

# Differentiation

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## Exercise 28A

Q. 1. Differentiate the following functions:

(i)  $x^{-3}$

(ii)  $\sqrt[3]{x}$

Answer : (i)  $x^{-3}$

$$\frac{d}{dx} x^n = nx^{n-1}$$

Differentiating w.r.t x,

$$\frac{d}{dx} x^{-3} = -3x^{-3-1}$$

$$= -3x^{-4}$$

(ii)  $\sqrt[3]{x} = x^{\frac{1}{3}}$

Formula:-

$$\frac{d}{dx} x^n = nx^{n-1}$$

Differentiating w.r.t x,

$$\frac{d}{dx} X^{\frac{1}{3}} = \frac{1}{3} X^{\frac{1}{3}-1}$$

$$= \frac{1}{3} X^{-\frac{2}{3}}$$

**Q. 2. Differentiate the following functions:**

(i)  $\frac{1}{X}$  (ii)  $\frac{1}{\sqrt{X}}$  (iii)  $\frac{1}{\sqrt[3]{X}}$

**Answer :** (i)  $\frac{1}{x} = X^{-1}$

**Formula:-**

$$\frac{d}{dx} X^n = nX^{n-1}$$



Differentiating w.r.t x,

$$\frac{d}{dx} X^{-1} = -1X^{-1-1}$$

$$= -X^{-2}$$

$$(ii) \frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$$

Formula:-

$$\frac{d}{dx} x^n = nx^{n-1}$$

Differentiating w.r.t x,

$$\frac{d}{dx} x^{-\frac{1}{2}} = -\frac{1}{2} x^{-\frac{1}{2}-1}$$

$$= -\frac{1}{2} x^{-\frac{3}{2}}$$

$$(iii) \frac{1}{\sqrt[3]{x}} = x^{-\frac{1}{3}}$$



Formula:-

$$\frac{d}{dx} x^n = nx^{n-1}$$

Differentiating w.r.t x,

$$\frac{d}{dx} x^{-\frac{1}{3}} = -\frac{1}{3} x^{-\frac{1}{3}-1}$$

$$= -\frac{1}{3} x^{-\frac{4}{3}}$$

**Q. 3. Differentiate the following functions:**

(i)  $3x^{-5}$

(ii)  $\frac{1}{5x}$

(iii)  $6\sqrt[3]{x^2}$

**Answer :** (i)  $3x^{-5}$

**Formula:-**

$$\frac{d}{dx} x^n = nx^{n-1}$$

Differentiating with respect to x,

$$\frac{d}{dx} 3x^{-5} = 3(-5)x^{-5-1}$$

$$= -15x^{-6}$$

(ii)  $1/5x = \frac{1}{5}x^{-1}$

**Formula:-**

$$\frac{d}{dx} x^n = nx^{n-1}$$

Differentiating with respect to x,



$$\frac{1}{5} \frac{d}{dx} X^{-1} = \frac{-1}{5} X^{-1-1}$$

$$= -\frac{1}{5} X^{-2}$$

$$(iii) 6. \sqrt[3]{X^2} = 6X^{\frac{2}{3}}$$

Formula:-

$$\frac{d}{dx} X^n = nX^{n-1}$$

Differentiating with respect to x,

$$\frac{d}{dx} 6X^{\frac{2}{3}} = 6 \times \frac{2}{3} X^{\frac{2}{3}-1}$$

$$= 4X^{-\frac{1}{3}}$$



**Q. 4 Differentiate the following functions:**

(i)  $6x^5 + 4x^3 - 3x^2 + 2x - 7$

(ii)

$$5x^{-3/2} + \frac{4}{\sqrt{x}} + \sqrt{x} - \frac{7}{x}$$

(iii)  $ax^3 + bx^2 + cx + d$ , where a, b, c, d are constants

**Answer :** (i)  $6x^5 + 4x^3 - 3x^2 + 2x - 7$

**Formula:-**

$$\frac{d}{dx} X^n = nX^{n-1}$$

Differentiating with respect to x,

$$\frac{d}{dx}(6x^5 + 4x^3 - 3x^2 + 2x - 7) = 30x^{5-1} + 12x^{3-1} - 6x^{2-1} + 2x^{1-1} + 0$$

$$= 30x^4 + 12x^2 - 6x^1 + 2x$$

$$(ii) 5x^{-3/2} + \frac{4}{\sqrt{x}} + \sqrt{x} - \frac{7}{x}$$

**Formula:-**

$$\frac{d}{dx}x^n = nx^{n-1}$$

Differentiating with respect to x,

$$\begin{aligned} \frac{d}{dx} \left( 5x^{-3/2} + \frac{4}{\sqrt{x}} + \sqrt{x} - \frac{7}{x} \right) \\ = 5 \times -\frac{3}{2} x^{-\frac{3}{2}-1} + 4 \times -\frac{1}{2} x^{-\frac{1}{2}-1} + \frac{1}{2} x^{\frac{1}{2}-1} - 7 \times -1 x^{-1-1} \\ = -\frac{15}{2} x^{-\frac{5}{2}} - 2x^{-\frac{3}{2}} + \frac{1}{2} x^{-\frac{1}{2}} + 7x^{-2} \end{aligned}$$

(iii)  $ax^3 + bx^2 + cx + d$ , where a, b, c, d are constants

**Formula:-**

$$\frac{d}{dx}x^n = nx^{n-1}$$

Differentiating with respect to x,

$$\frac{d}{dx}(ax^3 + bx^2 + cx + d) = 3ax^{3-1} + 2bx^{2-1} + cx^{1-1} + d \times 0$$

$$= 3ax^2 + 2bx + c$$

**Q. 5. Differentiate the following functions:**

(i)  $4x^3 + 3 \cdot 2^x + 6 \cdot \sqrt[8]{x^{-4}} + 5 \cot x$

(ii)  $\frac{x}{3} - \frac{3}{x} + \sqrt{x} - \frac{1}{\sqrt{x}} + x^2 - 2^x + 6x^{-2/3} - \frac{2}{3}x^6$

**Answer :**

(i)  $4x^3 + 3 \cdot 2^x + 6 \cdot \sqrt[8]{x^{-4}} + 5 \cot x$

$$= 4x^3 + 3 \cdot 2^x + 6x^{-\frac{1}{2}} + 5 \cot x$$

Formulae:

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{d}{dx} \cot x = -\operatorname{cosec}^2 x$$

$$\frac{d}{dx} a^x = \log_n(a) \times a^x$$

Differentiating with respect to  $x$ ,

$$\frac{d}{dx} (4x^3 + 3 \cdot 2^x + 6x^{-\frac{1}{2}} + 5 \cot x)$$

$$= 4 \cdot 3x^{3-1} + 3 \cdot \log_n(2) \cdot 2^x + 6 \times -\frac{1}{2} x^{-\frac{1}{2}-1} + 5 \times -\operatorname{cosec}^2 x$$

$$= 12x^2 + 3 \cdot \log_n(2) \cdot 2^x - 3x^{-\frac{3}{2}} - 5 \operatorname{cosec}^2 x$$

$$(ii) \frac{x}{3} - \frac{3}{x} + \sqrt{x} - \frac{1}{\sqrt{x}} + x^2 - 2^x + 6x^{-2/3} - \frac{2}{3}x^6$$

$$= \frac{x}{3} - 3x^{-1} + x^{\frac{1}{2}} - x^{-\frac{1}{2}} + x^2 - 2^x + 6x^{-2/3} - \frac{2}{3}x^6$$

$$\frac{d}{dx} x^n = nx^{n-1}$$

$$\frac{d}{dx} a^x = \log_n(a) \times a^x$$

Differentiating with respect to  $x$ ,

$$\frac{d}{dx} \left( \frac{x}{3} - 3x^{-1} + x^{\frac{1}{2}} - x^{-\frac{1}{2}} + x^2 - 2^x + 6x^{-\frac{2}{3}} - \frac{2}{3}x^6 \right)$$

$$= \frac{1}{3} - (-1) \times 3x^{-1-1} + \frac{1}{2}x^{\frac{1}{2}-1} - \left(-\frac{1}{2}\right)x^{-\frac{1}{2}-1} + 2x^{2-1} - \log(2) \cdot 2^x + 6\left(-\frac{2}{3}\right)x^{-\frac{2}{3}-1} - \frac{2}{3} \times 6x^{6-1}$$

$$= \frac{1}{3} + 3x^{-2} + \frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}} + 2x^1 - \log(2) \cdot 2^x - 4x^{-\frac{5}{3}} - 4x^5$$

**Q. 6. Differentiate the following functions:**

(i)  $4 \cot x - \frac{1}{2} \cos x + \frac{2}{\cos x} - \frac{3}{\sin x} + \frac{6 \cot x}{\operatorname{cosec} x} + 9$

(ii)  $-5 \tan x + 4 \tan x \cos x - 3 \cot x \sec x + 2 \sec x - 13$

**Answer : Formulae: -**

$$\frac{d}{dx} \cot x = -\operatorname{cosec}^2 x$$

$$\frac{d}{dx} \cos x = -\sin x$$

$$\frac{d}{dx} \sec x = \sec x \tan x$$

$$\frac{d}{dx} \operatorname{cosec} x = -\operatorname{cosec} x \cot x$$

$$\frac{d}{dx} \tan x = \sec^2 x$$

$$\frac{d}{dx} \sin x = \cos x$$

$$\frac{d}{dx} k = 0, k \text{ is constant}$$



$$(i) 4 \cot x - \frac{1}{2} \cos x + \frac{2}{\cos x} - \frac{3}{\sin x} + \frac{6 \cot x}{\operatorname{cosec} x} + 9$$

$$= 4 \cot x - \frac{1}{2} \cos x + 2 \sec x - 3 \operatorname{cosec} x + 6 \cos x + 9$$

Differentiating with respect to  $x$ ,

$$\frac{d}{dx} (4 \cot x - \frac{1}{2} \cos x + 2 \sec x - 3 \operatorname{cosec} x + 6 \cos x + 9)$$

$$= 4(-\operatorname{cosec}^2 x) - \frac{1}{2}(-\sin x) + 2 \sec x \times \tan x - 3(-\operatorname{cosec} x \times \cot x) + 6(-\sin x) + 0$$

$$= -4 \operatorname{cosec}^2 x + \frac{1}{2} \sin x + 2 \sec x \tan x + 3 \operatorname{cosec} x \cot x - 6 \sin x$$

$$(ii) -5 \tan x + 4 \tan x \cos x - 3 \cot x \sec x + 2 \sec x - 13$$

$$= -5 \tan x + 4 \sin x - 3 \operatorname{cosec} x + 2 \sec x - 13$$

Differentiating with respect to  $x$ ,

$$\frac{d}{dx} (-5 \tan x + 4 \sin x - 3 \operatorname{cosec} x + 2 \sec x - 13)$$

$$= -5 \sec^2 x + 4 \cos x - 3(-\operatorname{cosec} x \cot x) + 2 \sec x \tan x - 0$$

$$= -5 \sec^2 x + 4 \cos x + 3 \operatorname{cosec} x \cot x + 2 \sec x \tan x$$

**Q. 7 Differentiate the following functions:**

(i)  $(2x + 3)(3x - 5)$

(ii)  $x(1 + x)^3$

(iii)  $\left(\sqrt{x} + \frac{1}{x}\right)\left(x - \frac{1}{\sqrt{x}}\right)$

(iv)  $\left(x - \frac{1}{x}\right)^2$

(v)  $\left(x^2 - \frac{1}{x^2}\right)^3$

(vi)  $(2x^2 + 5x - 1)(x - 3)$

**Answer : Formula:**

$$\frac{d}{dx}f(g(x)) = \frac{d}{dg}f(g) \frac{d}{dx}g$$

**Chain rule -**

$$\frac{d}{dx}(uv) = u \frac{d}{dx}v + v \frac{d}{dx}u$$

Where u and v are the functions of x.

