

Exercise 11(D)

(i) $1 + 3 + 9 + 27 + \dots$ to 12 terms

(ii) $0.3 + 0.03 + 0.003 + 0.0003 + \dots$ to 8 terms.

(iii) $1 - 1/2 + 1/4 - 1/8 + \dots$ to 9 terms

(iv) $1 - 1/3 + 1/3^2 - 1/3^3 + \dots$ to n terms

(v) $\frac{x+y}{x-y} + 1 + \frac{x-y}{x+y} + \dots$ upto n terms

(vi) $\sqrt{3} + \frac{1}{\sqrt{3}} + \frac{1}{3\sqrt{3}} + \dots$ to n terms.

Solution:

(i) Given G.P: $1 + 3 + 9 + 27 + \dots$ to 12 terms

Here,

$$a = 1 \text{ and } r = 3/1 = 3 \text{ (} r > 1 \text{)}$$

Number of terms, $n = 12$

Hence,

$$S_n = a(r^n - 1) / r - 1$$

$$\Rightarrow S_{12} = (1)((3)^{12} - 1) / 3 - 1$$

$$= (3^{12} - 1) / 2$$

$$= (531441 - 1) / 2$$

$$= 531440 / 2$$

$$= 265720$$



(ii) Given G.P: $0.3 + 0.03 + 0.003 + 0.0003 + \dots$ to 8 terms

Here,

$$a = 0.3 \text{ and } r = 0.03/0.3 = 0.1 \text{ (} r < 1 \text{)}$$

Number of terms, $n = 8$

Hence,

$$S_n = a(1 - r^n) / 1 - r$$

$$\Rightarrow S_8 = (0.3)(1 - 0.1^8) / (1 - 0.1)$$

$$= 0.3(1 - 0.1^8) / 0.9$$

$$= (1 - 0.1^8) / 3$$

$$= 1/3(1 - (1/10)^8)$$

(iii) Given G.P: $1 - 1/2 + 1/4 - 1/8 + \dots$ to 9 terms

Here,

$$a = 1 \text{ and } r = (-1/2) / 1 = -1/2 \text{ (} |r| < 1 \text{)}$$

Number of terms, $n = 9$

Hence,

$$S_n = a(1 - r^n) / 1 - r$$

$$\Rightarrow S_9 = (1)(1 - (-1/2)^9) / (1 - (-1/2))$$

$$= (1 + (1/2)^9) / (3/2)$$

$$= 2/3 \times (1 + 1/512)$$

$$= \frac{2}{3} \times \left(\frac{513}{512}\right)$$

$$= \frac{171}{256}$$

(iv) Given G.P: $1 - \frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^3} + \dots$ to n terms

Here,

$$a = 1 \text{ and } r = \left(-\frac{1}{3}\right) / 1 = -\frac{1}{3} \quad (|r| < 1)$$

Number of terms is n

Hence,

$$S_n = a(1 - r^n) / 1 - r$$

$$S_n = \frac{1 \left(1 - \left(-\frac{1}{3}\right)^n\right)}{1 - \left(-\frac{1}{3}\right)}$$

$$= \frac{1 \left(1 - \left(-\frac{1}{3}\right)^n\right)}{1 + \frac{1}{3}}$$

$$= \frac{\left[1 - \left(-\frac{1}{3}\right)^n\right]}{\frac{4}{3}}$$

$$= \frac{3}{4} \left[1 - \left(-\frac{1}{3}\right)^n\right]$$



(v) Given G.P:

$$\frac{x+y}{x-y} + 1 + \frac{x-y}{x+y} + \dots \text{ upto } n \text{ terms}$$

Here,

$$a = (x+y) / (x-y) \text{ and } r = 1 / [(x+y) / (x-y)] = (x-y) / (x+y) \quad (|r| < 1)$$

