$$\frac{dy}{dx} = y\tan x - 2\sin x$$

Answer

Given Differential Equation :

$$\frac{dy}{dx} = y\tan x - 2\sin x$$

Formula :

i) $\int \tan x \, dx = \log|\sec x|$

ii) $a\log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) $2\sin x . \cos x = \sin 2x$

v) $\int \sin x \, dx = -\cos x$

vi) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$$\frac{dy}{dx} = y\tan x - 2\sin x$$

$\frac{dy}{dx} - y\tan x = -2\sin x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = -\tan x$ and $Q = -2\sin x$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int -\tan x\,dx}$$

$$= e^{-\log|\sec x|} \text{ ....}(\because \int \tan x\,dx = \log|\sec x|)$$

$$= e^{\log|\sec x|^{-1}} \text{ ....}(\because a\log b = \log b^a)$$

$$= e^{\log\left(\frac{1}{\sec x}\right)}$$

$$= e^{\log(\cos x)}$$

$$= \cos x \text{ ....}(\because a^{\log_a b} = b)$$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(\cos x) = \int (-2\sin x).(\cos x)dx + c$$

$$\therefore y.(\cos x) = -\int (2\sin x).(\cos x)dx + c$$

$$\therefore y.(\cos x) = -\int (\sin 2x)\,dx + c \text{ ....}(\because 2\sin x.\cos x = \sin 2x)$$

$$\therefore y.(\cos x) = \frac{\cos 2x}{2} + c \text{ ....}(\because \int \sin x\,dx = -\cos x)$$

Multiplying above equation by 2,

$$\therefore 2y.(\cos x) = \cos 2x + 2c$$

$\therefore 2y.(\cos x) = \cos 2x + C$ where, C=2c

Therefore, general solution is

$$2y.(\cos x) = \cos 2x + C$$

32. Question

Find the general solution for each of the following differential equations.

$$\frac{dy}{dx} = y\cot x = \sin 2x$$

Answer

Given Differential Equation :

$$\frac{dy}{dx} + y\cot x = \sin 2x$$

Formula :

i) $\int \cot x\, dx = \log|\sin x|$

ii) $a^{\log_a b} = b$

iii) $\int u.v\, dx = u.\int v\, dx - \int\left(\frac{du}{dx}.\int v\, dx\right) dx$

iv) $\int \sin x\, dx = -\cos x$

v) $\frac{d}{dx}(\sin x) = \cos x$

vi) $2\sin x.\cos x = \sin 2x$

vii) $\cos 2x = (\cos^2 x - \sin^2 x)$

viii) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\, dx}$$

Answer :

Given differential equation is

$\frac{dy}{dx} + y\cot x = \sin 2x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \cot x$ and $Q = \sin 2x$

Therefore, integrating factor is

$$\text{I. F.} = e^{\int P\,dx}$$

$$= e^{\int \cot x\,dx}$$

$$= e^{\log|\sin x|} \text{.........}\left(\because \int \cot x\,dx = \log|\sin x|\right)$$

$$= \sin x \text{.........}\left(\because a^{\log_a b} = b\right)$$

General solution is

$$y.(\text{I. F.}) = \int Q.(\text{I. F.})dx + c$$

$$\therefore y.(\sin x) = \int(\sin 2x).(\sin x)dx + c \text{.........eq(2)}$$

Let,

$$I = \int (\sin 2x).(\sin x)dx$$

Let, u=sin 2x & v=sin x

$$\therefore I = \sin 2x.\int \sin x\ dx - \int\left(\frac{d}{dt}(\sin 2x).\int \sin x\ dx\right)dx$$

$$\text{.........}\left(\because \int u.v\,dx = u.\int v\,dx - \int\left(\frac{du}{dx}.\int v\,dx\right)dx\right)$$

$$\therefore I = -\sin 2x.\cos x - \int\left((2\cos 2x).(-\cos x)\right)dx$$

$$\text{.........}\left(\because \int \sin x\,dx = -\cos x\ \&\ \frac{d}{dx}(\sin x) = \cos x\right)$$

$$\therefore I = -\sin 2x.\cos x + 2\int\left((\cos 2x).(\cos x)\right)dx$$

Again let, u=cos 2x & v=cos x

$$\therefore I = -\sin 2x.\cos x + 2\left\{\cos 2x.\int \cos x\ dx - \int\left(\frac{d}{dt}(\cos 2x).\int \cos x\ dx\right)dx\right\}$$

$$\text{.........}\left(\because \int u.v\,dx = u.\int v\,dx - \int\left(\frac{du}{dx}.\int v\,dx\right)dx\right)$$

$$\therefore I = -\sin 2x.\cos x + 2\left\{\cos 2x.\sin x - \int\left((-2\sin 2x).(\sin x)\right)dx\right\}$$