(i)  $(2x + 3)(3x - 5)$

Applying, Chain rule

Here,  $u = 2x + 3$

$v = 3x - 5$

$$\frac{d}{dx}(2x + 3)(3x - 5) = (2x + 3) \frac{d}{dx}(3x - 5) + (3x - 5) \frac{d}{dx}(2x + 3)$$

$$= (2x + 3)(3x^{1-1} + 0) + (3x - 5)(2x^{1-1} + 0)$$

$$= 6x + 9 + 6x - 10$$

$$= 12x - 1$$

(ii)  $x(1 + x)^3$

Applying, Chain rule

Here,  $u = x$

$$V = (1 + x)^3$$

$$\frac{d}{dx}x(1 + x)^3 = x \frac{d}{dx}(1 + x)^3 + (1 + x)^3 \frac{d}{dx}(x)$$

$$= x \times 3 \times (1 + x)^2 + (1 + x)^3(1)$$

$$= (1 + x)^2(3x + x + 1)$$

$$= (1 + x)^2(4x + 1)$$

$$(iii) \left(\sqrt{x} + \frac{1}{x}\right)\left(x - \frac{1}{\sqrt{x}}\right) = (x^{1/2} + x^{-1})(x - x^{-1/2})$$

Applying, Chain rule

Here,  $u = (x^{1/2} + x^{-1})$

$$V = (x - x^{-1/2})$$

$$\frac{d}{dx}(x^{1/2} + x^{-1})(x - x^{-1/2})$$

$$= (x^{1/2} + x^{-1}) \frac{d}{dx}(x - x^{-1/2}) + (x - x^{-1/2}) \frac{d}{dx}(x^{1/2} + x^{-1})$$

$$= (x^{1/2} + x^{-1})\left(1 + \frac{1}{2}x^{-3/2}\right) + (x - x^{-1/2})\left(\frac{1}{2}x^{-1/2} - x^{-2}\right)$$

$$= x^{1/2} + x^{-1} + \frac{1}{2}x^{-1} + \frac{1}{2}x^{-5/2} + \frac{1}{2}x^{1/2} - x^{-1} - \frac{1}{2}x^{-1} + x^{-5/2}$$

$$= \frac{3}{2}x^{1/2} + \frac{3}{2}x^{-5/2}$$



(iv)

$$\left(x - \frac{1}{x}\right)^2$$

Differentiation of composite function can be done by

$$\frac{d}{dx}f(g(x)) = \frac{d}{dg}f(g) \frac{d}{dx}g$$

$$\text{Here, } f(g) = g^2, g(x) = x - \frac{1}{x}$$

$$\frac{d}{dx}\left(x - \frac{1}{x}\right)^2 = 2g \times \left(1 + \frac{1}{x^2}\right)$$

$$= 2\left(x - \frac{1}{x}\right) \left(1 + \frac{1}{x^2}\right)$$

$$= 2\left(x + \frac{1}{x} - \frac{1}{x} + \frac{1}{x^3}\right)$$

$$= 2\left(x + \frac{1}{x^3}\right)$$

(v)

$$\left(x^2 - \frac{1}{x^2}\right)^3$$

Differentiation of composite function can be done by

$$\frac{d}{dx}f(g(x)) = \frac{d}{dg}f(g) \frac{d}{dx}g$$

$$\text{Here, } f(g) = g^3, g(x) = x^2 - \frac{1}{x^2}$$

$$\frac{d}{dx}\left(x^2 - \frac{1}{x^2}\right)^3 = 3g^2 \times \left(2x - \frac{2}{x^3}\right)$$



$$= 3\left(x^2 - \frac{1}{x^2}\right)^2 \left(2x - \frac{2}{x^3}\right)$$

$$= 3\left(2x^3 - \frac{2}{x} - \frac{2}{x} + \frac{2}{x^5}\right)$$

$$= 3\left(2x^3 - \frac{4}{x} + \frac{2}{x^5}\right)$$

(vi)  $(2x^2 + 5x - 1)(x - 3)$

Applying, Chain rule

Here,  $u = (2x^2 + 5x - 1)$

$V = (x - 3)$

$$\frac{d}{dx}(2x^2 + 5x - 1)(x - 3)$$

$$= (2x^2 + 5x - 1) \frac{d}{dx}(x - 3) + (x - 3) \frac{d}{dx}(2x^2 + 5x - 1)$$

$$= (2x^2 + 5x - 1) \times 1 + (x - 3)(4x + 5)$$

$$= 2x^2 + 5x - 1 + 4x^2 - 7x - 15$$

$$= 6x^2 - 2x - 16$$

**Q. 8. Differentiate the following functions:**

(i)  $\frac{3x^2 + 4x - 5}{x}$

(ii)  $\frac{(x^3 + 1)(x - 2)}{x^2}$

$$(iii) \frac{x-4}{2\sqrt{x}}$$

$$(iv) \frac{(1+x)\sqrt{x}}{\sqrt[3]{x}}$$

$$(v) \frac{ax^2 + bx + c}{\sqrt{x}}$$

$$(vi) \frac{a + b \cos x}{\sin x}$$

**Answer : Formula:**

$$\frac{d}{dx} \frac{u}{v} = \frac{v \frac{d}{dx} u - u \frac{d}{dx} v}{v^2}$$

$$(i) \frac{3x^2 + 4x - 5}{x}$$

Applying, quotient rule

$$\frac{d}{dx} \frac{3x^2 + 4x - 5}{x} = \frac{x \frac{d}{dx} (3x^2 + 4x - 5) - (3x^2 + 4x - 5) \frac{d}{dx} x}{x^2}$$

$$= \frac{x(6x+4) - (3x^2 + 4x - 5)1}{x^2}$$

$$= \frac{6x^2 + 4x - (3x^2 + 4x - 5)}{x^2}$$

$$= \frac{3x^2 + 5}{x^2}$$

$$(ii) \frac{(x^3 + 1)(x - 2)}{x^2}$$

Applying, quotient rule

$$\frac{d}{dx} \frac{(x^3 + 1)(x - 2)}{x^2} = \frac{x^2 \frac{d}{dx} (x^3 + 1)(x - 2) - (x^3 + 1)(x - 2) \frac{d}{dx} x^2}{x^4}$$

$$= \frac{x^2 \{(x^3 + 1) \frac{d}{dx} (x - 2) + (x - 2) \frac{d}{dx} (x^3 + 1)\} - (x^3 + 1)(x - 2) 2x}{x^4}$$

$$= \frac{x^2 \{(x^3+1)+(x-2)3x^2\} - (x^3+1)(x-2)2x}{x^4}$$

$$= \frac{x^2 \{x^3+1+3x^3-6x^2\} - 2(x^4+x)(x-2)}{x^4}$$

$$= \frac{4x^5-6x^4+x^2-2(x^5-2x^4+x^2-2x)}{x^4}$$

$$= \frac{2x^5-2x^4-x^2+4x}{x^4}$$

$$(iii) \frac{x-4}{2\sqrt{x}}$$

Applying, quotient rule

$$\frac{d}{dx} \frac{x-4}{2\sqrt{x}} = \frac{2\sqrt{x} \frac{d}{dx}(x-4) - (x-4) \frac{d}{dx} 2\sqrt{x}}{4x}$$

$$= \frac{2\sqrt{x} - (x-4)2 \cdot \frac{1}{2} x^{-\frac{1}{2}}}{4x}$$

$$= \frac{2\sqrt{x} - (x-4)x^{-\frac{1}{2}}}{4x}$$

$$= \frac{2\sqrt{x} - x^{\frac{1}{2}} + 4x^{-\frac{1}{2}}}{4x}$$

$$= \frac{\sqrt{x} + 4x^{-\frac{1}{2}}}{4x}$$

$$(iv) \frac{(1+x)\sqrt{x}}{\sqrt[3]{x}}$$

Applying, quotient rule

$$\frac{d}{dx} \frac{(1+x)\sqrt{x}}{\sqrt[3]{x}} = \frac{\sqrt[3]{x} \frac{d}{dx}(1+x)\sqrt{x} - (1+x)\sqrt{x} \frac{d}{dx} \sqrt[3]{x}}{x^{\frac{2}{3}}}$$

$$= \frac{\sqrt[3]{x} \left\{ (1+x) \frac{d}{dx} \sqrt{x} + \sqrt{x} \frac{d}{dx} (1+x) \right\} - (1+x)\sqrt{x} \cdot \frac{1}{3} x^{-\frac{2}{3}}}{x^{\frac{2}{3}}}$$



$$= \frac{\sqrt[3]{x} \left\{ (1+x) \cdot \frac{1}{2} x^{-\frac{1}{2}} + \sqrt{x} \right\} - (1+x) \cdot \frac{1}{3} x^{-\frac{1}{6}}}{x^{\frac{2}{3}}}$$

$$= \frac{\sqrt[3]{x} \left\{ \frac{1}{2} x^{-\frac{1}{2}} + \frac{1}{2} x^{\frac{1}{2}} + \sqrt{x} \right\} - \frac{1}{3} \left( x^{-\frac{1}{6}} + x^{\frac{5}{6}} \right)}{x^{\frac{2}{3}}}$$

$$= \frac{\sqrt[3]{x} \left\{ \frac{1}{2} x^{-\frac{1}{2}} + \frac{1}{2} x^{\frac{1}{2}} + \sqrt{x} \right\} - \frac{1}{3} \left( x^{-\frac{1}{6}} + x^{\frac{5}{6}} \right)}{x^{\frac{2}{3}}}$$

$$= \frac{\frac{1}{2} x^{-\frac{1}{6}} + \frac{1}{2} x^{\frac{5}{6}} + \sqrt{x} - \frac{1}{3} \left( x^{-\frac{1}{6}} + x^{\frac{5}{6}} \right)}{x^{\frac{2}{3}}}$$

$$= \frac{\frac{1}{6} x^{-\frac{1}{6}} + \frac{1}{6} x^{\frac{5}{6}} + \sqrt{x}}{x^{\frac{2}{3}}}$$

$$(v) \frac{ax^2 + bx + c}{\sqrt{x}}$$

Applying, quotient rule

$$\frac{d}{dx} \frac{ax^2 + bx + c}{\sqrt{x}} = \frac{\sqrt{x} \frac{d}{dx} (ax^2 + bx + c) - (ax^2 + bx + c) \frac{d}{dx} \sqrt{x}}{x}$$

$$= \frac{\sqrt{x}(2ax+b) - \frac{1}{2}(ax^2+bx+c)x^{-\frac{1}{2}}}{x}$$

$$= \frac{\frac{3}{2}ax^{\frac{3}{2}} + \frac{1}{2}bx^{\frac{1}{2}} - \frac{1}{2}cx^{-\frac{1}{2}}}{x}$$

$$(vi) \frac{a+b \cos x}{\sin x}$$

Applying, quotient rule

$$\frac{d}{dx} \frac{a + b \cos x}{\sin x} = \frac{\sin x \frac{d}{dx} (a + b \cos x) - (a + b \cos x) \frac{d}{dx} \sin x}{\sin^2 x}$$

$$= \frac{\sin x (-b \sin x) - (a + b \cos x) \cos x}{\sin^2 x}$$

$$= \frac{-b \sin^2 x - a \cos x - b \cos^2 x}{\sin^2 x}$$

$$= \frac{-b(1) - a \cos x}{\sin^2 x}$$

**Q. 9. Differentiate the following functions:**

(i) If  $y = 6x^5 - 4x^4 - 2x^2 + 5x - 9$ , find  $\frac{dy}{dx}$  at  $x = -1$ .

(ii) If  $y = (\sin x + \tan x)$ , find  $\frac{dy}{dx}$  at  $x = \frac{\pi}{3}$ .

(iii) If  $y = \frac{(2 - 3 \cos x)}{\sin x}$ , find  $\frac{dy}{dx}$  at  $x = \frac{\pi}{4}$ .

**Answer : Formulae:**

$$\frac{d}{dx} x^n = nx^{n-1}$$

$$\frac{d}{dx} \cot x = -\operatorname{cosec}^2 x$$

$$\frac{d}{dx} \operatorname{cosec} x = -\operatorname{cosec} x \cot x$$

$$\frac{d}{dx} \tan x = \sec^2 x$$

$$\frac{d}{dx} \sin x = \cos x$$

(i) If  $y = 6x^5 - 4x^4 - 2x^2 + 5x - 9$ , find  $\frac{dy}{dx}$  at  $x = -1$ .

Differentiating with respect to  $x$ ,



$$\frac{d}{dx}(6x^5 - 4x^4 - 2x^2 + 5x - 9)$$

$$= 30x^4 - 16x^3 - 4x + 5$$

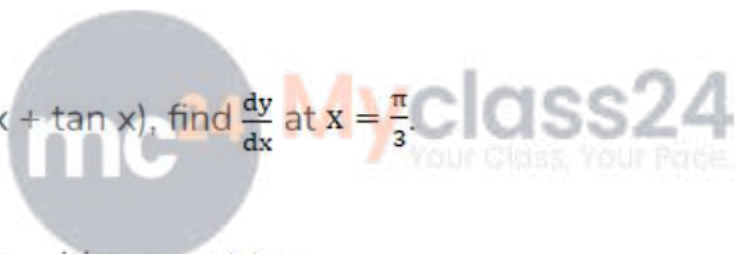
substituting  $x = -1$

$$\left(\frac{dy}{dx}\right)_{x=-1} = 30(-1)^4 - 16(-1)^3 - 4(-1) + 5$$

$$= 30 + 16 + 4 + 5$$

$$= 55$$

(ii) If  $y = (\sin x + \tan x)$ , find  $\frac{dy}{dx}$  at  $x = \frac{\pi}{3}$ .



Differentiating with respect to  $x$ ,

$$\frac{d}{dx}(\sin x + \tan x) = \cos x + \sec^2 x$$

Substituting  $x = \frac{\pi}{3}$

$$\left(\frac{dy}{dx}\right)_{x=\pi/3} = \cos \frac{\pi}{3} + \sec^2 \frac{\pi}{3}$$

$$= \frac{1}{2} + 4$$

$$= \frac{5}{2}$$

(iii) If  $y = \frac{(2-3\cos x)}{\sin x}$ , find  $\frac{dy}{dx}$  at  $x = \frac{\pi}{4}$ .

