NCERT Solutions for Class-XI Maths

Chapter-2 Exercise-Miscellaneous

NCERT Math Class 11

1. The relation f is defined by $f(x)=\begin{cases}x^2, & 0\le x\le 3\\ 3x, & 3\le x\le 10\end{cases}$

The relation g is defined by $g(x)=\begin{cases}x^2, 0\le x\le 2\\ 3x, 2\le x\le 10\end{cases}$

Show that f is a function and g is not a function.

1. The relation f is defined as

$$f(x)=\begin{cases}x^2, & 0\le x\le 3\\ 3x, & 3\le x\le 10\end{cases}$$

It is observed that for

$0\le x<3,\ f(x)=x^2$

$3<x\le 10,\ f(x)=3x$

Also, at $x=3, f(x)=3^2=9$ or $f(x)=3\times 3=9$ i.e., at $x=3, f(x)=9$

Therefore, for $0\le x\le 10$, the images of $f(x)$ are unique. Thus, the given relation is a function.

The relation g is defined as

$$g(x)=\begin{cases}x^2, 0\le x\le 2\\ 3x, 2\le x\le 10\end{cases}$$

It can be observed that for $x=2, g(x)=2^2=4$ and $g(x)=3\times 2=6$

Hence, element 2 of the domain of the relation g corresponds to two different images i.e., 4 and 6.

Hence, this relation is not a function.

2. If $f(x) = x^2$, find $\dfrac{f(1.1)-f(1)}{(1.1-1)}$

2. **Given:** $f(x) = x^2$

Then $\dfrac{f(1.1)-f(1)}{(1.1-1)}=\dfrac{(1.1)^2-(1)^2}{0.1}$

$= \frac{1.21-1}{0.1} = \frac{0.21}{0.1} = 2.1$

3. Find the domain of the function $f(x) = \frac{x^2+2x+1}{x^2-8x+12}$

3. The given function is $f(x) = \frac{x^2+2x+1}{x^2-8x+12}$

$f(x) = \frac{x^2+2x+1}{x^2-8x+12} = \frac{x^2+2x+1}{(x-6)(x-2)}$

It can be seen that function f is defined for all real numbers except at $x = 6$ and $x = 2$.

Hence, the domain of f is $\mathbf{R} - \{2, 6\}$.

4. Find the domain and the range of the real function f defined by $f(x) = \sqrt{(x-1)}$.

4. **Given:** $f(x) = \sqrt{(x-1)}$

$\sqrt{(x-1)}$ is defined for $x \geq 1$.

Hence, the domain of f is the set of all real numbers greater than or equal to 1 i.e., the domain of f = [1, ∞).

As $x \geq 1 \Rightarrow (x-1) \geq 0 \Rightarrow \sqrt{(x-1)} \geq 0$

Hence, the Range of f is the set of all real numbers greater than or equal to 0 i.e., the Range of f = [0, ∞).

5. Find the domain and the range of the real function f defined by

$$f(x) = |x-1|$$

5. The given real function is $f(x) = |x-1|$.

It is clear that $|x-1|$ is defined for all real numbers.

$\therefore$ Domain of $f = \mathbf{R}$

Also, for $x \in \mathbf{R}, |x-1|$ assumes all real numbers.

Hence, the range of f is the set of all non-negative real numbers.

6. Let $f = \left\{\left(x, \frac{x^2}{1+x^2}\right) : x\varepsilon R\right\}$ be a function from R into R. Determine the range of f.

6. **Given:** $f = \left\{\left(x, \frac{x^2}{1+x^2}\right) : x \in R\right\}$

$$f = \left\{ \left(x, \frac{x^2}{1+x^2} \right) : x \in R \right\}$$

$$f = \{(0,0), \left(\pm 0.5, \frac{1}{5}\right), \left(\pm 1, \frac{1}{2}\right), \left(\pm 1.5, \frac{9}{13}\right), \left(\pm 2, \frac{4}{5}\right), \left(3, \frac{9}{10}\right), \left(4, \frac{16}{17}\right), \ldots\ldots$$

As we know the range of f is the set of all second elements and in above series we can see that all the second elements are greater than or equal to 0 but are less than 1. Thus the range of f is [0, 1).

7. Let $f, g : \mathbf{R} \to \mathbf{R}$ be defined, respectively by $f(x) = x + 1, g(x) = 2x - 3$. Find $f + g, f - g$ and $\frac{f}{g}$.

7. $f, g : \mathbf{R} \to \mathbf{R}$ is defined as $f(x) = x + 1, g(x) = 2x - 3$

$(f + g)(x) = f(x) + g(x) = (x + 1) + (2x - 3) = 3x - 2$

$\therefore (f + g)(x) = 3x - 2$

$(f - g)(x) = f(x) - g(x) = (x + 1) - (2x - 3) = x + 1 - 2x + 3 = -x + 4$

$\therefore (f - g)(x) = -x + 4$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0, x \in \mathbf{R}$

$\therefore \left(\frac{f}{g}\right)(x) = \frac{x+1}{2x-3}, 2x - 3 \neq 0 \text{ or} 2x \neq 3$

$\therefore \left(\frac{f}{g}\right)(x) = \frac{x+1}{2x-3}, x \neq \frac{3}{2}$

8. Let f = {(1,1), (2,3), (0,–1), (–1, –3)} be a function from Z to Z defined by f(x) = ax + b, for some integers a, b. Determine a, b.

