EXERCISE 19.8

Evaluate the following integrals:

1. $\int \frac{1}{\sqrt{1-\cos 2x}}\,dx$

Solution:

Given

$$\int \frac{1}{\sqrt{1-\cos 2x}}\,dx$$

In the given equation $\cos 2x = \cos^2 x - \sin^2 x$

Also we know $\cos^2 x + \sin^2 x = 1$.

Substituting the values in the above equation we get

$$\Rightarrow \int \frac{1}{\sqrt{\sin^2 x + \cos^2 x - (-\sin^2 x + \cos^2 x)}}\,dx$$

$$\Rightarrow \int \frac{1}{\sqrt{\sin^2 x + \cos^2 x + \sin^2 x - \cos^2 x}}\,dx$$

$$\Rightarrow \int \frac{1}{\sqrt{2\sin^2 x}}\,dx$$

$$\Rightarrow \int \frac{1}{\sqrt{2}\sin x}\,dx$$

$$\frac{1}{\sqrt{2}} \int cosec\ x\,dx$$

$$\Rightarrow \frac{1}{\sqrt{2}} \int \csc x\,dx$$

$$\Rightarrow \frac{1}{\sqrt{2}} \log\left|\frac{\tan x}{2}\right| + c$$

3. $\int \frac{\sqrt{1+\cos 2x}}{\sqrt{1-\cos 2x}}\,dx$

Solution:

Given,

$$\int \frac{\sqrt{1+\cos 2x}}{\sqrt{1-\cos 2x}}\, dx$$

We know that

$1 + \cos 2x = 2 \cos^2 x$

$1 - \cos 2x = 2 \sin^2 x$

By substituting these formulae in the given equation we get

$$\Rightarrow \int \sqrt{\frac{2\cos^2 x}{2\sin^2 x}}\, dx$$

Again by applying standard formula, we get

$$\Rightarrow \int \sqrt{\cot^2 x}\, dx$$

By simplifying we get

$$\Rightarrow \int \cot x\, dx$$

$$\Rightarrow \log |\sin x| + c$$

4. $\displaystyle\int \frac{\sqrt{1-\cos x}}{\sqrt{1+\cos x}}\, dx$

Solution:

Given,

$$\int \frac{\sqrt{1-\cos x}}{\sqrt{1+\cos x}}\, dx$$

We know that

$1 - \cos x = 2\sin^2 \frac{x}{2}$

$1 + \cos x = 2\cos^2 \frac{x}{2}$

By substituting these formulae in the given equation we get

$\Rightarrow \int \sqrt{\tan^2 \frac{x}{2}}\, dx$

On simplification,

$\Rightarrow \int \tan \frac{x}{2}\, dx$

$\Rightarrow -2 \ln \left|\cos \frac{x}{2}\right| + c$

5. $\int \frac{\sec x}{\sec 2x}\, dx$

Solution:

Here first of all convert sec x in terms of cos x

We know

$\Rightarrow \sec x = \frac{1}{\cos x}, \sec 2x = \frac{1}{\cos 2x}$

Therefore the above equation becomes,

$\Rightarrow \frac{\frac{1}{\cos x}}{\frac{1}{\cos 2x}}$

$= \frac{\cos 2x}{\cos x}$

∴ The equation now becomes

$\Rightarrow \int \frac{\cos 2x}{\cos x}\, dx$

We know

Cos 2x = 2 $\cos^2 x$ - 1

∴ We can write the above equation as

$\Rightarrow \int \frac{2\cos^2 x - 1}{\cos x}\, dx$

$\Rightarrow \int 2 \cos x\, dx - \int \frac{1}{\cos x}\, dx$

$\Rightarrow 2 \sin x - \int \sec x \, dx$

$(\int \sec x \, dx = \ln|\sec x + \tan x| + c$

$\Rightarrow 2 \sin x - \log |\sec x + \tan x| + c$

6. $\int \frac{\cos 2x}{(\cos x + \sin x)^2} dx$

Solution:

Let

$$I = \int \frac{\cos 2x}{(\cos x + \sin x)^2} dx$$

By substituting the formula, we get

$$= \int \frac{\cos^2 x - \sin^2 x}{(\cos x + \sin x)^2} dx$$

On simplification, we get

$$= \int \frac{\cos x - \sin x}{\cos x + \sin x} dx$$

Put sin x + cos x = t

$$\Rightarrow -\sin x + \cos x = \frac{dt}{dx}$$

On rearranging

$$\Rightarrow (\cos x - \sin x) \, dx = dt$$

$$\therefore I = \int \frac{1}{t} dt$$

$$= \ln |t| + C$$

Now substitute the value of t, we get

$$= In \ |\cos x + \sin x| + C$$

7. $\int \frac{\sin(x-a)}{\sin(x-b)}$

Solution:

To solve these types of questions, it is better to eliminate the denominator.

$\Rightarrow \int \frac{\sin(x-a)}{\sin(x-b)} dx$

Add and subtract b in (x - a)

$\Rightarrow \int \frac{\sin(x-a+b-b)}{\sin(x-b)} dx$

$\Rightarrow \int \frac{\sin(x-b+b-a)}{\sin(x-b)}$

Numerator is of the form sin (A + B) = sin A cos B + cos A sin B

Where A = x - b; B = b - a

$\Rightarrow \int \frac{\sin(x-b)\cos(b-a)+\cos(x-b)\sin(b-a)}{\sin(x-b)} dx$

$\Rightarrow \int \frac{\sin(x-b)\cos(b-a)}{\sin(x-b)} dx + \int \frac{\cos(x-b)\sin(b-a)}{\sin(x-b)} dx$

$\Rightarrow \int \cos(b-a)\, dx + \int \cot(x-b)\sin(b-a)\, dx$

$\Rightarrow \cos(b-a) \int dx + \sin(b-a) \int \cot(x-b)\, dx$

As $\int \cot(x)\, dx = \ln|\sin x|$

$\Rightarrow$ Cos (b - a) x + sin (b - a) log |sin (x - b)|

Therefore,

= cos (b - a)x + sin(b - a) log |sin(x - b)| + c, where c is an arbitrary constant.