$\therefore \mathrm{I} = -\sin 2x . \cos x + 2\{\cos 2x . \sin x + 2 \int ((\sin 2x).(\sin x))\, dx\}$

$\therefore \mathrm{I} = -\sin 2x . \cos x + 2\{\cos 2x . \sin x + 2\mathrm{I}\}$

$\therefore \mathrm{I} = -\sin 2x . \cos x + 2 \cos 2x . \sin x + 4\mathrm{I}$

$\therefore \mathrm{I} - 4\mathrm{I} = -2 \sin x \cos x . \cos x + 2(\cos^2 x - \sin^2 x). \sin x$

.........$(\because \sin 2x = 2 \sin x . \cos x \ \& \cos 2x = (\cos^2 x - \sin^2 x))$

$\therefore -3\mathrm{I} = -2 \sin x \cos^2 x + 2 \sin x \cos^2 x - 2\sin^3 x$

$\therefore -3\mathrm{I} = -2\sin^3 x$

$\therefore \mathrm{I} = \frac{2}{3} \sin^3 x$

Substituting I in eq(2),

$\therefore y.(\sin x) = \frac{2}{3} \sin^3 x + c$

Therefore, general solution is

$y.(\sin x) = \frac{2}{3} sin^3 x + c$

33. Question

Find the general solution for each of the following differential equations.

$$\frac{dy}{dx} + 2y \tan x = \sin x$$

Answer

Given Differential Equation :

$$\frac{dy}{dx} + 2y \tan x = \sin x$$

Formula :

i) $\int \tan x\, dx = \log|\sec x|$

ii) $a \log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) $\int \left(\frac{-1}{x^2}\right) dx = \frac{1}{x}$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

<u>Answer</u> :

Given differential equation is

$\frac{dy}{dx} + 2y\tan x = \sin x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = 2\tan x$ and $Q = \sin x$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int 2\tan x\,dx}$$

$= e^{2\log|\sec x|}$ ....$(\because \int \tan x\,dx = \log|\sec x|)$

$= e^{\log|\sec x|^2}$ ....$(\because a\log b = \log b^a)$

$= \sec^2 x$ ....$\left(\because a^{\log_a b} = b\right)$

$$= \frac{1}{\cos^2 x}$$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$\therefore y.\left(\frac{1}{\cos^2 x}\right) = \int (\sin x).\left(\frac{1}{\cos^2 x}\right)dx + c$ ....eq(2)

Let,

$$I = \int (\sin x).\left(\frac{1}{\cos^2 x}\right)dx$$

Put, cos x=t => -sin x dx = dt

$$\therefore I = \int \left(\frac{-1}{t^2}\right)dt$$

$$\therefore I = \frac{1}{t} \,\ldots\ldots\ldots \left(\because \int \left(\frac{-1}{x^2}\right)dx = \frac{1}{x}\right)$$

$$\therefore I = \frac{1}{\cos x}$$

Substituting I in eq(2),

$$\therefore y.\left(\frac{1}{\cos^2 x}\right) = \frac{1}{\cos x} + c$$

Multiplying above equation by $\cos^2 x$,

$$\therefore y = \cos x + c(\cos^2 x)$$

Therefore, general solution is

$$y = \cos x + c(cos^2 x)$$

34. Question

Find the general solution for each of the following differential equations.

$$\frac{dy}{dx} + y\cot x = x^2\cot x + 2x$$

Answer

Given Differential Equation :

$$\frac{dy}{dx} + y\cot x = x^2\cot x + 2x$$

Formula :

i) $\int \cot x\,dx = \log|\sin x|$

ii) $a^{\log_a b} = b$

iii) $\int u.v\,dx = u.\int v\,dx - \int \left(\frac{du}{dx}.\int v\,dx\right)dx$

iv) $\int \cos x\,dx = \sin x$

v) $\frac{d}{dx}(x^n) = nx^{n-1}$

vi) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$\frac{dy}{dx} + y\cot x = x^2 \cot x + 2x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \cot x$ and $Q = x^2 \cot x + 2x$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \cot x\,dx}$$

