

EXERCISE 4.1

Find the cubes of the following numbers:

(i) 7 (ii) 12

(iii) 16 (iv) 21

(v) 40 (vi) 55

(vii) 100 (viii) 302

(ix) 301

Solution:

(i) 7

Cube of 7 is

$$7 = 7 \times 7 \times 7 = 343$$

(ii) 12

Cube of 12 is

$$12 = 12 \times 12 \times 12 = 1728$$

(iii) 16

Cube of 16 is

$$16 = 16 \times 16 \times 16 = 4096$$

(iv) 21

Cube of 21 is

$$21 = 21 \times 21 \times 21 = 9261$$

(v) 40

Cube of 40 is

$$40 = 40 \times 40 \times 40 = 64000$$

(vi) 55

Cube of 55 is

$$55 = 55 \times 55 \times 55 = 166375$$

(vii) 100

Cube of 100 is

$$100 = 100 \times 100 \times 100 = 1000000$$

(viii) 302

Cube of 302 is



$$302 = 302 \times 302 \times 302 = 27543608$$

(ix) 301

Cube of 301 is

$$301 = 301 \times 301 \times 301 = 27270901$$

1. Write the cubes of all natural numbers between 1 and 10 and verify the following statements:

(i) Cubes of all odd natural numbers are odd.

(ii) Cubes of all even natural numbers are even.

Solutions:

Firstly let us find the Cube of natural numbers up to 10

$$1^3 = 1 \times 1 \times 1 = 1$$

$$2^3 = 2 \times 2 \times 2 = 8$$

$$3^3 = 3 \times 3 \times 3 = 27$$

$$4^3 = 4 \times 4 \times 4 = 64$$

$$5^3 = 5 \times 5 \times 5 = 125$$

$$6^3 = 6 \times 6 \times 6 = 216$$

$$7^3 = 7 \times 7 \times 7 = 343$$

$$8^3 = 8 \times 8 \times 8 = 512$$

$$9^3 = 9 \times 9 \times 9 = 729$$

$$10^3 = 10 \times 10 \times 10 = 1000$$

∴ From the above results we can say that

(i) Cubes of all odd natural numbers are odd.

(ii) Cubes of all even natural numbers are even.

2. Observe the following pattern:

$$1^3 = 1$$

$$1^3 + 2^3 = (1+2)^2$$

$$1^3 + 2^3 + 3^3 = (1+2+3)^2$$

Write the next three rows and calculate the value of $1^3 + 2^3 + 3^3 + \dots + 9^3$ by the above pattern.

Solution:

According to given pattern,

$$1^3 + 2^3 + 3^3 + \dots + 9^3$$

$$1^3 + 2^3 + 3^3 + \dots + n^3 = (1+2+3+\dots+n)^2$$

So when $n = 10$

$$\begin{aligned} 1^3 + 2^3 + 3^3 + \dots + 9^3 + 10^3 &= (1+2+3+\dots+10)^2 \\ &= (55)^2 = 55 \times 55 = 3025 \end{aligned}$$

3. Write the cubes of 5 natural numbers which are multiples of 3 and verify the followings:

“The cube of a natural number which is a multiple of 3 is a multiple of 27”

Solution:

We know that the first 5 natural numbers which are multiple of 3 are 3, 6, 9, 12 and 15

So now, let us find the cube of 3, 6, 9, 12 and 15

$$3^3 = 3 \times 3 \times 3 = 27$$

$$6^3 = 6 \times 6 \times 6 = 216$$

$$9^3 = 9 \times 9 \times 9 = 729$$

$$12^3 = 12 \times 12 \times 12 = 1728$$

$$15^3 = 15 \times 15 \times 15 = 3375$$

We found that all the cubes are divisible by 27

\therefore “The cube of a natural number which is a multiple of 3 is a multiple of 27”

4. Write the cubes of 5 natural numbers which are of the form $3n + 1$ (e.g. 4, 7, 10, ...) and verify the following:

“The cube of a natural number of the form $3n+1$ is a natural number of the same form i.e. when divided by 3 it leaves the remainder 1”

Solution:

We know that the first 5 natural numbers in the form of $(3n + 1)$ are 4, 7, 10, 13 and 16

So now, let us find the cube of 4, 7, 10, 13 and 16

$$4^3 = 4 \times 4 \times 4 = 64$$

$$7^3 = 7 \times 7 \times 7 = 343$$

$$10^3 = 10 \times 10 \times 10 = 1000$$

$$13^3 = 13 \times 13 \times 13 = 2197$$

$$16^3 = 16 \times 16 \times 16 = 4096$$

We found that all these cubes when divided by ‘3’ leaves remainder 1.