Differentiating with respect to  $x$ ,

$$\frac{d}{dx}(2\operatorname{cosec} x - 3\cot x) = 2(-\operatorname{cosec} x \cot x) - 3(-\operatorname{cosec}^2 x)$$

Substituting  $x = \frac{\pi}{4}$

$$\left(\frac{dy}{dx}\right)_{x = \pi/4} = 2(-\operatorname{cosec} \frac{\pi}{4} \cot \frac{\pi}{4}) - 3(-\operatorname{cosec}^2 \frac{\pi}{4})$$

$$= -2 \times \sqrt{2} + 3 \times 2$$

$$= 6 - 2 \times \sqrt{2}$$

**Q. 10.**

If  $y = \left( \sqrt{x} + \frac{1}{\sqrt{x}} \right)$ , show that  $2x \cdot \frac{dy}{dx} + y = 2\sqrt{x}$ .

**Answer :** To show:

$$2x \cdot \frac{dy}{dx} + y = 2\sqrt{x}$$

Differentiating with respect to  $x$

$$\frac{dy}{dx} = \frac{d}{dx} \left( \sqrt{x} + \frac{1}{\sqrt{x}} \right) = \frac{1}{2\sqrt{x}} - \frac{1}{2x^{\frac{3}{2}}}$$

Now,

$$\text{LHS} = 2x \cdot \frac{dy}{dx} + y$$

$$\text{LHS} = 2x \times \left( \frac{1}{2\sqrt{x}} - \frac{1}{2x^{\frac{3}{2}}} \right) + \sqrt{x} + \frac{1}{\sqrt{x}}$$

$$\text{LHS} = \sqrt{x} - \frac{1}{\sqrt{x}} + \sqrt{x} + \frac{1}{\sqrt{x}}$$

$$\text{LHS} = 2\sqrt{x}$$

$$\therefore \text{LHS} = \text{RHS}$$



**Q. 11**

$$\text{If } y = \left( \sqrt{\frac{x}{a}} + \sqrt{\frac{a}{x}} \right), \text{ prove that } (2xy) \left( \frac{dy}{dx} \right) = \left( \frac{x}{a} - \frac{a}{x} \right).$$

**Answer :** To prove:

$$(2xy) \left( \frac{dy}{dx} \right) = \left( \frac{x}{a} - \frac{a}{x} \right).$$

Differentiating  $y$  with respect to  $x$

$$\frac{dy}{dx} = \frac{d}{dx} \left( \sqrt{\frac{x}{a}} + \sqrt{\frac{a}{x}} \right) = \frac{1}{2\sqrt{ax}} - \frac{\sqrt{a}}{2x^{\frac{3}{2}}}$$

Now,

$$\text{LHS} = (2xy) \left( \frac{dy}{dx} \right)$$

$$\text{LHS} = 2x \left( \sqrt{\frac{x}{a}} + \sqrt{\frac{a}{x}} \right) \left( \frac{1}{2\sqrt{ax}} - \frac{\sqrt{a}}{2x^2} \right)$$

$$\text{LHS} = \left( \sqrt{\frac{x}{a}} + \sqrt{\frac{a}{x}} \right) \left( \sqrt{\frac{x}{a}} - \sqrt{\frac{a}{x}} \right)$$

$$\text{LHS} = \left( \frac{x}{a} - \frac{a}{x} \right)$$

∴ LHS = RHS

Q. 12 If  $y = \sqrt{\frac{1 + \cos 2x}{1 - \cos 2x}}$ , find  $\frac{dy}{dx}$ .

Answer :

$$y = \sqrt{\frac{1 + \cos 2x}{1 - \cos 2x}}$$


**Formula:**

**Using double angle formula:**

$$\cos 2x = 2\cos^2 x - 1$$

$$= 1 - 2\sin^2 x$$

$$\therefore 1 + \cos 2x = 2\cos^2 x$$

$$1 - \cos 2x = 2\sin^2 x$$

$$\therefore y = \sqrt{\frac{2\cos^2 x}{2\sin^2 x}}$$

$$= \sqrt{\cot^2 x}$$

$$= \cot x$$

Differentiating y with respect to x

$$\frac{dy}{dx} = \frac{d}{dx}(\cot x)$$

$$= -\operatorname{cosec}^2 x$$

**Q. 13**

$$y = \frac{1 - \tan^2(x/2)}{1 + \tan^2(x/2)}, \text{ find } \frac{dy}{dx}$$

**Answer :** Formula:

Using Half angle formula,

$$\cos x = \frac{1 - \tan^2(x/2)}{1 + \tan^2(x/2)}$$

$$\therefore y = \cos x$$

Differentiating y with respect to x

$$\frac{dy}{dx} = \frac{d}{dx} \cos x$$

$$= -\sin x$$

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### Exercise 28B

**Q. 1. Find the derivation of each of the following from the first principle:**

**(ax + b)**

**Answer :** Let  $f(x) = ax + b$

We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \dots(i)$$

$$f(x) = ax + b$$

$$f(x + h) = a(x + h) + b$$

$$= ax + ah + b$$

Putting values in (i), we get

$$f'(x) = \lim_{h \rightarrow 0} \frac{ax + ah + b - (ax + b)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{ax + ah + b - ax - b}{h}$$

$$= \lim_{h \rightarrow 0} \frac{ah}{h}$$

$$= \lim_{h \rightarrow 0} a$$

$$f'(x) = a$$

Hence,  $f'(x) = a$



**Q. 2. Find the derivation of each of the following from the first principle:**

$$\left( ax^2 + \frac{b}{x} \right)$$

**Answer :**

$$\text{Let } f(x) = ax^2 + \frac{b}{x}$$

We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \dots(i)$$

$$f(x) = ax^2 + \frac{b}{x}$$

$$f(x+h) = a(x+h)^2 + \frac{b}{(x+h)}$$

Putting values in (i), we get

$$f'(x) = \lim_{h \rightarrow 0} \frac{\left[ a(x+h)^2 + \frac{b}{(x+h)} \right] - \left[ ax^2 + \frac{b}{x} \right]}{h}$$

$$= \lim_{h \rightarrow 0} \frac{a(x+h)^2 + \frac{b}{(x+h)} - ax^2 - \frac{b}{x}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{a[(x+h)^2 - x^2] + b \left[ \frac{1}{x+h} - \frac{1}{x} \right]}{h}$$

$$= \lim_{h \rightarrow 0} \frac{a[x^2 + h^2 + 2xh - x^2] + h \left[ \frac{x - (x+h)}{x(x+h)} \right]}{h}$$

$$= \lim_{h \rightarrow 0} \frac{a[h^2 + 2xh] + b \left[ \frac{x - x - h}{x(x+h)} \right]}{h}$$

$$= \lim_{h \rightarrow 0} \frac{a[h^2 + 2xh] + b \left[ \frac{-h}{x(x+h)} \right]}{h}$$

$$= \lim_{h \rightarrow 0} \left[ \frac{ah(h+2x)}{h} + \frac{b(-h)}{hx(x+h)} \right]$$

Taking 'h' common from both the numerator and denominator, we get

$$= \lim_{h \rightarrow 0} \left[ a(h+2x) - \frac{b}{x(x+h)} \right]$$

Putting  $h = 0$ , we get

$$= a[(0) + 2x] - \frac{b}{x(x+0)}$$

$$= 2ax - \frac{b}{x^2}$$

Hence,

$$f'(x) = 2ax - \frac{b}{x^2}$$

**Q. 3. Find the derivation of each of the following from the first principle:**

$$3x^2 + 2x - 5$$

**Answer :** Let  $f(x) = 3x^2 + 2x - 5$

We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \quad \dots (i)$$

$$f(x) = 3x^2 + 2x - 5$$

$$f(x+h) = 3(x+h)^2 + 2(x+h) - 5$$

$$= 3(x^2 + h^2 + 2xh) + 2x + 2h - 5$$

$$[\because (a+b)^2 = a^2 + b^2 + 2ab]$$

$$= 3x^2 + 3h^2 + 6xh + 2x + 2h - 5$$

Putting values in (i), we get

$$f'(x) = \lim_{h \rightarrow 0} \frac{3x^2 + 3h^2 + 6xh + 2x + 2h - 5 - (3x^2 + 2x - 5)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{3x^2 + 3h^2 + 6xh + 2x + 2h - 5 - 3x^2 - 2x + 5}{h}$$

$$= \lim_{h \rightarrow 0} \frac{3h^2 + 6xh + 2h}{h}$$

$$= \lim_{h \rightarrow 0} 3h + 6x + 2$$

Putting  $h = 0$ , we get

$$f'(x) = 3(0) + 6x + 2$$

$$= 6x + 2$$

Hence,  $f'(x) = 6x + 2$

**Q. 4 Find the derivation of each of the following from the first principle:**

$$x^3 - 2x^2 + x + 3$$

**Answer :** Let  $f(x) = x^3 - 2x^2 + x + 3$

We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \quad \dots(i)$$

$$f(x) = x^3 - 2x^2 + x + 3$$

$$f(x+h) = (x+h)^3 - 2(x+h)^2 + (x+h) + 3$$

Putting values in (i), we get

$$f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^3 - 2(x+h)^2 + (x+h) + 3 - [x^3 - 2x^2 + x + 3]}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(x+h)^3 - 2(x+h)^2 + (x+h) + 3 - x^3 + 2x^2 - x - 3}{h}$$

$$= \lim_{h \rightarrow 0} \frac{[(x+h)^3 - x^3] - 2[(x+h)^2 - x^2] + [x+h-x]}{h}$$

Using the identities:

$$(a + b)^3 = a^3 + b^3 + 3ab^2 + 3a^2b$$

$$(a + b)^2 = a^2 + b^2 + 2ab$$

$$= \lim_{h \rightarrow 0} \frac{[x^3 + h^3 + 3xh^2 + 3x^2h - x^3] - 2[x^2 + h^2 + 2xh - x^2] + h}{h}$$

$$= \lim_{h \rightarrow 0} \frac{[h^3 + 3xh^2 + 3x^2h] - 2[h^2 + 2xh] + h}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h[h^2 + 3xh + 3x^2] - 2h[h + 2x] + h}{h}$$

$$= \lim_{h \rightarrow 0} h^2 + 3xh + 3x^2 - 2h - 4x + 1$$

Putting  $h = 0$ , we get

$$f'(x) = (0)^2 + 2x(0) + 3x^2 - 2(0) - 4x + 1$$

$$= 3x^2 - 4x + 1$$

$$\text{Hence, } f'(x) = 3x^2 - 4x + 1$$



**Q. 5. Find the derivation of each of the following from the first principle:**

$$x^8$$

**Answer :** Let  $f(x) = x^8$

We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \dots(i)$$

$$f(x) = x^8$$

$$f(x + h) = (x + h)^8$$

Putting values in (i), we get

$$f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^8 - x^8}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(x+h)^8 - x^8}{(x+h) - x}$$

[Add and subtract x in denominator]

$$= \lim_{z \rightarrow x} \frac{z^8 - x^8}{z - x} \text{ where } z = x + h \text{ and } z \rightarrow x \text{ as } h \rightarrow 0$$

$$= 8x^{8-1} \left[ \because \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1} \right]$$

$$= 8x^7$$

Hence,  $f'(x) = 8x^7$

**Q. 6 Find the derivation of each of the following from the first principle:**

$$\frac{1}{x^3}$$



**Answer :**

$$\text{Let } f(x) = \frac{1}{x^3}$$

We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \dots (i)$$

$$f(x) = \frac{1}{x^3}$$

$$f(x+h) = \frac{1}{(x+h)^3}$$

Putting values in (i), we get

$$f'(x) = \lim_{h \rightarrow 0} \frac{\frac{1}{(x+h)^3} - \frac{1}{x^3}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(x+h)^{-3} - x^{-3}}{(x+h) - x}$$

[Add and subtract x in denominator]

$$= \lim_{z \rightarrow x} \frac{z^{-3} - x^{-3}}{z - x} \text{ where } z = x + h \text{ and } z \rightarrow x \text{ as } h \rightarrow 0$$

$$= (-3)x^{-3-1} \left[ \because \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1} \right]$$

$$= -3x^{-4}$$

$$= -\frac{3}{x^4}$$

Hence,

$$f'(x) = -\frac{3}{x^4}$$



**Q. 7. Find the derivation of each of the following from the first principle:**

$$\frac{1}{x^5}$$

**Answer :** Let ,

$$f(x) = \frac{1}{x^5}$$

We need to find the derivative of f(x) i.e. f'(x)

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \dots(i)$$

$$f(x) = \frac{1}{x^5}$$

$$f(x+h) = \frac{1}{(x+h)^5}$$

Putting values in (i), we get

$$f'(x) = \lim_{h \rightarrow 0} \frac{\frac{1}{(x+h)^5} - \frac{1}{x^5}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(x+h)^{-5} - x^{-5}}{(x+h) - x}$$