$= e^{\log|\sin x|}$ ....$(\because \int \cot x\,dx = \log|\sin x|)$

= sin x ....$(\because a^{\log_a b} = b)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(\sin x) = \int (x^2 \cot x + 2x).(\sin x)dx + c$$

$$\therefore y.(\sin x) = \int (x^2 \cot x.\sin x + 2x\sin x)\,dx + c$$

$$\therefore y.(\sin x) = \int \left(x^2 \frac{\cos x}{\sin x}.\sin x + 2x\sin x\right) dx + c$$

$$\therefore y.(\sin x) = \int (x^2 \cos x + 2x\sin x)\,dx + c$$

$\therefore y.(\sin x) = \int x^2 \cos x\ dx + \int 2x\sin x\,dx + c$ ....eq(2)

Let,

$$I = \int x^2 \cos x\ dx$$

Let, $u=x^2$ and $v=\cos x$

$$\therefore I = x^2.\int \cos x\ dx - \int \left(\frac{d}{dt}(x^2).\int \cos x\ dx\right) dx$$

.........$\left(\because\ \int u.v\,dx = u.\int v\,dx - \int\left(\frac{du}{dx}.\int v\,dx\right) dx\right)$

$$\therefore I = x^2.\sin x - \int 2x.\sin x\,dx$$

.........$\left(\because\ \int \cos x\,dx = \sin x\ \&\ \frac{d}{dx}(x^n) = nx^{n-1}\right)$

Substituting I in eq(2),

$$\therefore y.(\sin x) = x^2.\sin x - \int 2x.\sin x\,dx + \int 2x\sin x\,dx + c$$

$$\therefore y.(\sin x) = x^2.\sin x + c$$

Dividing above equation by sin x,

$$\therefore y = x^2 + \frac{c}{\sin x}$$

Therefore, general solution is

$$y = x^2 + c(cosec\ x)$$

35. Question

Find a particular solution satisfying the given condition for each of the following differential equations.

$x\frac{dy}{dx} + y = x^3$, given that y =1 when x =2

Answer

Given Differential Equation :

$$x\frac{dy}{dx}+y=x^3$$

Formula :

i) $\int \frac{1}{x}dx = \log x$

ii) $a^{\log_a b} = b$

iii) $\int x^n\,dx = \frac{x^{n+1}}{n+1}$

iv) General solution :

For the differential equation in the form of

$$\frac{dy}{dx}+Py=Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$$x\frac{dy}{dx}+y=x^3$$

Dividing above equation by x,

$\therefore \frac{dy}{dx}+\frac{1}{x}.y = x^2$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx}+Py=Q$$

Where, $P = \frac{1}{x}$ and $Q = x^2$

Therefore, integrating factor is

$\text{I.F.} = e^{\int P\,dx}$

$= e^{\int \frac{1}{x} dx}$

$= e^{\log x}$ ....$\left(\because \int \frac{1}{x} dx = \log x\right)$

$= x$ ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(\text{I.F.}) = \int Q.(\text{I.F.})dx + c$$

$$\therefore y.(x) = \int x^2.(x)dx + c$$

$$\therefore xy = \int x^3 dx + c$$

$\therefore xy = \frac{x^4}{4} + c$ ....$\left(\because \int x^n\,dx = \frac{x^{n+1}}{n+1}\right)$

Dividing above equation by x,

$$\therefore y = \frac{x^3}{4} + \frac{c}{x}$$

Therefore general equation is

$$y = \frac{x^3}{4} + \frac{c}{x}$$

For particular solution put y=1 and x=2 in above equation,

$$\therefore 1 = \frac{2^3}{4} + \frac{c}{2}$$

$$\therefore 1 = \frac{8}{4} + \frac{c}{2}$$

$$\therefore 1 = 2 + \frac{c}{2}$$

$$\therefore \frac{c}{2} = -1$$

$$\therefore c = -2$$

Therefore, particular solution is

$$y = \frac{x^3}{4} - \frac{2}{x}$$

36. Question

Find a particular solution satisfying the given condition for each of the following differential equations.