\therefore the statement “The cube of a natural number of the form $3n+1$ is a natural number of the same form i.e. when divided by 3 it leaves the remainder 1” is true.

5. Write the cubes 5 natural numbers of the form $3n+2$ (i.e. 5, 8, 11, ...) and verify the following:

“The cube of a natural number of the form $3n+2$ is a natural number of the same form i.e. when it is divided by 3 the remainder is 2”

Solution:

We know that the first 5 natural numbers in the form $(3n + 2)$ are 5, 8, 11, 14 and 17

So now, let us find the cubes of 5, 8, 11, 14 and 17

$$5^3 = 5 \times 5 \times 5 = 125$$

$$8^3 = 8 \times 8 \times 8 = 512$$

$$11^3 = 11 \times 11 \times 11 = 1331$$

$$14^3 = 14 \times 14 \times 14 = 2744$$

$$17^3 = 17 \times 17 \times 17 = 4913$$

We found that all these cubes when divided by '3' leaves remainder 2.

∴ the statement "The cube of a natural number of the form $3n+2$ is a natural number of the same form i.e. when it is dividend by 3 the remainder is 2' is true.

6. Write the cubes of 5 natural numbers of which are multiples of 7 and verity the following:

"The cube of a multiple of 7 is a multiple of 7^3 .

Solution:

The first 5 natural numbers which are multiple of 7 are 7, 14, 21, 28 and 35

So, the Cube of 7, 14, 21, 28 and 35

$$7^3 = 7 \times 7 \times 7 = 343$$

$$14^3 = 14 \times 14 \times 14 = 2744$$

$$21^3 = 21 \times 21 \times 21 = 9261$$

$$28^3 = 28 \times 28 \times 28 = 21952$$

$$35^3 = 35 \times 35 \times 35 = 42875$$

We found that all these cubes are multiples of $7^3(343)$ as well.

∴ The statement "The cube of a multiple of 7 is a multiple of 7^3 is true.

7. Which of the following are perfect cubes?

(i) 64 (ii) 216

(iii) 243 (iv) 1000

(v) 1728 (vi) 3087

(vii) 4608 (viii) 106480

(ix) 166375 (x) 456533

Solution:

(i) 64

First find the factors of 64

$$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6 = (2^2)^3 = 4^3$$

Hence, it's a perfect cube.

(ii) 216

First find the factors of 216

$$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^3 \times 3^3 = 6^3$$

Hence, it's a perfect cube.

(iii) 243

First find the factors of 243

$$243 = 3 \times 3 \times 3 \times 3 \times 3 = 3^5 = 3^3 \times 3^2$$

Hence, it's not a perfect cube.

(iv) 1000

First find the factors of 1000

$$1000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5 = 2^3 \times 5^3 = 10^3$$

Hence, it's a perfect cube.

(v) 1728

First find the factors of 1728

$$1728 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^6 \times 3^3 = (4 \times 3)^3 = 12^3$$

Hence, it's a perfect cube.

(vi) 3087

First find the factors of 3087

$$3087 = 3 \times 3 \times 7 \times 7 \times 7 = 3^2 \times 7^3$$

Hence, it's not a perfect cube.

(vii) 4608

First find the factors of 4608

$$4608 = 2 \times 2 \times 3 \times 113$$

Hence, it's not a perfect cube.

(viii) 106480

First find the factors of 106480

$$106480 = 2 \times 2 \times 2 \times 2 \times 5 \times 11 \times 11 \times 11$$

Hence, it's not a perfect cube.