[Add and subtract x in denominator]

$$= \lim_{z \rightarrow x} \frac{z^{-5} - x^{-5}}{z - x} \text{ where } z = x+h \text{ and } z \rightarrow x \text{ as } h \rightarrow 0$$

$$= (-5)x^{-5-1} \left[ \because \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1} \right]$$

$$= -5x^{-6}$$

$$= -\frac{5}{x^6}$$

Hence,

$$f'(x) = -\frac{5}{x^6}$$

**Q. 8. Find the derivation of each of the following from the first principle:**

$$\sqrt{ax+b}$$

**Answer :** Let

$$f(x) = \sqrt{ax+b}$$

We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \dots(i)$$

$$f(x) = \sqrt{ax + b}$$

$$f(x+h) = \sqrt{a(x+h) + b}$$

$$= \sqrt{ax + ah + b}$$

Putting values in (i), we get

$$f'(x) = \lim_{h \rightarrow 0} \frac{\sqrt{ax + ah + b} - \sqrt{ax + b}}{h}$$

Now rationalizing the numerator by multiplying and divide by the conjugate of

$$\begin{aligned} & \sqrt{ax + ah + b} - \sqrt{ax + b} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{ax + ah + b} - \sqrt{ax + b}}{h} \times \frac{\sqrt{ax + ah + b} + \sqrt{ax + b}}{\sqrt{ax + ah + b} + \sqrt{ax + b}} \end{aligned}$$

Using the formula:

$$(a + b)(a - b) = (a^2 - b^2)$$

$$= \lim_{h \rightarrow 0} \frac{(\sqrt{ax + ah + b})^2 - (\sqrt{ax + b})^2}{h(\sqrt{ax + ah + b} + \sqrt{ax + b})}$$

$$= \lim_{h \rightarrow 0} \frac{ax + ah + b - ax - b}{h(\sqrt{ax + ah + b} + \sqrt{ax + b})}$$

$$= \lim_{h \rightarrow 0} \frac{ah}{h(\sqrt{ax + ah + b} + \sqrt{ax + b})}$$

$$= \lim_{h \rightarrow 0} \frac{a}{\sqrt{ax + ah + b} + \sqrt{ax + b}}$$

Putting  $h = 0$ , we get

$$= \frac{a}{\sqrt{ax + a(0) + b} + \sqrt{ax + b}}$$

$$= \frac{a}{\sqrt{ax + b} + \sqrt{ax + b}}$$

$$= \frac{a}{2\sqrt{ax + b}}$$

Hence,

$$f'(x) = \frac{a}{2\sqrt{ax+b}}$$

**Q. 9. Find the derivation of each of the following from the first principle:**

$$\sqrt{5x - 4}$$

**Answer :** Let

$$f(x) = \sqrt{5x - 4}$$



We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \dots(i)$$

$$f(x) = \sqrt{5x - 4}$$

$$f(x + h) = \sqrt{5(x + h) - 4}$$

$$= \sqrt{5x + 5h - 4}$$

Putting values in (i), we get

$$f'(x) = \lim_{h \rightarrow 0} \frac{\sqrt{5x + 5h - 4} - \sqrt{5x - 4}}{h}$$

Now rationalizing the numerator by multiplying and divide by the conjugate of

$$\begin{aligned} & \sqrt{5x + 5h - 4} - \sqrt{5x - 4} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{5x + 5h - 4} - \sqrt{5x - 4}}{h} \times \frac{\sqrt{5x + 5h - 4} + \sqrt{5x - 4}}{\sqrt{5x + 5h - 4} + \sqrt{5x - 4}} \end{aligned}$$

**Using the formula:**

$$(a + b)(a - b) = (a^2 - b^2)$$

$$= \lim_{h \rightarrow 0} \frac{(\sqrt{5x + 5h - 4})^2 - (\sqrt{5x - 4})^2}{h(\sqrt{5x + 5h - 4} + \sqrt{5x - 4})}$$

$$= \lim_{h \rightarrow 0} \frac{5x + 5h - 4 - 5x + 4}{h(\sqrt{5x + 5h - 4} + \sqrt{5x - 4})}$$

$$= \lim_{h \rightarrow 0} \frac{5h}{h(\sqrt{5x + 5h - 4} + \sqrt{5x - 4})}$$

$$= \lim_{h \rightarrow 0} \frac{5}{\sqrt{5x + 5h - 4} + \sqrt{5x - 4}}$$

Putting  $h = 0$ , we get

$$= \frac{5}{\sqrt{5x + 5(0) - 4} + \sqrt{5x - 4}}$$

$$= \frac{5}{\sqrt{5x - 4} + \sqrt{5x - 4}}$$

$$= \frac{5}{2\sqrt{5x - 4}}$$

Hence,

$$f'(x) = \frac{5}{2\sqrt{5x-4}}$$

**Q. 10. Find the derivation of each of the following from the first principle:**

$$\frac{1}{\sqrt{x+2}}$$

**Answer :** Let

$$f(x) = \frac{1}{\sqrt{x+2}}$$

We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \dots (i)$$

$$f(x) = \frac{1}{\sqrt{x+2}}$$

$$f(x+h) = \frac{1}{\sqrt{x+h+2}}$$

Putting values in (i), we get

$$f'(x) = \lim_{h \rightarrow 0} \frac{\frac{1}{\sqrt{x+h+2}} - \frac{1}{\sqrt{x+2}}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{\sqrt{x+2} - \sqrt{x+h+2}}{(\sqrt{x+h+2})(\sqrt{x+2})}}{h}$$

Now rationalizing the numerator by multiplying and divide by the conjugate of

$$\sqrt{x+2} - \sqrt{x+h+2}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{x+2} - \sqrt{x+h+2}}{h(\sqrt{x+h+2})(\sqrt{x+2})} \times \frac{\sqrt{x+2} + \sqrt{x+h+2}}{\sqrt{x+2} + \sqrt{x+h+2}}$$

**Using the formula:**

$$(a + b)(a - b) = (a^2 - b^2)$$

$$= \lim_{h \rightarrow 0} \frac{(\sqrt{x+2})^2 - (\sqrt{x+h+2})^2}{h(\sqrt{x+h+2})(\sqrt{x+2})(\sqrt{x+2} + \sqrt{x+h+2})}$$

$$= \lim_{h \rightarrow 0} \frac{x+2 - x-h-2}{h(\sqrt{x+h+2})(\sqrt{x+2})(\sqrt{x+2} + \sqrt{x+h+2})}$$

$$= \lim_{h \rightarrow 0} \frac{-h}{h(\sqrt{x+h+2})(\sqrt{x+2})(\sqrt{x+2} + \sqrt{x+h+2})}$$

$$= \lim_{h \rightarrow 0} \frac{-1}{(\sqrt{x+h+2})(\sqrt{x+2})(\sqrt{x+2} + \sqrt{x+h+2})}$$

Putting  $h = 0$ , we get

$$= \frac{-1}{(\sqrt{x+0+2})(\sqrt{x+2})(\sqrt{x+2} + \sqrt{x+0+2})}$$

$$= \frac{-1}{(\sqrt{x+2})^2 (2\sqrt{x+2})}$$

$$= \frac{-1}{2(\sqrt{x+2})^3}$$

Hence,

$$f'(x) = \frac{-1}{2(\sqrt{x+2})^3}$$

**Q. 11. Find the derivation of each of the following from the first principle:**

$$\frac{1}{\sqrt{2x+3}}$$

**Answer :** Let

$$f(x) = \frac{1}{\sqrt{2x+3}}$$

We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \dots (i)$$

$$f(x) = \frac{1}{\sqrt{2x + 3}}$$

$$f(x + h) = \frac{1}{\sqrt{2x + 2h + 3}}$$

Putting values in (i), we get

$$f'(x) = \lim_{h \rightarrow 0} \frac{\frac{1}{\sqrt{2x + 2h + 3}} - \frac{1}{\sqrt{2x + 3}}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{\sqrt{2x + 3} - \sqrt{2x + 2h + 3}}{(\sqrt{2x + 2h + 3})(\sqrt{2x + 3})}}{h}$$

Now rationalizing the numerator by multiplying and divide by the conjugate of

$$\begin{aligned} & \sqrt{2x + 3} - \sqrt{2x + 2h + 3} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{2x + 3} - \sqrt{2x + 2h + 3}}{h(\sqrt{2x + 2h + 3})(\sqrt{2x + 3})} \times \frac{\sqrt{2x + 3} + \sqrt{2x + 2h + 3}}{\sqrt{2x + 3} + \sqrt{2x + 2h + 3}} \end{aligned}$$

**Using the formula:**

$$(a + b)(a - b) = (a^2 - b^2)$$

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{(\sqrt{2x + 3})^2 - (\sqrt{2x + 2h + 3})^2}{h(\sqrt{2x + 2h + 3})(\sqrt{2x + 3})(\sqrt{2x + 3} + \sqrt{2x + 2h + 3})} \\ &= \lim_{h \rightarrow 0} \frac{2x + 3 - 2x - 2h - 3}{h(\sqrt{2x + 2h + 3})(\sqrt{2x + 3})(\sqrt{2x + 3} + \sqrt{2x + 2h + 3})} \end{aligned}$$

$$= \lim_{h \rightarrow 0} \frac{-2h}{h(\sqrt{2x+2h+3})(\sqrt{2x+3})(\sqrt{2x+3} + \sqrt{2x+2h+3})}$$

$$= \lim_{h \rightarrow 0} \frac{-2}{(\sqrt{2x+2h+3})(\sqrt{2x+3})(\sqrt{2x+3} + \sqrt{2x+2h+3})}$$

Putting  $h = 0$ , we get

$$= \frac{-2}{(\sqrt{2x+0+3})(\sqrt{2x+3})(\sqrt{2x+3} + \sqrt{2x+0+3})}$$

$$= \frac{-2}{(\sqrt{2x+3})^2 (2\sqrt{2x+3})}$$

$$= \frac{-2}{2(\sqrt{2x+3})^3}$$

$$= \frac{-1}{(\sqrt{2x+3})^3}$$



Hence,

$$f'(x) = \frac{-1}{(\sqrt{2x+3})^3}$$

**Q. 12. Find the derivation of each of the following from the first principle:**

$$\frac{1}{\sqrt{6x-5}}$$

**Answer :** Let

$$f(x) = \frac{1}{\sqrt{6x-5}}$$

We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \dots (i)$$

$$f(x) = \frac{1}{\sqrt{6x - 5}}$$

$$f(x + h) = \frac{1}{\sqrt{6x + 6h - 5}}$$

Putting values in (i), we get

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{\frac{1}{\sqrt{6x + 6h - 5}} - \frac{1}{\sqrt{6x - 5}}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{\sqrt{6x - 5} - \sqrt{6x + 6h - 5}}{(\sqrt{6x + 6h - 5})(\sqrt{6x - 5})}}{h} \end{aligned}$$

Now rationalizing the numerator by multiplying and divide by the conjugate of  $\sqrt{6x - 5} - \sqrt{6x + 6h - 5}$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{6x - 5} - \sqrt{6x + 6h - 5}}{h(\sqrt{6x + 6h - 5})(\sqrt{6x - 5})} \times \frac{\sqrt{6x - 5} + \sqrt{6x + 6h - 5}}{\sqrt{6x - 5} + \sqrt{6x + 6h - 5}}$$

**Using the formula:**

$$(a + b)(a - b) = (a^2 - b^2)$$

$$= \lim_{h \rightarrow 0} \frac{(\sqrt{6x - 5})^2 - (\sqrt{6x + 6h - 5})^2}{h(\sqrt{6x + 6h - 5})(\sqrt{6x - 5})(\sqrt{6x - 5} + \sqrt{6x + 6h - 5})}$$

$$= \lim_{h \rightarrow 0} \frac{6x - 5 - 6x - 6h + 5}{h(\sqrt{6x + 6h - 5})(\sqrt{6x - 5})(\sqrt{6x - 5} + \sqrt{6x + 6h - 5})}$$

$$= \lim_{h \rightarrow 0} \frac{-6h}{h(\sqrt{6x + 6h - 5})(\sqrt{6x - 5})(\sqrt{6x - 5} + \sqrt{6x + 6h - 5})}$$

$$= \lim_{h \rightarrow 0} \frac{-6}{(\sqrt{6x+6h-5})(\sqrt{6x-5})(\sqrt{6x-5} + \sqrt{6x+6h-5})}$$

Putting  $h = 0$ , we get

$$= \frac{-6}{(\sqrt{6x+6(0)-5})(\sqrt{6x-5})(\sqrt{6x-5} + \sqrt{6x+6(0)-5})}$$

$$= \frac{-6}{(\sqrt{6x-5})^2 (2\sqrt{6x-5})}$$

$$= \frac{-6}{2(\sqrt{6x-5})^3}$$

$$= \frac{-3}{(\sqrt{6x-5})^3}$$

Hence,

$$f'(x) = \frac{-3}{(\sqrt{6x-5})^3}$$



**Q. 13. Find the derivation of each of the following from the first principle:**

$$\frac{1}{\sqrt{2-3x}}$$

**Answer :** Let

$$f(x) = \frac{1}{\sqrt{2-3x}}$$

We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h)-f(x)}{h} \dots(i)$$

$$f(x) = \frac{1}{\sqrt{2-3x}}$$

$$f(x+h) = \frac{1}{\sqrt{2-3(x+h)}} = \frac{1}{\sqrt{2-3x-3h}}$$

Putting values in (i), we get

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{\frac{1}{\sqrt{2-3x-3h}} - \frac{1}{\sqrt{2-3x}}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{\sqrt{2-3x} - \sqrt{2-3x-3h}}{\sqrt{2-3x-3h}(\sqrt{2-3x})}}{h} \end{aligned}$$