$\frac{dy}{dx} + y\cot x = 4x\,\mathrm{cosec}\,x$, given that $y = 0$ when $x = \frac{\pi}{2}$.

Answer

Given Differential Equation :

$$\frac{dy}{dx} + y.\cot x = 4x\,\mathrm{cosec}\,x$$

Formula :

i) $\int \cot x\,dx = \log|\sin x|$

ii) $a^{\log_a b} = b$

iii) $\int x^n\,dx = \frac{x^{n+1}}{n+1}$

iv) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$\frac{dy}{dx} + y.\cot x = 4x\,\mathrm{cosec}\,x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \cot x$ and $Q = 4x \operatorname{cosec} x$

Therefore, integrating factor is

$$\text{I. F.} = e^{\int P\,dx}$$

$$= e^{\int \cot x\,dx}$$

$$= e^{\log|\sin x|} \;\ldots\ldots\ldots \left(\because \int \cot x\,dx = \log|\sin x|\right)$$

$= \sin x$ .... $\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(\text{I. F.}) = \int Q.(\text{I. F.})dx + c$$

$$\therefore y.(\sin x) = \int (4x \operatorname{cosec} x).(\sin x)dx + c$$

$$\therefore y.(\sin x) = 4\int \left(x\,\frac{1}{\sin x}\right).(\sin x)dx + c$$

$$\therefore y.(\sin x) = 4\int (x)\,dx + c$$

$$\therefore y.(\sin x) = 4\frac{x^2}{2} + c \;\ldots\ldots\ldots \left(\because \int x^n\,dx = \frac{x^{n+1}}{n+1}\right)$$

$$\therefore y.(\sin x) = 2x^2 + c$$

Therefore general equation is

$$y.(\sin x) = 2x^2 + c$$

For particular solution put y=0 and $x = \frac{\pi}{2}$ in above equation,

$$\therefore 0 = 2\frac{\pi^2}{4} + c$$

$$\therefore 0 = \frac{\pi^2}{2} + c$$

$$\therefore c = -\frac{\pi^2}{2}$$

Therefore, particular solution is

$y.(\sin x) = 2x^2 - \frac{\pi^2}{2}$

37. Question

Find a particular solution satisfying the given condition for each of the following differential equations.

$\frac{dy}{dx} + 2xy = x$, given that y = 0 when x =0.

Answer

Given Differential Equation :

$$\frac{dy}{dx} + 2xy = x$$

Formula :

i) $\int x^n \, dx = \frac{x^{n+1}}{n+1}$

ii) $\int (e^{kx}) dx = \frac{e^{kx}}{k}$

iii) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P \, dx}$$

Answer :

Given differential equation is

$\frac{dy}{dx} + 2xy = x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = 2x$ and $Q = x$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int 2x\,dx}$$

$$= e^{2\frac{x^2}{2}} \text{.........}\left(\because \int x^n\,dx = \frac{x^{n+1}}{n+1}\right)$$

$$= e^{x^2}$$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.\left(e^{x^2}\right) = \int (x).\left(e^{x^2}\right)dx + c$$

$$\therefore y.\left(e^{x^2}\right) = \frac{1}{2}\int (2x).\left(e^{x^2}\right)dx + c \text{.........eq(2)}$$

Let,

$$I = \int (2x).\left(e^{x^2}\right)dx$$

Put, x^2=t => 2x dx = dt

$$\therefore I = \int (e^t)dt$$

$$\therefore I = e^t \text{.........}\left(\because \int \left(e^{kx}\right)dx = \frac{e^{kx}}{k}\right)$$

$$\therefore I = e^{x^2}$$

Substituting I in eq(2),

$$\therefore y.\left(e^{x^2}\right) = \frac{1}{2}.e^{x^2} + c$$

Therefore, general solution is

$$y.\left(e^{x^2}\right) = \frac{1}{2}.e^{x^2} + c$$

For particular solution put y=0 and x=0 in above equation,

$$\therefore 0 = \frac{1}{2}.e^0 + c$$

$$\therefore 0 = \frac{1}{2} + c$$

$$\therefore c = -\frac{1}{2}$$

Substituting c in general solution,

$$y.\left(e^{x^2}\right) = \frac{1}{2}.e^{x^2} - \frac{1}{2}$$

Multiplying above equation by $\frac{2}{e^{x^2}}$

$$\therefore 2y = 1 - e^{-x^2}$$

Therefore, particular solution is

$$2y = 1 - e^{-x^2}$$

38. Question

Find a particular solution satisfying the given condition for each of the following differential equations.

