EXERCISE 4.4

Find a natural number whose square diminished by 84 is equal to thrice of 8 more than the given number.

Solution:

Let the natural number = 'x'.
According to the question,
We get the equation,
$x^2 - 84 = 3(x+8)$
$x^2 - 84 = 3x + 24$
$x^2 - 3x - 84 - 24 = 0$
$x^2 - 3x - 108 = 0$
$x^2 - 12x + 9x - 108 = 0$
$x(x - 12) + 9(x - 12) = 0$
$(x + 9)(x - 12)$
$\Rightarrow x = -9$ and $x = 12$
Since, natural numbers cannot be negative.
The number is 12.

1. A natural number, when increased by 12, equals 160 times its reciprocal. Find the number.

Solution:

Let the natural number = x
When the number increased by 12 = x + 12
Reciprocal of the number = 1/x
According to the question, we have,
x + 12 = 160 times of reciprocal of x
$x + 12 = 160/x$
$x(x + 12) = 160$
$x^2 + 12x - 160 = 0$
$x^2 + 20x - 8x - 160 = 0$
$x(x + 20) - 8(x + 20) = 0$
$(x + 20)(x - 8) = 0$
$x + 20 = 0$ or $x - 8 = 0$
$x = -20$ or $x = 8$
Since, natural numbers cannot be negative.
The required number = x = 8

2. A train, travelling at a uniform speed for 360 km, would have taken 48 minutes less to travel the same distance if its speed were 5 km/h more. Find the original speed of the train.

Solution:

Let original speed of train = x km/h
We know,
Time = distance/speed

According to the question, we have,
Time taken by train = 360/x hour
And, Time taken by train its speed increase 5 km/h = 360/(x + 5)
It is given that,
Time taken by train in first - time taken by train in 2nd case = 48 min = 48/60 hour
$360/x - 360/(x+5) = 48/60 = 4/5$
$360(1/x - 1/(x+5)) = 4/5$
$360 \times 5/4\ (5/(x^2+5x)) = 1$
$450 \times 5 = x^2 + 5x$
$x^2 + 5x - 2250 = 0$
$x = (-5 \pm \sqrt{}\ (25+9000))/2$
$= (-5 \pm \sqrt{}\ (9025))/2$
$= (-5 \pm 95)/2$
$= -50, 45$
But $x \neq -50$ because speed cannot be negative
So, x = 45 km/h
Hence, original speed of train = 45 km/h

3. If Zeba were younger by 5 years than what she really is, then the square of her age (in years) would have been 11 more than five times her actual age. What is her age now?
Solution:

Let Zeba's age = x
According to the question,
$(x-5)^2 = 11 + 5x$
$x^2 + 25 - 10x = 11 + 5x$
$x^2 - 15x + 14 = 0$
$x^2 - 14x - x + 14 = 0$
$x(x-14) - 1(x-14) = 0$
$x = 1$ or $x = 14$
We have to neglect 1 as 5 years younger than 1 cannot happen.
Therefore, Zeba's present age = 14 years.