Now rationalizing the numerator by multiplying and divide by the conjugate of  $\sqrt{2-3x} - \sqrt{2-3x-3h}$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{2-3x} - \sqrt{2-3x-3h}}{h\sqrt{2-3x-3h}(\sqrt{2-3x})} \times \frac{\sqrt{2-3x} + \sqrt{2-3x-3h}}{\sqrt{2-3x} + \sqrt{2-3x-3h}}$$

**Using the formula:**

$$(a+b)(a-b) = (a^2 - b^2)$$

$$= \lim_{h \rightarrow 0} \frac{(\sqrt{2-3x})^2 - (\sqrt{2-3x-3h})^2}{h(\sqrt{2-3x-3h})(\sqrt{2-3x})(\sqrt{2-3x} + \sqrt{2-3x-3h})}$$

$$= \lim_{h \rightarrow 0} \frac{2-3x-2+3x+3h}{h(\sqrt{2-3x-3h})(\sqrt{2-3x})(\sqrt{2-3x} + \sqrt{2-3x-3h})}$$

$$= \lim_{h \rightarrow 0} \frac{3h}{h(\sqrt{2-3x-3h})(\sqrt{2-3x})(\sqrt{2-3x} + \sqrt{2-3x-3h})}$$

$$= \lim_{h \rightarrow 0} \frac{3}{(\sqrt{2-3x-3h})(\sqrt{2-3x})(\sqrt{2-3x} + \sqrt{2-3x-3h})}$$

Putting  $h = 0$ , we get

$$\begin{aligned} &= \frac{3}{(\sqrt{2-3x-3(0)})(\sqrt{2-3x})(\sqrt{2-3x} + \sqrt{2-3x-3(0)})} \\ &= \frac{3}{(\sqrt{2-3x})^2 (2\sqrt{2-3x})} \\ &= \frac{3}{2(\sqrt{2-3x})^3} \end{aligned}$$

Hence,

$$f'(x) = \frac{3}{2(\sqrt{2-3x})^3}$$

**Q. 14. Find the derivation of each of the following from the first principle:**

$$\frac{2x+3}{3x+2}$$



**Answer :** Let,

$$f(x) = \frac{2x+3}{3x+2}$$

We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \dots(i)$$

$$f(x) = \frac{2x+3}{3x+2}$$

$$f(x+h) = \frac{2(x+h)+3}{3(x+h)+2} = \frac{2x+2h+3}{3x+3h+2}$$

Putting values in (i), we get

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{\frac{2x + 2h + 3}{3x + 3h + 2} - \frac{2x + 3}{3x + 2}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\frac{(2x + 2h + 3)(3x + 2) - (2x + 3)(3x + 3h + 2)}{(3x + 3h + 2)(3x + 2)}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{6x^2 + 4x + 6xh + 4h + 9x + 6 - [6x^2 + 6xh + 4x + 9x + 9h + 6]}{h((3x + 3h + 2)(3x + 2))} \\
 &= \lim_{h \rightarrow 0} \frac{6x^2 + 4x + 6xh + 4h + 9x + 6 - 6x^2 - 6xh - 4x - 9x - 9h - 6}{h((3x + 3h + 2)(3x + 2))} \\
 &= \lim_{h \rightarrow 0} \frac{-5h}{h((3x + 3h + 2)(3x + 2))} \\
 &= \lim_{h \rightarrow 0} \frac{-5}{((3x + 3h + 2)(3x + 2))}
 \end{aligned}$$

Putting  $h = 0$ , we get

$$\begin{aligned}
 &= \frac{-5}{((3x + 3(0) + 2)(3x + 2))} \\
 &= \frac{-5}{(3x + 2)(3x + 2)} \\
 &= \frac{-5}{(3x + 2)^2}
 \end{aligned}$$

Hence,

$$f'(x) = \frac{-5}{(3x+2)^2}$$

**Q. 15. Find the derivation of each of the following from the first principle:**

$$\frac{5-x}{5+x}$$

**Answer :** Let

$$f(x) = \frac{5-x}{5+x}$$

We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \dots(i)$$

$$f(x) = \frac{5-x}{5+x}$$

$$f(x+h) = \frac{5-(x+h)}{5+(x+h)} = \frac{5-x-h}{5+x+h}$$

Putting values in (i), we get

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{\frac{5-x-h}{5+x+h} - \frac{5-x}{5+x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{(5-x-h)(5+x) - (5-x)(5+x+h)}{h(5+x+h)(5+x)} \\ &= \lim_{h \rightarrow 0} \frac{25 + 5x - 5x - x^2 - 5h - xh - [25 + 5x + 5h - 5x - x^2 - xh]}{h(5+x+h)(5+x)} \\ &= \lim_{h \rightarrow 0} \frac{25 - x^2 - 5h - xh - 25 - 5h + x^2 + xh}{h(5+x+h)(5+x)} \\ &= \lim_{h \rightarrow 0} \frac{-10h}{h(5+x+h)(5+x)} \\ &= \lim_{h \rightarrow 0} \frac{-10}{(5+x+h)(5+x)} \end{aligned}$$

Putting  $h = 0$ , we get

$$= \frac{-10}{(5+x+0)(5+x)}$$

$$= \frac{-10}{(5+x)(5+x)}$$

$$= \frac{-10}{(5+x)^2}$$

Hence,

$$f'(x) = \frac{-10}{(5+x)^2}$$

**Q. 16. Find the derivation of each of the following from the first principle:**

$$\frac{x^2 + 1}{x}, x \neq 0$$

**Answer :** Let

$$f(x) = \frac{x^2 + 1}{x}$$



We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \dots (i)$$

$$f(x) = \frac{x^2 + 1}{x}$$

$$f(x+h) = \frac{(x+h)^2 + 1}{x+h} = \frac{x^2 + h^2 + 2xh + 1}{x+h}$$

Putting values in (i), we get

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{\frac{x^2 + h^2 + 2xh + 1}{x+h} - \frac{x^2 + 1}{x}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\frac{(x^2 + h^2 + 2xh + 1)(x) - (x^2 + 1)(x+h)}{(x+h)(x)}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{x^3 + xh^2 + 2x^2h + x - [x^3 + x^2h + x + h]}{h(x+h)(x)} \\
 &= \lim_{h \rightarrow 0} \frac{x^3 + xh^2 + 2x^2h + x - x^3 - x^2h - x - h}{h(x+h)(x)} \\
 &= \lim_{h \rightarrow 0} \frac{xh^2 + x^2h - h}{h(x+h)(x)} \\
 &= \lim_{h \rightarrow 0} \frac{xh + x^2 - 1}{(x+h)(x)}
 \end{aligned}$$

Putting  $h = 0$ , we get

$$\begin{aligned}
 &= \frac{x(0) + x^2 - 1}{(x+0)(x)} \\
 &= \frac{x^2 - 1}{(x)^2}
 \end{aligned}$$

Hence,

$$f'(x) = \frac{x^2 - 1}{x^2}$$

**Q. 17. Find the derivation of each of the following from the first principle:**

$$\sqrt{\cos 3x}$$

**Answer :** Let

$$f(x) = \sqrt{\cos 3x}$$

We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \dots (i)$$

$$f(x) = \sqrt{\cos 3x}$$

$$f(x+h) = \sqrt{\cos 3(x+h)}$$

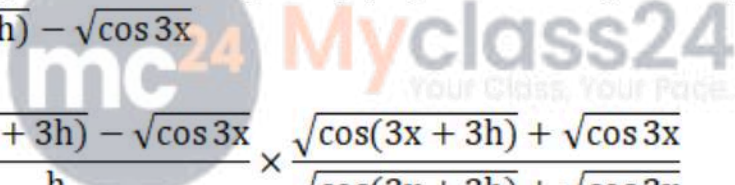
$$= \sqrt{\cos(3x+3h)}$$

Putting values in (i), we get

$$f'(x) = \lim_{h \rightarrow 0} \frac{\sqrt{\cos(3x+3h)} - \sqrt{\cos 3x}}{h}$$

Now rationalizing the numerator by multiplying and divide by the conjugate

of  $\sqrt{\cos(3x+3h)} - \sqrt{\cos 3x}$


$$= \lim_{h \rightarrow 0} \frac{\sqrt{\cos(3x+3h)} - \sqrt{\cos 3x}}{h} \times \frac{\sqrt{\cos(3x+3h)} + \sqrt{\cos 3x}}{\sqrt{\cos(3x+3h)} + \sqrt{\cos 3x}}$$

**Using the formula:**

$$(a+b)(a-b) = (a^2 - b^2)$$

$$= \lim_{h \rightarrow 0} \frac{(\sqrt{\cos(3x+3h)})^2 - (\sqrt{\cos 3x})^2}{h(\sqrt{\cos(3x+3h)} + \sqrt{\cos 3x})}$$

$$= \lim_{h \rightarrow 0} \frac{\cos(3x+3h) - \cos 3x}{h(\sqrt{\cos(3x+3h)} + \sqrt{\cos 3x})}$$

**Using the formula:**

$$\cos A - \cos B = -2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$

$$\begin{aligned}
&= \lim_{h \rightarrow 0} \frac{-2 \sin \frac{3x+3h+3x}{2} \sin \frac{3x+3h-3x}{2}}{h(\sqrt{\cos(3x+3h)} + \sqrt{\cos 3x})} \\
&= \lim_{h \rightarrow 0} \frac{-2 \sin \frac{6x+3h}{2} \sin \frac{3h}{2}}{h\sqrt{\cos(3x+3h)} + \sqrt{\cos 3x}} \\
&= -2 \lim_{h \rightarrow 0} \frac{\sin \frac{3h}{2}}{\frac{3h}{2}} \times \frac{3}{2} \lim_{h \rightarrow 0} \sin\left(\frac{6x+3h}{2}\right) \times \lim_{h \rightarrow 0} \frac{1}{\sqrt{\cos(3x+3h)} + \sqrt{\cos 3x}}
\end{aligned}$$

[Here, we multiply and divide by  $\frac{3}{2}$ ]

$$\begin{aligned}
&= -2 \times \frac{3}{2} \lim_{h \rightarrow 0} \frac{\sin \frac{3h}{2}}{\frac{3h}{2}} \times \lim_{h \rightarrow 0} \sin\left(\frac{6x+3h}{2}\right) \times \lim_{h \rightarrow 0} \frac{1}{\sqrt{\cos(3x+3h)} + \sqrt{\cos 3x}} \\
&= -3 \times (1) \times \lim_{h \rightarrow 0} \sin\left(\frac{6x+3h}{2}\right) \times \lim_{h \rightarrow 0} \frac{1}{\sqrt{\cos(3x+3h)} + \sqrt{\cos 3x}}
\end{aligned}$$

$$\left[ \because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right]$$

Putting  $h = 0$ , we get

$$\begin{aligned}
&= -3 \times \sin\left[\frac{6x+3(0)}{2}\right] \times \frac{1}{\sqrt{\cos(3x+3(0))} + \sqrt{\cos 3x}} \\
&= -3 \sin 3x \times \frac{1}{2\sqrt{\cos 3x}} \\
&= -\frac{3 \sin 3x}{2(\cos 3x)^{\frac{1}{2}}}
\end{aligned}$$

Hence,

$$f'(x) = -\frac{3 \sin 3x}{2(\cos 3x)^{\frac{3}{2}}}$$

**Q. 18. Find the derivation of each of the following from the first principle:**

$$\sqrt{\sec x}$$

**Answer :** Let

$$f(x) = \sqrt{\sec x}$$

We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \dots(i)$$

$$f(x) = \sqrt{\sec x}$$

$$f(x+h) = \sqrt{\sec(x+h)}$$

Putting values in (i), we get

$$f'(x) = \lim_{h \rightarrow 0} \frac{\sqrt{\sec(x+h)} - \sqrt{\sec x}}{h}$$

Now rationalizing the numerator by multiplying and divide by the conjugate of  $\sqrt{\sec(x+h)} - \sqrt{\sec x}$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{\sec(x+h)} - \sqrt{\sec x}}{h} \times \frac{\sqrt{\sec(x+h)} + \sqrt{\sec x}}{\sqrt{\sec(x+h)} + \sqrt{\sec x}}$$

**Using the formula:**

$$(a+b)(a-b) = (a^2 - b^2)$$

$$= \lim_{h \rightarrow 0} \frac{(\sqrt{\sec(x+h)})^2 - (\sqrt{\sec x})^2}{h(\sqrt{\sec(x+h)} + \sqrt{\sec x})}$$

$$\begin{aligned}
&= \lim_{h \rightarrow 0} \frac{\sec(x+h) - \sec(x)}{h(\sqrt{\sec(x+h)} + \sqrt{\sec x})} \\
&= \lim_{h \rightarrow 0} \frac{\frac{1}{\cos(x+h)} - \frac{1}{\cos x}}{h(\sqrt{\sec(x+h)} + \sqrt{\sec x})} \\
&= \lim_{h \rightarrow 0} \frac{\frac{\cos x - \cos(x+h)}{\cos(x+h)\cos x}}{h(\sqrt{\sec(x+h)} + \sqrt{\sec x})} \\
&= \lim_{h \rightarrow 0} \frac{\cos x - \cos(x+h)}{h(\cos(x+h)\cos x)(\sqrt{\sec(x+h)} + \sqrt{\sec x})}
\end{aligned}$$