$\frac{dy}{dx} + 2y = e^{-2x} \sin x$, given that $y = 0$, when $x = 0$.

Answer

Given Differential Equation :

$$\frac{dy}{dx} + 2y = e^{-2x}.\sin x$$

Formula :

i) $\int 1\, dx = x$

ii) $\int (\sin x) dx = -\cos x$

iii) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$\text{I.F.} = e^{\int P\,dx}$

Answer :

Given differential equation is

$\frac{dy}{dx} + 2y = e^{-2x}.\sin x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = 2$ and $Q = e^{-2x}.\sin x$

Therefore, integrating factor is

$\text{I.F.} = e^{\int P\,dx}$

$= e^{\int 2\,dx}$

$= e^{2x}$ ....($\because \int 1\,dx = x$)

General solution is

$$y.(\text{I.F.}) = \int Q.(\text{I.F.})dx + c$$

$$\therefore y.(e^{2x}) = \int (e^{-2x}.\sin x).(e^{2x})dx + c$$

$$\therefore y.(e^{2x}) = \int \left(\frac{1}{e^{2x}}.\sin x\right).(e^{2x})dx + c$$

$$\therefore y.(e^{2x}) = \int (\sin x)dx + c$$

$\therefore y.(e^{2x}) = -\cos x + c$ ....($\because \int(\sin x)dx = -\cos x$)

Therefore, general solution is

$y.(e^{2x}) = -\cos x + c$

For particular solution put y=0 and x=0 in above equation,

$\therefore 0 = -\cos 0 + c$

$\therefore 0 = -1 + c$

$\therefore c = 1$

Substituting c in general solution,

$$y.(e^{2x}) = -\cos x + 1$$

Therefore, particular solution is

$$y.(e^{2x}) = -\cos x + 1$$

39. Question

Find a particular solution satisfying the given condition for each of the following differential equations.

$(1+x^2)\frac{dy}{dx} + 2xy = 4x^2$, given that y = 0 when x = 0.

Answer

Given Differential Equation :

$$(1+x^2)\frac{dy}{dx} + 2xy = 4x^2$$

Formula :

i) $\int \frac{f(x)}{f'(x)} dx = \log f(x)$

ii) $\int x^n \, dx = \frac{x^{n+1}}{n+1}$

iii) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$$(1+x^2)\frac{dy}{dx} + 2xy = 4x^2$$

Dividing above equation by $(1+x^2)$,

$\therefore \frac{dy}{dx}+\frac{2x}{(1+x^2)}y=\frac{4x^2}{(1+x^2)}$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx}+Py=Q$$

Where, $P=\frac{2x}{(1+x^2)}$ and $Q=\frac{4x^2}{(1+x^2)}$

Therefore, integrating factor is

$$I.F.=\ e^{\int P\,dx}$$

$$=e^{\int\frac{2x}{(1+x^2)}dx}$$

Let, $f(x)=(1+x^2)\ \therefore f'(x)=2x$

$$\therefore I.F.=e^{\log(1+x^2)}\ ....\left(\because\ \int\frac{f(x)}{f'(x)}dx=\log f(x)\right)$$

$$=(1+x^2)$$

General solution is

$$y.(I.F.)=\int Q.(I.F.)dx\ +c$$

$$\therefore y.(1+x^2)=\int\left(\frac{4x^2}{(1+x^2)}\right).(1+x^2)dx\ +c$$

$$\therefore y.(1+x^2)=4\int x^2dx\ +c$$

$$\therefore y.(1+x^2)=4\frac{x^3}{3}+c\ ....\left(\because\ \int x^n\,dx=\frac{x^{n+1}}{n+1}\right)$$

Therefore, general solution is

$$y.(1+x^2)=4\frac{x^3}{3}+c$$

For particular solution put y=0 and x=0 in above equation,

$$\therefore 0=0+c$$

$$\therefore c=0$$

Substituting c in general solution,

$$\therefore y.(1+x^2) = 4\frac{x^3}{3}$$

Dividing above equation by $(1+x^2)$,

$$\therefore y = \frac{4x^3}{3(1+x^2)}$$

Therefore, particular solution is

$$y = \frac{4x^3}{3(1+x^2)}$$

40. Question

Find a particular solution satisfying the given condition for each of the following differential equations.