8. **Given:** f = {(1,1), (2,3), (0,–1), (–1, –3)}

f(x) = ax + b

(1,1) Є f ⇒ for x=1, f(x) = 1

⇒ 1 = a(1) + b

⇒ a+b =1 …. (1)

Similarly, (0,-1) Є f ⇒ for x=0, f(x) = -1

⇒ -1 = a(0) + b

⇒ b =-1

⇒ a-1 =1 (from 1)

⇒ a =2.

Hence a=2 and b=-1.

9. Let R be a relation from $\mathbf{N}$ to $\mathbf{N}$ defined by $\mathbf{R}=\{(a,b): a,b \in \mathbf{N} \text{ and } a = b^2\}$. Are the following true?

(i) $(a,a) \in \mathbf{R}$, for all $a \in \mathbf{N}$

(ii) $(a,b) \in R$, implies $(b,a) \in R$

(iii) $(a,b) \in R, (b,c) \in R$ implies $(a,c) \in R$.

Justify your answer in each case.

9. $R=\{(a,b): a,b \in \mathbf{N} \text{ and } a=b^2\}$

(i) It can be seen that $2 \in \mathbf{N}$; however, $2 \neq 2^2 = 4$.

Therefore, the statement " $(a,a) \in \mathbf{R}$, for all $a \in \mathbf{N}''$ is not true.

(ii) It can be seen that $(9,3) \in \mathbf{N}$ because $9,3 \in \mathbf{N}$ and $9=3^2$. Now, 3 $\neq 9^2 = 81$; therefore, $(3,9) \notin \mathbf{N}$

Therefore, the statement " $(a,b) \in R$, implies $(b,a) \in R''$ is not true.

(iii) It can be seen that $(9,3) \in R, (16,4) \in R$ because $9,3,16,4 \in \mathbf{N}$ and $9=3^2$ and $16=4^2$.

Now, $9 \neq 4^2 = 16$; therefore, $(9,4) \notin \mathbf{N}$

Therefore, the statement " $(a,b) \in R, (b,c) \in R$ implies $(a,c) \in R''$ is not true.

10. Let A ={1,2,3,4}, B = {1,5,9,11,15,16} and f = {(1,5), (2,9), (3,1), (4,5), (2,11)}

Are the following true?

(i) f is a relation from A to B (ii) f is a function from A to B.

Justify your answer in each case.

10. **Given:** A ={1,2,3,4}, B = {1,5,9,11,15,16} and f = {(1,5), (2,9), (3,1), (4,5), (2,11)}

Now, A × B = {(1,1), (1,5), (1,9), (1,11), (1,15), (1,16), (2,1), (2,5), (2,9), (2,11), (2,15), (2,16), (3,1), (3,5), (3,9), (3,11), (3,15), (3,16), (4,1), (4,5), (4,9), (4,11), (4,15), (4,16).

(i) f is a relation from A to B.

f = {(1,5), (2,9), (3,1), (4,5), (2,11)}.

A relation from a non empty set A to a non empty set B is a subset of the Cartesian product A × B.

And we can see f is a subset of A × B.

Hence f is a relation from A to B statement is true.

(ii) f is a function from A to B.

f = {(1,5), (2,9), (3,1), (4,5), (2,11)}.

As we observe that same first element i.e. 2 corresponds to two different images that is 9 and 11. Thus f is not a function from A to B.

11. Let f be the subset of $\mathbf{Z}\times\mathbf{Z}$ defined by $f=\{(ab,a+b):a,b\in\mathbf{Z}\}$. Is f a function from $\mathbf{Z}$ to $\mathbf{Z}$: justify your answer.

11. The relation f is defined as $f=\{(ab,a+b):a,b\in\mathbf{Z}\}$

We know that a relation f from a set A to a set B is said to be a function if every element of set A has unique images in set B.

Since $2,6,-2,-6\in\mathbf{Z},(2\times6,2+6),(-2\times-6,-2+(-6))\in f$ i.e., $(12,8),(12,-8)\in f$

It can be seen that the same first element i.e., 12 corresponds to two different images i.e., 8 and -8. Thus, relation f is not a function.

12. Let A = {9,10,11,12,13} and let $f : A\rightarrow N$ be defined by f (n) = the highest prime factor of n. Find the range of f.

12. **Given:** A = {9,10,11,12,13}

$f : A\rightarrow N$ be defined by f (n) = the highest prime factor of n.

Prime factor of 9 = 3

Prime factor of 10 = 2,5

Prime factor of 11 = 11

Prime factor of 12 = 2,3

Prime factor of 13 = 13

f (n) = the highest prime factor of n.

Hence,

f(9) = the highest prime factor of 9 = 3

f(10) = the highest prime factor of 10 = 5

f(11) = the highest prime factor of 11 = 11

f(12) = the highest prime factor of 12 = 3

f(13) = the highest prime factor of 13 = 13

As the range of f is the set of all f(n), where n Є A.

Thus, the range of f is: {3, 5, 11, 13}.