Using the formula:

$$\begin{aligned}
\cos A - \cos B &= 2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{B-A}{2}\right) \\
&= \lim_{h \rightarrow 0} \frac{2 \sin \frac{x+(x+h)}{2} \sin \frac{(x+h)-x}{2}}{h(\cos(x+h)\cos x)(\sqrt{\sec(x+h)} + \sqrt{\sec x})} \\
&= \lim_{h \rightarrow 0} \frac{2 \sin \frac{2x+h}{2} \sin \frac{h}{2}}{h(\cos(x+h)\cos x)(\sqrt{\sec(x+h)} + \sqrt{\sec x})} \\
&= 2 \lim_{h \rightarrow 0} \frac{\sin \frac{h}{2}}{\frac{h}{2}} \\
&\quad \times \frac{1}{2} \lim_{h \rightarrow 0} \sin\left(\frac{2x+h}{2}\right) \\
&\quad \times \lim_{h \rightarrow 0} \frac{1}{(\cos(x+h)\cos x)(\sqrt{\sec(x+h)} + \sqrt{\sec x})}
\end{aligned}$$

[Here, we multiply and divide by  $\frac{1}{2}$ ]

$$\begin{aligned}
&= 2 \times \frac{1}{2} \lim_{h \rightarrow 0} \frac{\sin \frac{h}{2}}{\frac{h}{2}} \\
&\quad \times \lim_{h \rightarrow 0} \sin\left(x + \frac{h}{2}\right) \times \lim_{h \rightarrow 0} \frac{1}{(\cos(x+h) \cos x)(\sqrt{\sec(x+h)} + \sqrt{\sec x})} \\
&= (1) \times \lim_{h \rightarrow 0} \sin\left(x + \frac{h}{2}\right) \times \lim_{h \rightarrow 0} \frac{1}{(\cos(x+h) \cos x)(\sqrt{\sec(x+h)} + \sqrt{\sec x})}
\end{aligned}$$

$$\left[ \because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right]$$

Putting  $h = 0$ , we get

$$\begin{aligned}
&= \sin\left[x + \frac{0}{2}\right] \times \frac{1}{\cos(x+0) \cos x (\sqrt{\sec(x+0)} + \sqrt{\sec x})} \\
&= \sin x \times \frac{1}{\cos x \cos x (\sqrt{\sec x} + \sqrt{\sec x})} \\
&= \frac{\sin x}{\cos^2 x (2\sqrt{\sec x})} \\
&= \frac{\sin x}{\cos x} \times \frac{1}{\cos x} \times \frac{1}{2\sqrt{\sec x}} \\
&= \tan x \times \sec x \times \frac{1}{2\sqrt{\sec x}} \left[ \because \frac{\sin x}{\cos x} = \tan x \right] \& \left[ \frac{1}{\cos x} = \sec x \right] \\
&= \frac{1}{2} \tan x \sqrt{\sec x}
\end{aligned}$$

Hence,

$$f'(x) = \frac{1}{2} \tan x \sqrt{\sec x}$$

**Q. 19. Find the derivation of each of the following from the first principle:**

**$\tan^2 x$**

**Answer :** Let  $f(x) = \tan^2 x$

We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \dots (i)$$

$$f(x) = \tan^2 x$$

$$f(x + h) = \tan^2(x + h)$$

Putting values in (i), we get

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{\tan^2(x + h) - \tan^2 x}{h} \\ &= \lim_{h \rightarrow 0} \frac{[\tan(x + h) - \tan x][\tan(x + h) + \tan x]}{h} \end{aligned}$$

**Using:**

$$\tan x = \frac{\sin x}{\cos x}$$

$$= \lim_{h \rightarrow 0} \frac{\left[ \frac{\sin(x + h)}{\cos(x + h)} - \frac{\sin x}{\cos x} \right] \left[ \frac{\sin(x + h)}{\cos(x + h)} + \frac{\sin x}{\cos x} \right]}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\left[ \frac{\sin(x + h) \cos x - \sin x \cos(x + h)}{\cos(x + h) \cos x} \right] \left[ \frac{\sin(x + h) \cos x + \sin x \cos(x + h)}{\cos(x + h) \cos x} \right]}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\{\sin[(x + h) - x]\} \{\sin[(x + h) + x]\}}{h [\cos^2(x + h) \cos^2 x]}$$

[ $\because \sin A \cos B - \sin B \cos A = \sin(A - B)$ ]

$$\& \sin A \cos B + \sin B \cos A = \sin(A + B)]$$

$$= \lim_{h \rightarrow 0} \frac{[\sin h][\sin(2x + h)]}{h[\cos^2(x + h) \cos^2 x]}$$

$$= \frac{1}{\cos^2 x} \lim_{h \rightarrow 0} \frac{\sin h}{h} \times \lim_{h \rightarrow 0} \sin(2x + h) \times \lim_{h \rightarrow 0} \frac{1}{\cos^2(x + h)}$$

$$= \frac{1}{\cos^2 x} \times (1) \times \lim_{h \rightarrow 0} \sin(2x + h) \times \lim_{h \rightarrow 0} \frac{1}{\cos^2(x + h)}$$

$$\left[ \because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right]$$

Putting  $h = 0$ , we get

$$= \frac{1}{\cos^2 x} \times \sin(2x + 0) \times \frac{1}{\cos^2(x + 0)}$$

$$= \frac{1}{\cos^2 x} \times \sin 2x \times \frac{1}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} \times 2 \sin x \cos x \times \sec^2 x$$

$$[\because \sin 2x = 2 \sin x \cos x]$$

$$= 2 \frac{\sin x}{\cos x} \times \sec^2 x \left[ \because \frac{1}{\cos x} = \sec x \right]$$

$$= 2 \tan x \sec^2 x$$

$$\left[ \because \frac{\sin x}{\cos x} = \tan x \right]$$

Hence,  $f'(x) = 2 \tan x \sec^2 x$

**Q. 20. Find the derivation of each of the following from the first principle:**

**$\sin(2x + 3)$**

**Answer :** Let  $f(x) = \sin(2x + 3)$

We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \dots(i)$$

$$f(x) = \sin(2x + 3)$$

$$f(x + h) = \sin[2(x + h) + 3]$$

Putting values in (i), we get

$$f'(x) = \lim_{h \rightarrow 0} \frac{\sin[2(x + h) + 3] - \sin(2x + 3)}{h}$$

**Using the formula:**

$$\sin A - \sin B = 2 \sin \frac{A - B}{2} \cos \frac{A + B}{2}$$

$$= \lim_{h \rightarrow 0} \frac{2 \sin \frac{2(x + h) + 3 - (2x + 3)}{2} \cos \frac{2(x + h) + 3 + 2x + 3}{2}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2 \sin \frac{2x + 2h + 3 - 2x - 3}{2} \cos \frac{2x + 2h + 6 + 2x}{2}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2 \sin \frac{2h}{2} \cos \frac{4x + 2h + 6}{2}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2 \sin(h) \cos(2x + h + 3)}{h}$$

$$= 2 \lim_{h \rightarrow 0} \frac{\sin h}{h} \times \lim_{h \rightarrow 0} \cos(2x + h + 3)$$

$$= 2(1) \times \lim_{h \rightarrow 0} \cos(2x + h + 3)$$

$$\left[ \because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right]$$

Putting  $h = 0$ , we get

$$= 2\cos(2x + 0 + 3)$$

$$= 2\cos(2x + 3)$$

Hence,  $f'(x) = 2\cos(2x + 3)$

**Q. 21. Find the derivation of each of the following from the first principle:**

**$\tan(3x + 1)$**

**Answer :** Let  $f(x) = \tan(3x + 1)$

We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \dots(i)$$

$$f(x) = \tan(3x + 1)$$

$$f(x + h) = \tan[3(x + h) + 1]$$

Putting values in (i), we get

$$f'(x) = \lim_{h \rightarrow 0} \frac{\tan[3(x + h) + 1] - \tan[3x + 1]}{h}$$

**Using the formula:**

$$\tan A - \tan B = \frac{\sin(A - B)}{\cos A \cos B}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{\sin[3(x + h) + 1 - (3x + 1)]}{\cos[3(x + h) + 1] \cos[3x + 1]}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{\sin[3x + 3h + 1 - 3x - 1]}{\cos[3(x + h) + 1] \cos[3x + 1]}}{h}$$

$$\begin{aligned}
&= \lim_{h \rightarrow 0} \frac{\sin 3h}{h[\cos[3(x+h) + 1]\cos[3x + 1]]} \\
&= \lim_{h \rightarrow 0} \frac{\sin 3h}{h} \times \lim_{h \rightarrow 0} \frac{1}{\cos[3(x+h) + 1]\cos[3x + 1]} \\
&= \lim_{h \rightarrow 0} \frac{\sin 3h}{3h} \times 3 \times \lim_{h \rightarrow 0} \frac{1}{\cos[3(x+h) + 1]\cos[3x + 1]} \\
&= 3(1) \times \lim_{h \rightarrow 0} \frac{1}{\cos[3(x+h) + 1]\cos[3x + 1]}
\end{aligned}$$

$$\left[ \because \lim_{x \rightarrow 0} \frac{\sin 3x}{3x} = 1 \right]$$

Putting  $h = 0$ , we get

$$\begin{aligned}
&= 3 \times \frac{1}{\cos[3(x+0) + 1]\cos[3x + 1]} \\
&= \frac{3}{\cos[3x + 1]\cos[3x + 1]} \\
&= \frac{3}{\cos^2(3x + 1)} \\
&= 3\sec^2(3x + 1) \left[ \because \frac{1}{\cos x} = \sec x \right]
\end{aligned}$$

Hence,  $f'(x) = 3\sec^2(3x + 1)$

### Exercise 28C

**Q. 1. Differentiate:**

$$x^2 \sin x$$

**Answer :** To find: Differentiation of  $x^2 \sin x$

**Formula used:** (i)  $(uv)' = u'v + uv'$  (Leibnitz or product rule)

(ii)

$$\frac{dx^n}{dx} = nx^{n-1}$$

(iii)

$$\frac{d\sin x}{dx} = \cos x$$

Let us take  $u = x^2$  and  $v = \sin x$

$$u' = \frac{du}{dx} = \frac{d(x^2)}{dx} = 2x$$

$$v' = \frac{dv}{dx} = \frac{d(\sin x)}{dx} = \cos x$$

Putting the above obtained values in the formula:-

$$(uv)' = u'v + uv'$$

$$(x^2 \sin x)' = 2x \times \sin x + x^2 \times \cos x$$

$$= 2x \sin x + x^2 \cos x$$

$$\text{Ans) } 2x \sin x + x^2 \cos x$$

**Q. 2. Differentiate:**

**$e^x \cos x$**

**Answer :** To find: Differentiation of  $e^x \cos x$

**Formula used: (i)**  $(uv)' = u'v + uv'$  (Leibnitz or product rule)

(ii)

$$\frac{de^x}{dx} = e^x$$

(iii)

$$\frac{d\cos x}{dx} = -\sin x$$

Let us take  $u = e^x$  and  $v = \cos x$

$$u' = \frac{du}{dx} = \frac{de^x}{dx} = e^x$$

$$v' = \frac{dv}{dx} = \frac{d\cos x}{dx} = -\sin x$$

**Putting the above obtained values in the formula:-**

$$(uv)' = u'v + uv'$$

$$(e^x \cos x)' = e^x \times \cos x + e^x \times -\sin x$$

$$= e^x \cos x - e^x \sin x$$

$$= e^x (\cos x - \sin x)$$

**Ans)**  $e^x (\cos x - \sin x)$

**Q. 3. Differentiate:**

$e^x \cot x$

**Answer :** To find: Differentiation of  $e^x \cot x$

**Formula used:** (i)  $(uv)' = u'v + uv'$  (Leibnitz or product rule)

(ii)

$$\frac{de^x}{dx} = e^x$$

(iii)

$$\frac{d\cot x}{dx} = -\operatorname{cosec}^2 x$$

Let us take  $u = e^x$  and  $v = \cot x$

$$u' = \frac{du}{dx} = \frac{de^x}{dx} = e^x$$

$$v' = \frac{dv}{dx} = \frac{d\cot x}{dx} = -\operatorname{cosec}^2 x$$



**Putting the above obtained values in the formula:-**

$$(uv)' = u'v + uv'$$

$$(e^x \cot x)' = e^x \cot x + e^x \times -\operatorname{cosec}^2 x$$

$$= e^x \cot x - e^x \operatorname{cosec}^2 x$$

$$= e^x (\cot x - \operatorname{cosec}^2 x)$$

**Ans)**  $e^x (\cot x - \operatorname{cosec}^2 x)$

**Q. 4. Differentiate:**

**$x^n \cot x$**

**Answer :** To find: Differentiation of  $x^n \cot x$

Formula used: (i)  $(uv)' = u'v + uv'$  (Leibnitz or product rule)