(ix) 166375

First find the factors of 166375

$$166375 = 5 \times 5 \times 5 \times 11 \times 11 \times 11 = 5^3 \times 11^3 = 55^3$$

Hence, it's a perfect cube.

(x) 456533

First find the factors of 456533

$$456533 = 11 \times 11 \times 11 \times 7 \times 7 \times 7 = 11^3 \times 7^3 = 77^3$$

Hence, it's a perfect cube.

8. Which of the following are cubes of even natural numbers?**216, 512, 729, 1000, 3375, 13824****Solution:**

(i) $216 = 2^3 \times 3^3 = 6^3$

It's a cube of even natural number.

(ii) $512 = 2^9 = (2^3)^3 = 8^3$

It's a cube of even natural number.

(iii) $729 = 3^3 \times 3^3 = 9^3$

It's not a cube of even natural number.

(iv) $1000 = 10^3$

It's a cube of even natural number.

(v) $3375 = 3^3 \times 5^3 = 15^3$

It's not a cube of even natural number.

(vi) $13824 = 2^9 \times 3^3 = (2^3)^3 \times 3^3 = 8^3 \times 3^3 = 24^3$

It's a cube of even natural number.

9. Which of the following are cubes of odd natural numbers?**125, 343, 1728, 4096, 32768, 6859****Solution:**

(i) $125 = 5 \times 5 \times 5 = 5^3$

It's a cube of odd natural number.

(ii) $343 = 7 \times 7 \times 7 = 7^3$

It's a cube of odd natural number.

(iii) $1728 = 2^6 \times 3^3 = 4^3 \times 3^3 = 12^3$

It's not a cube of odd natural number. As 12 is even number.

(iv) $4096 = 2^{12} = (2^6)^2 = 64^2$

Its not a cube of odd natural number. As 64 is an even number.

(v) $32768 = 2^{15} = (2^5)^3 = 32^3$

It's not a cube of odd natural number. As 32 is an even number.

(vi) $6859 = 19 \times 19 \times 19 = 19^3$

It's a cube of odd natural number.

10. What is the smallest number by which the following numbers must be multiplied, so that the products are perfect cubes?

(i) 675 (ii) 1323

(iii) 2560 (iv) 7803

(v) 107811 (vi) 35721

Solution:

(i) 675

First find the factors of 675

$$\begin{aligned} 675 &= 3 \times 3 \times 3 \times 5 \times 5 \\ &= 3^3 \times 5^2 \end{aligned}$$

∴ To make a perfect cube we need to multiply the product by 5.

(ii) 1323

First find the factors of 1323

$$\begin{aligned} 1323 &= 3 \times 3 \times 3 \times 7 \times 7 \\ &= 3^3 \times 7^2 \end{aligned}$$

∴ To make a perfect cube we need to multiply the product by 7.

(iii) 2560

First find the factors of 2560

$$\begin{aligned} 2560 &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \\ &= 2^8 \times 5 \end{aligned}$$

∴ To make a perfect cube we need to multiply the product by $5 \times 5 = 25$.

(iv) 7803

First find the factors of 7803

$$\begin{aligned} 7803 &= 3 \times 3 \times 3 \times 17 \times 17 \\ &= 3^3 \times 17^2 \end{aligned}$$

∴ To make a perfect cube we need to multiply the product by 17.

(v) 107811

First find the factors of 107811

$$\begin{aligned} 107811 &= 3 \times 3 \times 3 \times 3 \times 11 \times 11 \times 11 \\ &= 3^4 \times 11^3 \end{aligned}$$

∴ To make a perfect cube we need to multiply the product by $3 \times 3 = 9$.

(vi) 35721

First find the factors of 35721

$$\begin{aligned} 35721 &= 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 7 \times 7 \\ &= 3^3 \times 3^3 \times 7^2 \end{aligned}$$

∴ To make a perfect cube we need to multiply the product by 7.