$x\frac{dy}{dx} - y = \log x$, given that $y = 0$ when $x = 1$.

Answer

Given Differential Equation :

$$x\frac{dy}{dx} - y = \log x$$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $a \log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) $\int u.v\, dx = u.\int v\, dx - \int \left(\frac{du}{dx}.\int v\, dx\right) dx$

v) $\int e^{kx}\, dx = \frac{e^{kx}}{k}$

vi) $\frac{d}{dx}(kx) = k$

vii) $\log 1 = 0$

viii) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

<u>Answer</u> :

Given differential equation is

$$x\frac{dy}{dx} - y = \log x$$

Dividing above equation by x,

$$\therefore \frac{dy}{dx} - \frac{1}{x}y = \frac{\log x}{x} \text{ ....eq(1)}$$

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{-1}{x}$ and $Q = \frac{\log x}{x}$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{-1}{x}dx}$$

$$= e^{-\log(x)} \text{ ....} \left(\because \int \frac{1}{x}dx = \log x\right)$$

$$= e^{\log x^{-1}} \text{ ....} (\because a\log b = \log b^{a})$$

$$= e^{\log\left(\frac{1}{x}\right)}$$

$$= \frac{1}{x} \text{ ....} \left(\because a^{\log_a b} = b\right)$$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.\left(\frac{1}{x}\right) = \int \left(\frac{\log x}{x}\right).\left(\frac{1}{x}\right)dx + c \text{ ....eq(2)}$$

Let,

$$I = \int \left(\frac{\log x}{x}\right).\left(\frac{1}{x}\right) dx$$

Put, log x =t => x=e^t

Therefore, (1/x) dx = dt

$$\therefore I = \int \left(\frac{t}{e^t}\right) dt$$

$$\therefore I = \int t.e^{-t}\, dt$$

Let, u=t and v=e^{-t}

$$\therefore I = t.\int e^{-t}\, dt - \int \left(\frac{d}{dt}(t).\int e^{-t}\, dt\right) dt$$

$$\text{.........}\left(\because \int u.v\, dx = u.\int v\, dx - \int \left(\frac{du}{dx}.\int v\, dx\right) dx\right)$$

$$\therefore I = -t.e^{-t} - \int \left((1).(-e^{-t})\right) dt$$

$$\text{.........}\left(\because \int e^{kx}\, dx = \frac{e^{kx}}{k}\ \&\ \frac{d}{dx}(kx) = k\right)$$

$$\therefore I = -t.e^{-t} - e^{-t}\ \text{.........}\left(\because \int e^{kx}\, dx = \frac{e^{kx}}{k}\right)$$

$$\therefore I = -\frac{\log x}{x} - \frac{1}{x}$$

Substituting I in eq(2),

$$\therefore y.\left(\frac{1}{x}\right) = -\frac{\log x}{x} - \frac{1}{x} + c$$

Multiplying above equation by x,

$$\therefore y = -\log x - 1 + cx$$

Therefore, general solution is

$$y = -\log x - 1 + cx$$

For particular solution put y=0 and x=1 in above equation,

$$\therefore 0 = -\log 1 - 1 + c$$

$\therefore c = 1$ ....($\because \log 1 = 0$)

Substituting c in general solution,

$\therefore y = -\log x - 1 + x$

$\therefore y = x - \log x - 1$

Therefore, particular solution is

$y = x - \log x - 1$

41. Question

Find a particular solution satisfying the given condition for each of the following differential equations.

$\frac{dy}{dx} + y \tan x = 2x + x^2 \tan x$, given that y = 1 when x = 0.