(ii)

$$\frac{dx^n}{dx} = nx^{n-1}$$



(iii)

$$\frac{d \cot x}{dx} = -\operatorname{cosec}^2 x$$

Let us take  $u = x^n$  and  $v = \cot x$

$$u' = \frac{du}{dx} = \frac{dx^n}{dx} = nx^{n-1}$$

$$v' = \frac{dv}{dx} = \frac{d \cot x}{dx} = -\operatorname{cosec}^2 x$$

Putting the above obtained values in the formula :-

$$(uv)' = u'v + uv'$$

$$(x^n \cot x)' = nx^{n-1} \times \cot x + x^n \times -\operatorname{cosec}^2 x$$

$$= nx^{n-1} \cot x - x^n \operatorname{cosec}^2 x$$

$$= x^n (nx^{-1}\cot x - \operatorname{cosec}^2 x)$$

$$\text{Ans) } x^n (nx^{-1}\cot x - \operatorname{cosec}^2 x)$$

**Q. 5. Differentiate:**

$$x^3 \sec x$$

**Answer :** To find: Differentiation of  $x^3 \sec x$

**Formula used:** (i)  $(uv)' = u'v + uv'$  (Leibnitz or product rule)

(ii)

$$\frac{dx^n}{dx} = nx^{n-1}$$

(iii)

$$\frac{d\sec x}{dx} = \sec x \tan x$$

Let us take  $u = x^3$  and  $v = \sec x$

$$u' = \frac{du}{dx} = \frac{dx^3}{dx} = 3x^2$$

$$v' = \frac{dv}{dx} = \frac{d\sec x}{dx} = \sec x \tan x$$

Putting the above obtained values in the formula :-

$$(uv)' = u'v + uv'$$

$$(x^3 \sec x)' = 3x^2 \times \sec x + x^3 \times \sec x \tan x$$

$$= 3x^2 \sec x + x^3 \sec x \tan x$$

$$= x^2 \sec x (3 + x \tan x)$$

$$\text{Ans) } x^2 \sec x (3 + x \tan x)$$

**Q. 6. Differentiate:**

$$(x^2 + 3x + 1) \sin x$$

**Answer :** To find: Differentiation of  $(x^2 + 3x + 1) \sin x$

**Formula used:** (i)  $(uv)' = u'v + uv'$  (Leibnitz or product rule)

(ii)

$$\frac{dx^n}{dx} = nx^{n-1}$$

(iii)

$$\frac{d\sin x}{dx} = \cos x$$

Let us take  $u = x^2 + 3x + 1$  and  $v = \sin x$

$$u' = \frac{du}{dx} = \frac{d(x^2 + 3x + 1)}{dx} = 2x + 3$$

$$v' = \frac{dv}{dx} = \frac{d\sin x}{dx} = \cos x$$

**Putting the above obtained values in the formula :-**

$$(uv)' = u'v + uv'$$

$$[(x^2 + 3x + 1) \sin x]' = (2x + 3) \times \sin x + (x^2 + 3x + 1) \times \cos x$$

$$= \sin x (2x + 3) + \cos x (x^2 + 3x + 1)$$

$$\text{Ans) } (2x + 3) \sin x + (x^2 + 3x + 1) \cos x$$

**Q. 7. Differentiate:**

$x^4 \tan x$

**Answer :** To find: Differentiation of  $x^4 \tan x$

**Formula used:** (i)  $(uv)' = u'v + uv'$  (Leibnitz or product rule)

(ii)

$$\frac{dx^n}{dx} = nx^{n-1}$$

(iii)

$$\frac{d \tan x}{dx} = \sec^2 x$$

Let us take  $u = x^4$  and  $v = \tan x$

$$u' = \frac{du}{dx} = \frac{dx^4}{dx} = 4x^3$$

$$v' = \frac{dv}{dx} = \frac{d \tan x}{dx} = \sec^2 x$$

Putting the above obtained values in the formula:-

$$(uv)' = u'v + uv'$$

$$(x^4 \tan x)' = 4x^3 \times \tan x + x^4 \times \sec^2 x$$

$$= 4x^3 \tan x + x^4 \sec^2 x$$

$$= x^3 (4 \tan x + x \sec^2 x)$$

$$\text{Ans } x^3 (4 \tan x + x \sec^2 x)$$

**Q. 8. Differentiate:**

$$(3x - 5)(4x^2 - 3 + e^x)$$

**Answer :** To find: Differentiation of  $(3x - 5)(4x^2 - 3 + e^x)$

**Formula used:** (i)  $(uv)' = u'v + uv'$  (Leibnitz or product rule)

(ii)

$$\frac{dx^n}{dx} = nx^{n-1}$$

(iii)

$$\frac{de^x}{dx} = e^x$$

Let us take  $u = (3x - 5)$  and  $v = (4x^2 - 3 + e^x)$

$$u' = \frac{du}{dx} = \frac{d(3x - 5)}{dx} = 3$$

$$v' = \frac{dv}{dx} = \frac{d(4x^2 - 3 + e^x)}{dx} = (8x + e^x)$$

Putting the above obtained values in the formula :-

$$(uv)' = u'v + uv'$$

$$[(3x - 5)(4x^2 - 3 + e^x)]' = 3 \times (4x^2 - 3 + e^x) + (3x - 5) \times (8x + e^x)$$

$$= 12x^2 - 9 + 3e^x + 24x^2 + 3xe^x - 40x - 5e^x$$

$$= 36x^2 + x(3e^x - 40) - 9 - 2e^x$$

$$\text{Ans) } 36x^2 + x(3e^x - 40) - 9 - 2e^x$$

**Q. 9. Differentiate:**

$$(x^2 - 4x + 5)(x^3 - 2)$$

**Answer : To find:** Differentiation of  $(x^2 - 4x + 5)(x^3 - 2)$

**Formula used:** (i)  $(uv)' = u'v + uv'$  (Leibnitz or product rule)

(ii)

$$\frac{dx^n}{dx} = nx^{n-1}$$

Let us take  $u = (x^2 - 4x + 5)$  and  $v = (x^3 - 2)$

$$u' = \frac{du}{dx} = \frac{d(x^2 - 4x + 5)}{dx} = 2x - 4$$

$$v' = \frac{dv}{dx} = \frac{d(x^3 - 2)}{dx} = 3x^2$$

**Putting the above obtained values in the formula:-**

$$(uv)' = u'v + uv'$$

$$(x^2 - 4x + 5)(x^3 - 2)]' = (2x - 4) \times (x^3 - 2) + (x^2 - 4x + 5) \times (3x^2)$$

$$= 2x^4 - 4x - 4x^3 + 8 + 3x^4 - 12x^3 + 15x^2$$

$$= 5x^4 - 16x^3 + 15x^2 - 4x + 8$$

Ans)  $5x^4 - 16x^3 + 15x^2 - 4x + 8$

**Q. 10. Differentiate:**

$$(x^2 + 2x - 3)(x^2 + 7x + 5)$$

**Answer :** To find: Differentiation of  $(x^2 + 2x - 3)(x^2 + 7x + 5)$

**Formula used:** (i)  $(uv)' = u'v + uv'$  (Leibnitz or product rule)

(ii)

$$\frac{dx^n}{dx} = nx^{n-1}$$

Let us take  $u = (x^2 + 2x - 3)$  and  $v = (x^2 + 7x + 5)$

$$u' = \frac{du}{dx} = \frac{d(x^2 + 2x - 3)}{dx} = 2x + 2$$

$$v' = \frac{dv}{dx} = \frac{d(x^2 + 7x + 5)}{dx} = 2x + 7$$

Putting the above obtained values in the formula :-

$$(uv)' = u'v + uv'$$

$$[(x^2 + 2x - 3)(x^2 + 7x + 5)]'$$

$$= (2x + 2) \times (x^2 + 7x + 5) + (x^2 + 2x - 3) \times (2x + 7)$$

$$= 2x^3 + 14x^2 + 10x + 2x^2 + 14x + 10 + 2x^3 + 7x^2 + 4x^2 + 14x - 6x - 21$$

$$= 4x^3 + 27x^2 + 32x - 11$$

Ans)  $4x^3 + 27x^2 + 32x - 11$

**Q. 11. Differentiate:**

$$(\tan x + \sec x)(\cot x + \operatorname{cosec} x)$$

**Answer :** To find: Differentiation of  $(\tan x + \sec x) (\cot x + \operatorname{cosec} x)$

**Formula used:** (i)  $(uv)' = u'v + uv'$  (Leibnitz or product rule)

(ii)

$$\frac{d \tan x}{dx} = \sec^2 x$$

(iii)

$$\frac{d \sec x}{dx} = \sec x \tan x$$

(iv)

$$\frac{d \cot x}{dx} = -\operatorname{cosec}^2 x$$

(v)

$$\frac{d \operatorname{cosec} x}{dx} = -\operatorname{cosec} x \cot x$$

Let us take  $u = (\tan x + \sec x)$  and  $v = (\cot x + \operatorname{cosec} x)$

$$u' = \frac{du}{dx} = \frac{d(\tan x + \sec x)}{dx} = \sec^2 x + \sec x \tan x = \sec x (\sec x + \tan x)$$

$$v' = \frac{dv}{dx} = \frac{d(\cot x + \operatorname{cosec} x)}{dx}$$

$$= -\operatorname{cosec}^2 x + (-\operatorname{cosec} x \cot x) = -\operatorname{cosec} x (\operatorname{cosec} x + \cot x)$$

**Putting the above obtained values in the formula:-**

$$(uv)' = u'v + uv'$$

$$[(\tan x + \sec x) (\cot x + \operatorname{cosec} x)]'$$

$$= [\sec x (\sec x + \tan x)] \times [(\cot x + \operatorname{cosec} x)] + [(\tan x + \sec x)] \times [-\operatorname{cosec} x (\operatorname{cosec} x + \cot x)]$$

$$= (\sec x + \tan x) [\sec x(\cot x + \operatorname{cosec} x) - \operatorname{cosec} x(\operatorname{cosec} x + \cot x)]$$

$$= (\sec x + \tan x) (\sec x - \operatorname{cosec} x) (\cot x + \operatorname{cosec} x)$$

Ans)  $(\sec x + \tan x) (\sec x - \operatorname{cosec} x) (\cot x + \operatorname{cosec} x)$

**Q. 12. Differentiate:**

$(x^3 \cos x - 2^x \tan x)$

**Answer :** To find: Differentiation of  $(x^3 \cos x - 2^x \tan x)$

**Formula used: (i)**  $(uv)' = u'v + uv'$  (Leibnitz or product rule)

(ii)

$$\frac{dx^n}{dx} = nx^{n-1}$$

(iii)

$$\frac{d\cos x}{dx} = -\sin x$$

(iv)

$$\frac{da^x}{dx} = a^x \log a$$



(v)

$$\frac{d\tan x}{dx} = \sec^2 x$$

Here we have two function  $(x^3 \cos x)$  and  $(2^x \tan x)$

We have two differentiate them separately

Let us assume  $g(x) = (x^3 \cos x)$

And  $h(x) = (2^x \tan x)$

Therefore,  $f(x) = g(x) - h(x)$

$\Rightarrow f'(x) = g'(x) - h'(x) \dots$  (i)

Applying product rule on  $g(x)$

Let us take  $u = x^3$  and  $v = \cos x$

$$u' = \frac{du}{dx} = \frac{d(x^3)}{dx} = 3x^2$$

$$v' = \frac{dv}{dx} = \frac{d(\cos x)}{dx} = -\sin x$$

**Putting the above obtained values in the formula:-**

$$(uv)' = u'v + uv'$$

$$[x^3 \cos x]' = 3x^2 \times \cos x + x^3 \times -\sin x$$

$$= 3x^2 \cos x - x^3 \sin x$$

$$= x^2 (3 \cos x - x \sin x)$$

$$g'(x) = x^2 (3 \cos x - x \sin x)$$

Applying product rule on h(x)

Let us take  $u = 2^x$  and  $v = \tan x$

$$u' = \frac{du}{dx} = \frac{d(2^x)}{dx} = 2^x \log 2$$

$$v' = \frac{dv}{dx} = \frac{d(\tan x)}{dx} = \sec^2 x$$

**Putting the above obtained values in the formula:-**

$$(uv)' = u'v + uv'$$

$$[2^x \tan x]' = 2^x \log 2 \times \tan x + 2^x \times \sec^2 x$$

$$= 2^x (\log 2 \tan x + \sec^2 x)$$

$$h'(x) = 2^x (\log 2 \tan x + \sec^2 x)$$

Putting the above obtained values in eqn. (i)

$$f'(x) = x^2 (3 \cos x - x \sin x) - 2^x (\log 2 \tan x + \sec^2 x)$$

$$\text{Ans) } x^2 (3 \cos x - x \sin x) - 2^x (\log 2 \tan x + \sec^2 x)$$

## Exercise 28D

Q. 1. Differentiate

$$\frac{2^x}{x}$$

Answer :