11. By which smallest number must the following numbers be divided so that the quotient is a perfect cube?

(i) 675 (ii) 8640

(iii) 1600 (iv) 8788

(v) 7803 (vi) 107811

(vii) 35721 (viii) 243000

Solution:

(i) 675

First find the prime factors of 675

$$\begin{aligned} 675 &= 3 \times 3 \times 3 \times 5 \times 5 \\ &= 3^3 \times 5^2 \end{aligned}$$

Since 675 is not a perfect cube.

To make the quotient a perfect cube we divide it by $5^2 = 25$, which gives 27 as quotient where, 27 is a perfect cube.

∴ 25 is the required smallest number.

(ii) 8640

First find the prime factors of 8640

$$\begin{aligned} 8640 &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \\ &= 2^3 \times 2^3 \times 3^3 \times 5 \end{aligned}$$

Since 8640 is not a perfect cube.

To make the quotient a perfect cube we divide it by 5, which gives 1728 as quotient and we know that 1728 is a perfect cube.

∴ 5 is the required smallest number.

(iii) 1600

First find the prime factors of 1600

$$\begin{aligned} 1600 &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \\ &= 2^3 \times 2^3 \times 5^2 \end{aligned}$$

Since 1600 is not a perfect cube.

To make the quotient a perfect cube we divide it by $5^2 = 25$, which gives 64 as quotient and we know that 64 is a perfect cube

∴ 25 is the required smallest number.

(iv) 8788

First find the prime factors of 8788

$$8788 = 2 \times 2 \times 13 \times 13 \times 13 \\ = 2^2 \times 13^3$$

Since 8788 is not a perfect cube.

To make the quotient a perfect cube we divide it by 4, which gives 2197 as quotient and we know that 2197 is a perfect cube

\therefore 4 is the required smallest number.

(v) 7803

First find the prime factors of 7803

$$7803 = 3 \times 3 \times 3 \times 17 \times 17 \\ = 3^3 \times 17^2$$

Since 7803 is not a perfect cube.

To make the quotient a perfect cube we divide it by $17^2 = 289$, which gives 27 as quotient and we know that 27 is a perfect cube

\therefore 289 is the required smallest number.

(vi) 107811

First find the prime factors of 107811

$$107811 = 3 \times 3 \times 3 \times 3 \times 11 \times 11 \times 11 \\ = 3^3 \times 11^3 \times 3$$

Since 107811 is not a perfect cube.

To make the quotient a perfect cube we divide it by 3, which gives 35937 as quotient and we know that 35937 is a perfect cube.

\therefore 3 is the required smallest number.

(vii) 35721

First find the prime factors of 35721

$$35721 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 7 \times 7 \\ = 3^3 \times 3^3 \times 7^2$$

Since 35721 is not a perfect cube.

To make the quotient a perfect cube we divide it by $7^2 = 49$, which gives 729 as quotient and we know that 729 is a perfect cube

\therefore 49 is the required smallest number.

(viii) 243000

First find the prime factors of 243000

$$243000 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5$$

$$= 2^3 \times 3^3 \times 5^3 \times 3^2$$

Since 243000 is not a perfect cube.

To make the quotient a perfect cube we divide it by $3^2 = 9$, which gives 27000 as quotient and we know that 27000 is a perfect cube

$\therefore 9$ is the required smallest number.

12. Prove that if a number is trebled then its cube is 27 time the cube of the given number.

Solution:

Let us consider a number as a

So the cube of the assumed number is $= a^3$

Now, the number is trebled $= 3 \times a = 3a$

So the cube of new number $= (3a)^3 = 27a^3$

\therefore New cube is 27 times of the original cube.

Hence, proved.

13. What happens to the cube of a number if the number is multiplied by

(i) 3?

(ii) 4?

(iii) 5?

Solution:

(i) 3?

Let us consider the number as a

So its cube will be $= a^3$

According to the question, the number is multiplied by 3

New number becomes $= 3a$

So the cube of new number will be $= (3a)^3 = 27a^3$

Hence, number will become 27 times the cube of the number.

(ii) 4?

Let us consider the number as a

So its cube will be $= a^3$

According to the question, the number is multiplied by 4

New number becomes $= 4a$

So the cube of new number will be $= (4a)^3 = 64a^3$

Hence, number will become 64 times the cube of the number.