Answer

Given Differential Equation :

$$\frac{dy}{dx} + y\tan x = 2x + x^2 \tan x$$

Formula :

i) $\int \tan x\, dx = \log|\sec x|$

ii) $a^{\log_a b} = b$

iii) $\int u.v\, dx = u.\int v\, dx - \int \left(\frac{du}{dx}.\int v\, dx\right) dx$

iv) $\int \sec x.\tan x\, dx = \sec x$

v) $\frac{d}{dx}(x^n) = nx^{n-1}$

vi) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$\text{I.F.} = e^{\int P\,dx}$

<u>Answer</u> :

Given differential equation is

$\frac{dy}{dx} + y\tan x = 2x + x^2 \tan x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \tan x$ and $Q = 2x + x^2 \tan x$

Therefore, integrating factor is

$\text{I.F.} = e^{\int P\,dx}$

$= e^{\int \tan x\,dx}$

$= e^{\log|\sec x|}$ ....($\because \int \tan x\,dx = \log|\sec x|$)

$= \sec x$ ....($\because a^{\log_a b} = b$)

General solution is

$$y.(\text{I.F.}) = \int Q.(\text{I.F.})dx + c$$

$$\therefore y.(\sec x) = \int (2x + x^2 \tan x).(\sec x)dx + c$$

$$\therefore y.(\sec x) = \int (x^2 \tan x.\sec x + 2x\sec x)\,dx + c$$

$\therefore y.(\sec x) = \int x^2 \tan x.\sec x\ dx + \int 2x\sec x\,dx + c$ ....eq(2)

Let,

$$I = \int x^2 \tan x.\sec x\ dx$$

Let, $u=x^2$ and v= tan x. sec x

$$\therefore I = x^2.\int \sec x.\tan x\ dx - \int\left(\frac{d}{dt}(x^2).\int \sec x.\tan x\ dx\right) dx$$

.........$\left(\because \int u.v\,dx = u.\int v\,dx - \int\left(\frac{du}{dx}.\int v\,dx\right) dx\right)$

$$\therefore I = x^2.\sec x - \int 2x.\sec x\ dx$$

$$\text{.........}\left(\because \int \sec x.\tan x\,dx = \sec x\ \&\ \frac{d}{dx}(x^n) = nx^{n-1}\right)$$

Substituting I in eq(2),

$$\therefore y.(\sec x) = x^2.\sec x - \int 2x.\sec x\ dx + \int 2x \sec x\,dx + c$$

$$\therefore y.(\sec x) = x^2.\sec x + c$$

$$\therefore y.\left(\frac{1}{\cos x}\right) = x^2.\left(\frac{1}{\cos x}\right) + c$$

Multiplying above equation by cos x,

$$\therefore y = x^2 + c.(\cos x)$$

Therefore, general solution is

$$y = x^2 + c.(\cos x)$$

For particular solution put y=1 and x=0 in above equation,

$$\therefore 1 = 0 + c$$

$$\therefore c = 1$$

Substituting c in general solution,

$$\therefore y = x^2 + \cos x$$

Therefore, particular solution is

$$y = x^2 + \cos x$$

42. Question

A curve passes through the origin and the slope of the tangent to the curve at any point ($\mathcal{X}$,) is equal to the sum of the coordinates of the point. Find the equation of the curve.

Answer

<u>Formula</u> :

i) $\int 1\ dx = x$

ii) $\int u.v\ dx = u.\int v\ dx - \int \left(\frac{du}{dx}.\int v\ dx\right)dx$

iii) $\int e^{kx}\ dx = \frac{e^{kx}}{k}$

iv) $\frac{d}{dx}(x^n) = nx^{n-1}$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

The slope of the tangent to the curve $= \frac{dy}{dx}$

The slope of the tangent to the curve is equal to the sum of the coordinates of the point.

$$\therefore \frac{dy}{dx} = x + y$$

Therefore differential equation is

$$\therefore \frac{dy}{dx} = x + y$$

$\therefore \frac{dy}{dx} - y = x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = -1$ and $Q = x$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int -1\,dx}$$

$= e^{-x}$ ....$(\because \int 1\,dx = x)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$\therefore y.(e^{-x}) = \int (x).(e^{-x})dx + c$ ....eq(2)

Let,

$$I = \int (x).(e^{-x})dx$$

Let, u=x and v= e^{-x}

$$\therefore I = x.\int e^{-x}\,dx - \int \left(\frac{d}{dx}(x).\int e^{-x}\,dx\right) dx$$

.........$\left(\because \int u.v\,dx = u.\int v\,dx - \int \left(\frac{du}{dx}.\int v\,dx\right) dx\right)$

$$\therefore I = -x.e^{-x} - \int (1).(-e^{-x})\,dx$$

.........$\left(\because \int e^{kx}\,dx = \frac{e^{kx}}{k} \ \&\ \frac{d}{dx}(x^n) = nx^{n-1}\right)$

