Chapter 14. Rectilinear Figures [Quadrilaterals: Parallelogram, Rectangle, Rhombus, Square and Trapezium]

Exercise 14(A)

Solution 1:

The sum of the interior angle=4 times the sum of the exterior angles.

Therefore the sum of the interior angles = $4\times360^{\circ} = 1440^{\circ}$.

Now we have

$$\begin{aligned}(2n-4)\times90^0 &= 1440^0\\ 2n-4 &= 16\\ 2n &= 20\\ n &= 10\end{aligned}$$

Thus the number of sides in the polygon is 10.

Solution 2:

Let the angles of the pentagon are 4x, 8x, 6x, 4x and 5x.

Thus we can write

$$\begin{aligned}4x+8x+6x+4x+5x &= 540^0\\ 27x &= 540^0\\ x &= 20^0\end{aligned}$$

Hence the angles of the pentagon are:

$4\times20^{\circ} = 80^{\circ}, 8\times20^{\circ} = 160^{\circ}, 6\times20^{\circ} = 120^{\circ}, 4\times20^{\circ} = 80^{\circ}, 5\times20^{\circ} = 100^{\circ}$

Solution 3:

Let the measure of each equal angles are x.

Then we can write

$$\begin{aligned}140^0+5x &= (2\times6-4)\times90^0\\ 140^0+5x &= 720^0\\ 5x &= 580^0\\ x &= 116^0\end{aligned}$$

Therefore the measure of each equal angles are 116°

Solution 4:

Let the number of sides of the polygon is *n* and there are k angles with measure 195^{O}.

Therefore we can write:

$$\begin{aligned}5\times90^0+k\times195^0 &= (2n-4)90^0\\ 180^0n-195^0k &= 450^0-360^0\\ 180^0n-195^0k &= 90^0\\ 12n-13k &= 6\end{aligned}$$

In this linear equation n and k must be integer. Therefore to satisfy this equation the minimum value of

k must be 6 to get n as integer.
Hence the number of sides are: 5 + 6 = 11.

Solution 5:

Let the measure of each equal angles are x.

Then we can write:

$$3\times 132^0 + 4x = (2\times 7 - 4)90^0$$
$$4x = 900^0 - 396$$
$$4x = 504$$
$$x = 126^0$$

Thus the measure of each equal angles are 126°.

Solution 6:

Let the measure of each equal sides of the polygon is x.

Then we can write:

$$142^0 + 176^0 + 6x = (2\times 8 - 4)90^0$$
$$6x = 1080^0 - 318^0$$
$$6x = 762^0$$
$$x = 127^0$$

Thus the measure of each equal angles are 127°.

Solution 7:

Let the measure of the angles are 3x, 4x and 5x.

Thus

$$\angle A + \angle B + \angle C + \angle D + \angle E = 540^0$$
$$3x + (\angle B + \angle C) + 4x + 5x = 540^0$$
$$12x + 180^0 = 540^0$$
$$12x = 360^0$$
$$x = 30^0$$

Thus the measure of angle E will be $4\times 30^\circ = 120^\circ$

Solution 8:

(i)

Let each angle of measure x degree.

Therefore measure of each angle will be:

$$x = 180^0 - 2\times 15^0 = 150^0$$

(ii)

Let each angle of measure x degree.

Therefore measure of each exterior angle will be:

$$\begin{aligned} x &= 180^0 - 150^0 \\ &= 30^0 \end{aligned}$$

(iii)

Let the number of each sides is n.

Now we can write

$$\begin{aligned} n\cdot 150^0 &= (2n-4)\times 90^0 \\ 180^0 n - 150^0 n &= 360^0 \\ 30^0 n &= 360^0 \\ n &= 12 \end{aligned}$$

Thus the number of sides are 12.

Solution 9:

Let measure of each interior and exterior angles are 3k and 2k.

Let number of sides of the polygon is n.

Now we can write:

$$\begin{aligned} n\cdot 3k &= (2n-4)\times 90^0 \\ 3nk &= (2n-4)90^0 \quad \ldots(1) \end{aligned}$$

Again

$$\begin{aligned} n\cdot 2k &= 360^0 \\ nk &= 180^0 \end{aligned}$$

From (1)

$$\begin{aligned} 3\cdot 180^0 &= (2n-4)90^0 \\ 3 &= n-2 \\ n &= 5 \end{aligned}$$

Thus the number of sides of the polygon is 5.

Solution 10:

For (n-1) sided regular polygon:

Let measure of each angle is x.

Therefore

$$(n-1)x = (2(n-1)-4)90^0$$
$$x = \frac{n-3}{n-1}180^0$$

For (n+1) sided regular polygon:

Let measure of each angle is y.

Therefore

$$(n+2)y = (2(n+2)-4)90^0$$
$$y = \frac{n}{n+2}180^0$$

Now we have

$$y - x = 6^0$$
$$\frac{n}{n+2}180^0 - \frac{n-3}{n-1}180^0 = 6^0$$
$$\frac{n}{n+2} - \frac{n-3}{n-1} = \frac{1}{30}$$
$$30n(n-1) - 30(n-3)(n+2) = (n+2)(n-1)$$
$$-30n + 30n + 180 = n^2 + n - 2$$
$$n^2 + n - 182 = 0$$
$$(n-13)(n+14) = 0$$
$$n = 13, -14$$

Thus the value of n is 13.

Solution 11:

(i)

Let the measure of each exterior angle is x and the number of sides is n.

Therefore we can write:

$$n = \frac{360^0}{x}$$

Now we have

$$x + x + 90^0 = 180^0$$
$$2x = 90^0$$
$$x = 45^0$$

(ii)

Thus the number of sides in the polygon is:

$$n = \frac{360^0}{45^0}$$
$$= 8$$