(iii) 5?

Let us consider the number as a

So its cube will be = a^3

According to the question, the number is multiplied by 5

New number becomes = $5a$

So the cube of new number will be = $(5a)^3 = 125a^3$

Hence, number will become 125 times the cube of the number.

14. Find the volume of a cube, one face of which has an area of 64m^2 .

Solution:

We know that the given area of one face of cube = 64 m^2

Let the length of edge of cube be 'a' metre

$$a^2 = 64$$

$$a = \sqrt{64}$$

$$= 8\text{m}$$

Now, volume of cube = a^3

$$a^3 = 8^3 = 8 \times 8 \times 8$$

$$= 512\text{m}^3$$

∴ Volume of a cube is 512m^3

15. Find the volume of a cube whose surface area is 384m^2 .

Solution:

We know that the surface area of cube = 384 m^2

Let us consider the length of each edge of cube be 'a' meter

$$6a^2 = 384$$

$$a^2 = 384/6$$

$$= 64$$

$$a = \sqrt{64}$$

$$= 8\text{m}$$

Now, volume of cube = a^3

$$a^3 = 8^3 = 8 \times 8 \times 8$$

$$= 512\text{m}^3$$

∴ Volume of a cube is 512m^3

16. Evaluate the following:

(i) $\{(5^2 + 12^2)^{1/2}\}^3$

(ii) $\{(6^2 + 8^2)^{1/2}\}^3$

Solution:

(i) $\{(5^2 + 12^2)^{1/2}\}^3$

When simplified above equation we get,

$$\begin{aligned} & \{(25 + 144)^{1/2}\}^3 \\ & \{(169)^{1/2}\}^3 \\ & \{(13^2)^{1/2}\}^3 \\ & (13)^3 \\ & 2197 \end{aligned}$$

(ii) $\{(6^2 + 8^2)^{1/2}\}^3$

When simplified above equation we get,

$$\begin{aligned} & \{(36 + 64)^{1/2}\}^3 \\ & \{(100)^{1/2}\}^3 \\ & \{(10^2)^{1/2}\}^3 \\ & (10)^3 \\ & 1000 \end{aligned}$$

17. Write the units digit of the cube of each of the following numbers:

31, 109, 388, 4276, 5922, 77774, 44447, 125125125

Solution:

31

To find unit digit of cube of a number we perform the cube of unit digit only.

Unit digit of 31 is 1

Cube of 1 = $1^3 = 1$

∴ Unit digit of cube of 31 is always 1

109

To find unit digit of cube of a number we perform the cube of unit digit only.

Unit digit of 109 is = 9

Cube of 9 = $9^3 = 729$

∴ Unit digit of cube of 109 is always 9

388

To find unit digit of cube of a number we perform the cube of unit digit only.

Unit digit of 388 is = 8

Cube of 8 = $8^3 = 512$

∴ Unit digit of cube of 388 is always 2

4276

To find unit digit of cube of a number we perform the cube of unit digit only.

Unit digit of 4276 is = 6

Cube of 6 = $6^3 = 216$

∴ Unit digit of cube of 4276 is always 6

5922

To find unit digit of cube of a number we perform the cube of unit digit only.

Unit digit of 5922 is = 2

Cube of 2 = $2^3 = 8$

∴ Unit digit of cube of 5922 is always 8

77774

To find unit digit of cube of a number we perform the cube of unit digit only.

Unit digit of 77774 is = 4

Cube of 4 = $4^3 = 64$

∴ Unit digit of cube of 77774 is always 4

44447

To find unit digit of cube of a number we perform the cube of unit digit only.

Unit digit of 44447 is = 7

Cube of 7 = $7^3 = 343$

∴ Unit digit of cube of 44447 is always 3

125125125

To find unit digit of cube of a number we perform the cube of unit digit only.