$\therefore I = -x.e^{-x} - e^{-x}$ ....$\left(\because \int e^{kx}\,dx = \frac{e^{kx}}{k}\right)$

Substituting I in eq(2),

$$\therefore y.(e^{-x}) = -x.e^{-x} - e^{-x} + c$$

Dividing above equation by e^{-x},

$$\therefore y = -x - 1 + c.e^{x}$$

Therefore, general solution is

$$y + x + 1 = c.e^{x}$$

The curve passes through origin , therefore the above equation satisfies for x=0 and y=0,

$$\therefore 0 + 0 + 1 = c.e^{0}$$

$$\therefore c = 1$$

Substituting c in general solution,

$$\therefore y + x + 1 = e^{x}$$

Therefore, equation of the curve is

$$y + x + 1 = e^{x}$$

43. Question

A curve passes through the point (0, 2) and the sum of the coordinates of any point on the curve exceeds the magnitude of the slope of the tangent to the curve at that point by 5. Find the equation of the curve.

Answer

Formula :

i) $\int 1\, dx = x$

ii) $\int u.v\, dx = u.\int v\, dx - \int \left(\frac{du}{dx}.\int v\, dx\right) dx$

iii) $\int e^{kx}\, dx = \frac{e^{kx}}{k}$

iv) $\frac{d}{dx}(x^n) = nx^{n-1}$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\, dx}$$

Answer :

The slope of the tangent to the curve $= \frac{dy}{dx}$

The sum of the coordinates of any point on the curve exceeds the magnitude of the slope of the tangent to the curve at the given point by 5.

$$\therefore 5 + \frac{dy}{dx} = x + y$$

Therefore differential equation is

$$\therefore 5 + \frac{dy}{dx} = x + y$$

$\therefore \frac{dy}{dx} - y = x - 5$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx}+Py=Q$$

Where, $P=-1$ and $Q=x-5$

Therefore, integrating factor is

$$I.F.=\ e^{\int P\,dx}$$

$$=e^{\int -1\,dx}$$

$$=e^{-x}\ ....(\because\ \int 1\,dx=x)$$

General solution is

$$y.(I.F.)=\int Q.(I.F.)dx+c$$

$\therefore y.(e^{-x})=\int(x-5).(e^{-x})dx+c$ ....eq(2)

Let,

$$I=\int (x-5).(e^{-x})dx$$

Let, u=x-5 and v= e^{-x}

$$\therefore I=(x-5).\int e^{-x}\,dx-\int\left(\frac{d}{dt}(x-5).\int e^{-x}\,dx\right)dx$$

$$........\left(\because\ \int u.v\,dx=u.\int v\,dx-\int\left(\frac{du}{dx}.\int v\,dx\right)dx\right)$$

$$\therefore I=-(x-5).e^{-x}-\int(1).(-e^{-x})\,dx$$

$$........\left(\because\ \int e^{kx}\,dx=\frac{e^{kx}}{k}\ \&\ \frac{d}{dx}(x^n)=nx^{n-1}\right)$$

$$\therefore I=-(x-5).e^{-x}-e^{-x}\ ....\left(\because\ \int e^{kx}\,dx=\frac{e^{kx}}{k}\right)$$

Substituting I in eq(2),

$$\therefore y.(e^{-x})=-(x-5).e^{-x}-e^{-x}+c$$

Dividing above equation by e^{-x},

$$\therefore y=-(x-5)-1+c.e^{x}$$

$\therefore y = -x + 5 - 1 + c.e^x$

$\therefore y = -x + 4 + c.e^x$

Therefore, general solution is

$y = -x + 4 + c.e^x$

The curve passes through point (0,2) , therefore the above equation satisfies for x=0 and y=2,

$\therefore 2 = -0 + 4 + c.e^0$

$\therefore c = -2$

Substituting c in general solution,

$\therefore y = -x + 4 - 2e^x$

Therefore, equation of the curve is

$y = 4 - x - 2e^x$

44. Question

Find the general solution for each of the following differential equations.

$ydx - (x + 2y^2)dy = 0$

Answer

Given Differential Equation :

$ydx - (x + 2y^2)dy = 0$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

iii) $a \log b = \log b^a$

iv) $a^{\log_a b} = b$

v) General solution :

For the differential equation in the form of

$\frac{dx}{dy} + Px = Q$

General solution is given by,