$

$\Rightarrow x^2+13x-30=0$

$\Rightarrow x^2+(15-2)x+30=0$

$\Rightarrow x^2+15x-2x+30=0$

$\Rightarrow x(x+15)-2(x+15)=0$

$\Rightarrow (x-2)(x+15)=0$

$\Rightarrow x-2=0 \text{ or } x+15=0$

$\Rightarrow x=2 \text{ or } x=-15$

$\Rightarrow x=2$ ($\because$ Width cannot be negative)

Thus, the width of the path is 2 m.

63. The sum of the areas of two squares is 640 m^2. If the difference in their perimeter be 64m, find the sides of the two square

Sol:

Let the length of the side of the first and the second square be x and y. respectively.

According to the question:

$x^2+y^2=640$ ....(i)

Also,

$4x-4y=64$

$\Rightarrow x-y=16$

$\Rightarrow x=16+y$

Putting the value of x in (i), we get:

$x^2+y^2=640$

$\Rightarrow (16+y)^2+y^2=640$

$\Rightarrow 256+32y+y^2+y^2=640$

$\Rightarrow 2y^2+32y-384=0$

$\Rightarrow y^2+16y-192=0$

$\Rightarrow y^2+(24-8)y-192=0$

$\Rightarrow y^2+24y-8y-192=0$

$\Rightarrow y(y+24)-8(y+24)=0$

$\Rightarrow (y+24)(y-8)=0$

$\Rightarrow y=-24 \ or \ y=8$

$\therefore y=8$ ($\because$ Side cannot be negative)

$\therefore x=16+y=16+8=24 \ m$

Thus, the sides of the squares are 8 m and 24 m.

64. The length of a rectangle is thrice as long as the side of a square. The side of the square is 4 cm more than the width of the rectangle. Their areas being equal, find the dimensions.

Sol:

Let the breadth of rectangle be x cm.

According to the question:

Side of the square $=(x+4)cm$

Length of the rectangle = $\{3(x+4)\}$ cm

It is given that the areas of the rectangle and square are same.

$\therefore 3(x+4)\times x=(x+4)^2$

$\Rightarrow 3x^2+12x=(x+4)^2$

$\Rightarrow 3x^2+12x=x^2+8x+16$

$\Rightarrow 2x^2+4x-16=0$

$\Rightarrow x^2+2x-8=0$

$\Rightarrow x^2+(4-2)x-8=0$

$\Rightarrow x^2+4x-2x-8=0$

$\Rightarrow x(x+4)-2(x+4)=0$

$\Rightarrow (x+4)(x-2)=0$

$\Rightarrow x=-4 \ or \ x=2$

$\therefore x=2$ ($\because$ The value of x cannot be negative)

Thus, the breadth of the rectangle is 2 cm and length is $\{3(2+4)=18\}$ cm.

Also, the side of the square is 6 cm.

65. A farmer prepares rectangular vegetable garden of area 180 sq meters. With 39 meters of barbed wire, he can fence the three sides of the garden, leaving one of the longer sides unfenced. Find the dimensions of the garden.

Sol:

Let the length and breadth of the rectangular garden be x and y meter, respectively.

Given:

$xy=180 \ sq \ m$ ...(i) and