Unit digit of 125125125 is = 5

Cube of 5 = $5^3 = 125$

∴ Unit digit of cube of 125125125 is always 5

18. Find the cubes of the following numbers by column method:

(i) 35

(ii) 56

(iii) 72

Solution:

(i) 35

We have, $a = 3$ and $b = 5$

Column I a^3	Column II $3 \times a^2 \times b$	Column III $3 \times a \times b^2$	Column IV b^3
$3^3 = 27$	$3 \times 9 \times 5 = 135$	$3 \times 3 \times 25 = 225$	$5^3 = 125$
+15	+23	+12	12 <u>5</u>
<u>42</u>	<u>158</u>	<u>237</u>	
42	8	7	5

∴ The cube of 35 is 42875

(ii) 56

We have, $a = 5$ and $b = 6$

Column I a^3	Column II $3 \times a^2 \times b$	Column III $3 \times a \times b^2$	Column IV b^3
$5^3 = 125$	$3 \times 25 \times 6 = 450$	$3 \times 5 \times 36 = 540$	$6^3 = 216$
+50	+56	+21	<u>12</u> 6
<u>175</u>	50 <u>6</u>	56 <u>1</u>	
175	6	1	6

∴ The cube of 56 is 175616

(iii) 72

We have, $a = 7$ and $b = 2$

Column I a^3	Column II $3 \times a^2 \times b$	Column III $3 \times a \times b^2$	Column IV b^3
$7^3 = 343$	$3 \times 49 \times 2 = 294$	$3 \times 7 \times 4 = 84$	$2^3 = 8$
+30	+8	+0	<u>8</u>
<u>373</u>	30 <u>2</u>	8 <u>4</u>	
373	2	4	8

∴ The cube of 72 is 373248

19. Which of the following numbers are not perfect cubes?

(i) 64

(ii) 216

(iii) 243

(iv) 1728

Solution:

(i) 64

Firstly let us find the prime factors of 64

$$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

$$= 2^3 \times 2^3$$

$$= 4^3$$

Hence, it's a perfect cube.

(ii) 216

Firstly let us find the prime factors of 216

$$\begin{aligned} 216 &= 2 \times 2 \times 2 \times 3 \times 3 \times 3 \\ &= 2^3 \times 3^3 \\ &= 6^3 \end{aligned}$$

Hence, it's a perfect cube.

(iii) 243

Firstly let us find the prime factors of 243

$$\begin{aligned} 243 &= 3 \times 3 \times 3 \times 3 \times 3 \\ &= 3^3 \times 3^2 \end{aligned}$$

Hence, it's not a perfect cube.

(iv) 1728

Firstly let us find the prime factors of 1728

$$\begin{aligned} 1728 &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \\ &= 2^3 \times 2^3 \times 3^3 \\ &= 12^3 \end{aligned}$$

Hence, it's a perfect cube.

20. For each of the non-perfect cubes in Q. No 20 find the smallest number by which it must be

(a) Multiplied so that the product is a perfect cube.

(b) Divided so that the quotient is a perfect cube.

Solution:

Only non-perfect cube in previous question was = 243

(a) Multiplied so that the product is a perfect cube.

Firstly let us find the prime factors of 243

$$243 = 3 \times 3 \times 3 \times 3 \times 3 = 3^3 \times 3^2$$

Hence, to make it a perfect cube we should multiply it by 3.

(b) Divided so that the quotient is a perfect cube.

Firstly let us find the prime factors of 243

$$243 = 3 \times 3 \times 3 \times 3 \times 3 = 3^3 \times 3^2$$

Hence, to make it a perfect cube we have to divide it by 9.

21. By taking three different, values of n verify the truth of the following statements:

(i) If n is even, then n^3 is also even.

(ii) If n is odd, then n^3 is also odd.

(iii) If n leaves remainder 1 when divided by 3, then n^3 also leaves 1 as remainder when divided by 3.

(iv) If a natural number n is of the form $3p+2$ then n^3 also a number of the same type.

Solution:

(i) If n is even, then n^3 is also even.

Let us consider three even natural numbers 2, 4, 6

So now, Cubes of 2, 4 and 6 are

$$2^3 = 8$$

$$4^3 = 64$$

$$6^3 = 216$$

Hence, we can see that all cubes are even in nature.

Statement is verified.

(ii) If n is odd, then n^3 is also odd.