Number of terms = n

Hence,

$$S_n = a(1 - r^n) / 1 - r$$

$$\begin{aligned}
 S_n &= \frac{\frac{x+y}{x-y} \left(1 - \left(\frac{x-y}{x+y} \right)^n \right)}{1 - \left(\frac{x-y}{x+y} \right)} \\
 &= \frac{\frac{x+y}{x-y} \left(1 - \left(\frac{x-y}{x+y} \right)^n \right)}{\frac{x+y-x+y}{x+y}} \\
 &= \frac{\frac{x+y}{x-y} \left(1 - \left(\frac{x-y}{x+y} \right)^n \right)}{\frac{2y}{x+y}} \\
 &= \frac{(x+y)^2 \left(1 - \left(\frac{x-y}{x+y} \right)^n \right)}{2y(x-y)}
 \end{aligned}$$

(vi) Given G.P:

$$\sqrt{3} + \frac{1}{\sqrt{3}} + \frac{1}{3\sqrt{3}} + \dots \text{to } n \text{ terms.}$$

Here,

$$a = \sqrt{3} \text{ and } r = \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{3} \quad (|r| < 1)$$

Number of terms = n

Hence,

$$S_n = a(1 - r^n) / 1 - r$$

$$S_n = \frac{\sqrt{3} \left(1 - \left(\frac{1}{3} \right)^n \right)}{1 - \frac{1}{3}} = \frac{\sqrt{3} \left(1 - \frac{1}{3^n} \right)}{\frac{2}{3}}$$

$$= \frac{3\sqrt{3}}{2} \left(1 - \frac{1}{3^n} \right)$$

2. How many terms of the geometric progression $1 + 4 + 16 + 64 + \dots$ must be added to get sum equal to 5461?

Solution:

Given G.P: $1 + 4 + 16 + 64 + \dots$

Here,

$$a = 1 \text{ and } r = 4/1 = 4 \quad (r > 1)$$

And,

$$S_n = 5461$$

We know that,

$$S_n = a(r^n - 1) / r - 1$$

$$\Rightarrow S_n = (1)((4)^n - 1) / 4 - 1$$

$$= (4^n - 1) / 3$$

$$5461 = (4^n - 1) / 3$$

$$16383 = 4^n - 1$$

$$4^n = 16384$$

$$4^n = 4^7$$

$$n = 7$$

Therefore, 7 terms of the G.P must be added to get a sum of 5461.

3. The first term of a G.P. is 27 and its 8th term is 1/81. Find the sum of its first 10 terms.

Solution:

Given,

First term (a) of a G.P = 27

And, 8th term = $t_8 = ar^{8-1} = 1/81$

$$(27)r^7 = 1/81$$

$$r^7 = 1/(81 \times 27)$$

$$r^7 = (1/3)^7$$

$$r = 1/3 \quad (r < 1)$$

$$S_n = a(1 - r^n) / 1 - r$$

Now,

Sum of first 10 terms = S_{10}

$$S_{10} = \frac{27 \left(1 - \left(\frac{1}{3} \right)^{10} \right)}{1 - \frac{1}{3}} = \frac{27 \left(1 - \frac{1}{3^{10}} \right)}{\frac{2}{3}}$$

$$= \frac{81}{2} \left(1 - \frac{1}{3^{10}} \right)$$

4. A boy spends Rs.10 on first day, Rs.20 on second day, Rs.40 on third day and so on. Find how much, in all, will he spend in 12 days?

Solution:

Given,

Amount spent on 1st day = Rs 10

Amount spent on 2nd day = Rs 20

And amount spent on 3rd day = Rs 40

It's seen that,

10, 20, 40, forms a G.P with first term, $a = 10$ and common ratio, $r = 20/10 = 2$ ($r > 1$)

The number of days, $n = 12$

Hence, the sum of money spend in 12 days is the sum of 12 terms of the G.P.

$$S_n = a(r^n - 1) / r - 1$$

$$S_{12} = (10)(2^{12} - 1) / 2 - 1 = 10(2^{12} - 1) = 10(4096 - 1) = 10 \times 4095 = 40950$$

Therefore, the amount spent by him in 12 days is Rs 40950

5. The 4th term and the 7th term of a G.P. are 1/27 and 1/729 respectively. Find the sum of n terms of the G.P.

Solution:

Given,

$$t_4 = 1/27 \text{ and } t_7 = 1/729$$

We know that,

$$t_n = ar^{n-1}$$

So,

$$t_4 = ar^{4-1} = ar^3 = 1/27 \dots (1)$$

$$t_7 = ar^{7-1} = ar^6 = 1/729 \dots (2)$$

Dividing (2) by (1) we get,

$$ar^6 / ar^3 = (1/729) / (1/27)$$

$$r^3 = (1/3)^3$$

$$r = 1/3 \text{ (} r < 1 \text{)}$$

In (1)

$$a \times 1/27 = 1/27$$

$$a = 1$$

Hence,

$$S_n = a(1 - r^n) / 1 - r$$

$$S_n = (1 - (1/3)^n) / 1 - (1/3)$$

$$= (1 - (1/3)^n) / (2/3)$$

$$= 3/2 (1 - (1/3)^n)$$

6. A geometric progression has common ratio = 3 and last term = 486. If the sum of its terms is 728; find its first term.

Solution:

Given,

For a G.P.,

$$r = 3, l = 486 \text{ and } S_n = 728$$

$$\frac{l r - a}{r - 1} = 728$$

$$\frac{486 \times 3 - a}{3 - 1} = 728$$

$$\frac{1458 - a}{2} = 728$$

$$1458 - a = 728 \times 2 = 1456$$

$$\text{Thus, } a = 2$$

7. Find the sum of G.P.: 3, 6, 12,, 1536.

Solution:

Given G.P: 3, 6, 12,, 1536

Here,

$$a = 3, l = 1536 \text{ and } r = 6/3 = 2$$

So,

$$\begin{aligned} \text{The sum of terms} &= (lr - a) / (r - 1) \\ &= (1536 \times 2 - 3) / (2 - 1) \\ &= 3072 - 3 \\ &= 3069 \end{aligned}$$

8. How many terms of the series $2 + 6 + 18 + \dots$ must be taken to make the sum equal to 728?

Solution:

Given G.P: $2 + 6 + 18 + \dots$

Here,

$$a = 2 \text{ and } r = 6/2 = 3$$

Also given,

$$S_n = 728$$

$$S_n = a(r^n - 1) / r - 1$$

$$728 = (2)(3^n - 1) / 3 - 1 = 3^n - 1$$

$$729 = 3^n$$

$$3^6 = 3^n$$

$$n = 6$$

Therefore, 6 terms must be taken to make the sum equal to 728.

9. In a G.P., the ratio between the sum of first three terms and that of the first six terms is 125:152.

Find its common ratio.

Solution:

Given,

$$\frac{a(r^3 - 1)}{r - 1} : \frac{a(r^6 - 1)}{r - 1} = 125 : 152$$

$$\frac{\frac{a(r^3 - 1)}{r - 1}}{\frac{a(r^6 - 1)}{r - 1}} = \frac{125}{152}$$

$$\frac{(r^3 - 1)}{(r^6 - 1)} = \frac{125}{152}$$

$$\frac{r^3 - 1}{(r^3)^2 - (1)^2} = \frac{125}{152}$$

$$\frac{r^3 - 1}{(r^3 - 1)(r^3 + 1)} = \frac{125}{152}$$

$$\frac{1}{r^3 + 1} = \frac{125}{152}$$

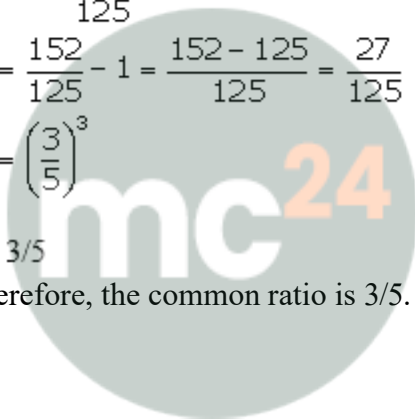
$$r^3 + 1 = \frac{152}{125}$$

$$r^3 = \frac{152}{125} - 1 = \frac{152 - 125}{125} = \frac{27}{125}$$

$$r^3 = \left(\frac{3}{5}\right)^3$$

$$r = 3/5$$

Therefore, the common ratio is 3/5.



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