Let us consider three odd natural numbers 3, 5, 7

So now, cubes of 3, 5 and 7 are

$$3^3 = 27$$

$$5^3 = 125$$

$$7^3 = 343$$

Hence, we can see that all cubes are odd in nature.

Statement is verified.

(iii) If n leaves remainder 1 when divided by 3, then n^3 also leaves 1 as remainder when divided by 3.

Let us consider three natural numbers of the form $(3n+1)$ are 4, 7 and 10

So now, cube of 4, 7, 10 are

$$4^3 = 64$$

$$7^3 = 343$$

$$10^3 = 1000$$

We can see that if we divide these numbers by 3, we get 1 as remainder in each case.

Hence, statement is verified.

(iv) If a natural number n is of the form $3p+2$ then n^3 also a number of the same type.

Let us consider three natural numbers of the form $(3p+2)$ are 5, 8 and 11

So now, cube of 5, 8 and 10 are

$$5^3 = 125$$

$$8^3 = 512$$

$$11^3 = 1331$$

Now, we try to write these cubes in form of $(3p + 2)$

$$125 = 3 \times 41 + 2$$

$$512 = 3 \times 170 + 2$$

$$1331 = 3 \times 443 + 2$$

Hence, statement is verified.

22. Write true (T) or false (F) for the following statements:

(i) 392 is a perfect cube.

(ii) 8640 is not a perfect cube.

(iii) No cube can end with exactly two zeros.

(iv) There is no perfect cube which ends in 4.

(v) For an integer a , a^3 is always greater than a^2 .

(vi) If a and b are integers such that $a^2 > b^2$, then $a^3 > b^3$.

(vii) If a divides b , then a^3 divides b^3 .

(viii) If a^2 ends in 9, then a^3 ends in 7.

(ix) If a^2 ends in an even number of zeros, then a^3 ends in 25.

(x) If a^2 ends in an even number of zeros, then a^3 ends in an odd number of zeros.

Solution:

(i) 392 is a perfect cube.

Firstly let's find the prime factors of $392 = 2 \times 2 \times 2 \times 7 \times 7 = 2^3 \times 7^2$

Hence the statement is False.

(ii) 8640 is not a perfect cube.

Prime factors of $8640 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 = 2^3 \times 2^3 \times 3^3 \times 5$

Hence the statement is True

(iii) No cube can end with exactly two zeros.

Statement is True.

Because a perfect cube always have zeros in multiple of 3.

(iv) There is no perfect cube which ends in 4.

We know 64 is a perfect cube $= 4 \times 4 \times 4$ and it ends with 4.

Hence the statement is False.

(v) For an integer a , a^3 is always greater than a^2 .

Statement is False.

Because in case of negative integers ,

$$(-2)^2 = 4 \text{ and } (-2)^3 = -8$$

(vi) If a and b are integers such that $a^2 > b^2$, then $a^3 > b^3$.

Statement is False.

In case of negative integers,

$$(-5)^2 > (-4)^2 = 25 > 16$$

But, $(-5)^3 > (-4)^3 = -125 > -64$ is not true.

(vii) If a divides b, then a^3 divides b^3 .

Statement is True.

If a divides b

$$b/a = k, \text{ so } b = ak$$

$$b^3/a^3 = (ak)^3/a^3 = a^3k^3/a^3 = k^3,$$

For each value of b and a its true.

(viii) If a^2 ends in 9, then a^3 ends in 7.

Statement is False.

Let $a = 7$

$$7^2 = 49 \text{ and } 7^3 = 343$$

(ix) If a^2 ends in an even number of zeros, then a^3 ends in 25.

Statement is False.

Since, when $a = 20$

$$a^2 = 20^2 = 400 \text{ and } a^3 = 8000 \text{ (} a^3 \text{ doesn't end with 25)}$$

(x) If a^2 ends in an even number of zeros, then a^3 ends in an odd number of zeros.

Statement is False.

Since, when $a = 100$

$$a^2 = 100^2 = 10000 \text{ and } a^3 = 100^3 = 1000000 \text{ (} a^3 \text{ doesn't end with odd number of zeros)}$$