To find: Differentiation of  $\frac{2^x}{x}$

Formula used: (i)  $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)

$$(ii) \frac{da^x}{dx} = a^x \log a$$

Let us take  $u = 2^x$  and  $v = x$

$$u' = \frac{du}{dx} = \frac{d(2^x)}{dx} = 2^x \log 2$$

$$v' = \frac{dv}{dx} = \frac{d(x)}{dx} = 1$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left(\frac{2^x}{x}\right)' = \frac{2^x \log 2 \times x - 2^x \times 1}{(x)^2}$$

$$= \frac{2^x(x \log 2 - 1)}{x^2}$$

$$\text{Ans) } = \frac{2^x(x \log 2 - 1)}{x^2}$$

**Q. 2. Differentiate**

$$\frac{\log x}{x}$$

**Answer :**

To find: Differentiation of  $\frac{\log x}{x}$

Formula used: (i)  $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)

$$(ii) \frac{d \log x}{dx} = \frac{1}{x}$$

Let us take  $u = \log x$  and  $v = x$

$$u' = \frac{du}{dx} = \frac{d(\log x)}{dx} = \frac{1}{x}$$

$$v' = \frac{dv}{dx} = \frac{d(x)}{dx} = 1$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left(\frac{\log x}{x}\right)' = \frac{\frac{1}{x} \times x - \log x \times 1}{(x)^2}$$

$$= \frac{1 - \log x}{x^2}$$

$$\text{Ans) } = \frac{1 - \log x}{x^2}$$

**Q. 3. Differentiate**

$$\frac{e^x}{(1+x)}$$

**Answer :**

To find: Differentiation of  $\frac{e^x}{(1+x)}$

Formula used: (i)  $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)

$$(ii) \frac{de^x}{dx} = e^x$$

Let us take  $u = e^x$  and  $v = (1+x)$

$$u' = \frac{du}{dx} = \frac{d(e^x)}{dx} = e^x$$

$$v' = \frac{dv}{dx} = \frac{d(1+x)}{dx} = 1$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left(\frac{e^x}{(1+x)}\right)' = \frac{e^x \times (1+x) - e^x \times 1}{(1+x)^2}$$

$$= \frac{xe^x}{(1+x)^2}$$

$$\text{Ans) } = \frac{xe^x}{(1+x)^2}$$

**Q. 4. Differentiate**

$$\frac{e^x}{(1+x^2)}$$

**Answer :**

To find: Differentiation of  $\frac{e^x}{(1+x^2)}$

Formula used: (i)  $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)

$$(ii) \frac{de^x}{dx} = e^x$$

$$(iii) \frac{dx^n}{dx} = nx^{n-1}$$

Let us take  $u = e^x$  and  $v = (1+x^2)$

$$u' = \frac{du}{dx} = \frac{d(e^x)}{dx} = e^x$$

$$v' = \frac{dv}{dx} = \frac{d(1+x^2)}{dx} = 2x$$

**Putting the above obtained values in the formula:-**

$$\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left(\frac{e^x}{(1+x^2)}\right)' = \frac{e^x \times (1+x^2) - e^x \times 2x}{(1+x^2)^2}$$

$$= \frac{e^x(x^2-2x+1)}{(1+x^2)^2}$$

$$= \frac{e^x(x-1)^2}{(1+x^2)^2}$$



$$\text{Ans) } = \frac{e^x(x-1)^2}{(1+x^2)^2}$$

**Q. 5. Differentiate**

$$\left( \frac{2x^2 - 4}{3x^2 + 7} \right)$$

**Answer :**

To find: Differentiation of  $\frac{(2x^2-4)}{(3x^2+7)}$

Formula used: (i)  $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)

$$(ii) \frac{dx^n}{dx} = nx^{n-1}$$

Let us take  $u = (2x^2 - 4)$  and  $v = (3x^2 + 7)$

$$u' = \frac{du}{dx} = \frac{d(2x^2 - 4)}{dx} = 4x$$

$$v' = \frac{dv}{dx} = \frac{d(3x^2 + 7)}{dx} = 6x$$

**Putting the above obtained values in the formula:-**

$$\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{(2x^2-4)}{(3x^2+7)}\right]' = \frac{4x \times (3x^2+7) - (2x^2-4) \times 6x}{(3x^2+7)^2}$$

$$= \frac{12x^3 + 28x - 12x^3 + 24x}{(3x^2+7)^2}$$

$$= \frac{52x}{(3x^2+7)^2}$$

$$\text{Ans) } = \frac{52x}{(3x^2+7)^2}$$

**Q. 6. Differentiate**

$$\left( \frac{x^2 + 3x - 1}{x + 2} \right)$$

**Answer :**

To find: Differentiation of  $\left( \frac{x^2 + 3x - 1}{x + 2} \right)$

Formula used: (i)  $\left( \frac{u}{v} \right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)

$$(ii) \frac{dx^n}{dx} = nx^{n-1}$$

Let us take  $u = (x^2 + 3x - 1)$  and  $v = (x + 2)$

$$u' = \frac{du}{dx} = \frac{d(x^2 + 3x - 1)}{dx} = 2x + 3$$

$$v' = \frac{dv}{dx} = \frac{d(x + 2)}{dx} = 1$$

**Putting the above obtained values in the formula:-**

$$\left( \frac{u}{v} \right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left( \frac{x^2 + 3x - 1}{x + 2} \right)' = \frac{(2x + 3) \times (x + 2) - (x^2 + 3x - 1) \times 1}{(x + 2)^2}$$

$$= \frac{2x^2 + 7x + 6 - x^2 - 3x + 1}{(x + 2)^2}$$

$$= \frac{x^2 + 4x + 7}{(x + 2)^2}$$

$$\text{Ans) } = \frac{x^2 + 4x + 7}{(x + 2)^2}$$

**Q. 7. Differentiate**

$$\frac{(x^2 - 1)}{(x^2 + 7x + 1)}$$

**Answer :**

To find: Differentiation of  $\frac{(x^2 - 1)}{(x^2 + 7x + 1)}$

Formula used: (i)  $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)

$$(ii) \frac{dx^n}{dx} = nx^{n-1}$$

Let us take  $u = (x^2 - 1)$  and  $v = (x^2 + 7x + 1)$

$$u' = \frac{du}{dx} = \frac{d(x^2 - 1)}{dx} = 2x$$

$$v' = \frac{dv}{dx} = \frac{d(x^2 + 7x + 1)}{dx} = 2x + 7$$

**Putting the above obtained values in the formula:-**

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[ \frac{(x^2-1)}{(x^2+7x+1)} \right]' = \frac{2x \times (x^2+7x+1) - (x^2-1) \times (2x+7)}{(x^2+7x+1)^2}$$

$$= \frac{2x^3 + 14x^2 + 2x - 2x^3 - 7x^2 + 2x + 7}{(x^2+7x+1)^2}$$

$$= \frac{7x^2 + 4x + 7}{(x^2+7x+1)^2}$$

$$\text{Ans) } = \frac{7x^2 + 4x + 7}{(x^2+7x+1)^2}$$

**Q. 8. Differentiate**

$$\left( \frac{5x^2 + 6x + 7}{2x^2 + 3x + 4} \right)$$

**Answer :**

To find: Differentiation of  $\left( \frac{5x^2 + 6x + 7}{2x^2 + 3x + 4} \right)$

Formula used: (i)  $\left( \frac{u}{v} \right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)

$$(ii) \frac{dx^n}{dx} = nx^{n-1}$$

Let us take  $u = (5x^2 + 6x + 7)$  and  $v = (2x^2 + 3x + 4)$

$$u' = \frac{du}{dx} = \frac{d(5x^2 + 6x + 7)}{dx} = 10x + 6$$

$$v' = \frac{dv}{dx} = \frac{d(2x^2 + 3x + 4)}{dx} = 4x + 3$$

**Putting the above obtained values in the formula:-**

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left(\frac{5x^2 + 6x + 7}{2x^2 + 3x + 4}\right)' = \frac{(10x+6) \times (2x^2+3x+4) - (5x^2+6x+7) \times (4x+3)}{(2x^2 + 3x + 4)^2}$$

$$= \frac{20x^3 + 30x^2 + 40x + 12x^2 + 18x + 24 - 20x^3 - 15x^2 - 24x^2 - 18x - 28x - 21}{(2x^2 + 3x + 4)^2}$$

$$= \frac{3x^2 + 12x + 3}{(2x^2 + 3x + 4)^2}$$

$$= \frac{3(x^2 + 4x + 1)}{(2x^2 + 3x + 4)^2}$$

$$\text{Ans) } = \frac{3(x^2 + 4x + 1)}{(2x^2 + 3x + 4)^2}$$

Q. 9. Differentiate

$$\frac{x}{(a^2 + x^2)}$$

Answer :

To find: Differentiation of  $\frac{x}{(a^2 + x^2)}$

Formula used: (i)  $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)

$$(ii) \frac{dx^n}{dx} = nx^{n-1}$$

Let us take  $u = (x)$  and  $v = (a^2 + x^2)$

$$u' = \frac{du}{dx} = \frac{d(x)}{dx} = 1$$

$$v' = \frac{dv}{dx} = \frac{d(a^2+x^2)}{dx} = 2x$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{x}{(a^2+x^2)}\right]' = \frac{1 \times (a^2+x^2) - (x) \times (2x)}{(a^2+x^2)^2}$$

$$= \frac{a^2+x^2 - 2x^2}{(a^2+x^2)^2}$$

$$= \frac{a^2 - x^2}{(a^2+x^2)^2}$$

$$\text{Ans) } = \frac{a^2 - x^2}{(a^2+x^2)^2}$$



**Q. 10. Differentiate**

$$\frac{x^4}{\sin x}$$

**Answer :**

To find: Differentiation of  $\frac{x^4}{\sin x}$

Formula used: (i)  $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)

$$(ii) \frac{dx^n}{dx} = nx^{n-1}$$

$$(iii) \frac{d\sin x}{dx} = \cos x$$

Let us take  $u = (x^4)$  and  $v = (\sin x)$

$$u' = \frac{du}{dx} = \frac{d(x^4)}{dx} = 4x^3$$

$$v' = \frac{dv}{dx} = \frac{d(\sin x)}{dx} = \cos x$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{x^4}{\sin x}\right]' = \frac{4x^3 \times (\sin x) - (x^4) \times (\cos x)}{(\sin x)^2}$$

$$= \frac{x^3[4(\sin x) - x(\cos x)]}{(\sin x)^2}$$

$$\text{Ans) } = \frac{x^3[4(\sin x) - x(\cos x)]}{(\sin x)^2}$$

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**Q. 11. Differentiate**

$$\frac{\sqrt{a} + \sqrt{x}}{\sqrt{a} - \sqrt{x}}$$

**Answer :**

To find: Differentiation of  $\frac{\sqrt{a} + \sqrt{x}}{\sqrt{a} - \sqrt{x}}$

Formula used: (i)  $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)

$$(ii) \frac{dx^n}{dx} = nx^{n-1}$$

Let us take  $u = (\sqrt{a} + \sqrt{x})$  and  $v = (\sqrt{a} - \sqrt{x})$

$$u' = \frac{du}{dx} = \frac{d(\sqrt{a} + \sqrt{x})}{dx} = \frac{1}{2\sqrt{x}}$$

$$v' = \frac{dv}{dx} = \frac{d(\sqrt{a} - \sqrt{x})}{dx} = -\frac{1}{2\sqrt{x}}$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{\sqrt{a} + \sqrt{x}}{\sqrt{a} - \sqrt{x}}\right]' = \frac{\frac{1}{2\sqrt{x}} \times (\sqrt{a} - \sqrt{x}) - (\sqrt{a} + \sqrt{x}) \times -\frac{1}{2\sqrt{x}}}{(\sqrt{a} - \sqrt{x})^2}$$

$$= \frac{\frac{\sqrt{a}}{2\sqrt{x}} - \frac{1}{2} + \frac{\sqrt{a}}{2\sqrt{x}} + \frac{1}{2}}{(\sqrt{a} - \sqrt{x})^2}$$

$$= \frac{\sqrt{a}}{\sqrt{x}(\sqrt{a} - \sqrt{x})^2}$$

$$\text{Ans) } = \frac{\sqrt{a}}{\sqrt{x}(\sqrt{a} - \sqrt{x})^2}$$

**Q. 12. Differentiate**

$$\frac{\cos x}{\log x}$$

**Answer :**

To find: Differentiation of  $\frac{\cos x}{\log x}$

Formula used: (i)  $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)

$$(ii) \frac{d \cos x}{dx} = -\sin x$$

$$(iii) \frac{d \log x}{dx} = \frac{1}{x}$$

Let us take  $u = (\cos x)$  and  $v = (\log x)$

$$u' = \frac{du}{dx} = \frac{d(\cos x)}{dx} = -\sin x$$

$$v' = \frac{dv}{dx} = \frac{d(\log x)}{dx} = \frac{1}{x}$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{\cos x}{\log x}\right]' = \frac{-\sin x \times (\log x) - (\cos x) \times \left(\frac{1}{x}\right)}{(\log x)^2}$$

$$= \frac{-x \sin x (\log x) - (\cos x)}{x (\log x)^2}$$

$$\text{Ans) } = \frac{-x \sin x (\log x) - (\cos x)}{x (\log x)^2}$$

**Q. 13. Differentiate**

$$\frac{2 \cot x}{\sqrt{x}}$$

**Answer :**

To find: Differentiation of  $\frac{2 \cot x}{\sqrt{x}}$

Formula used: (i)  $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)

$$(ii) \frac{d \cot x}{dx} = -\operatorname{cosec}^2 x$$

$$(iii) \frac{dx^n}{dx} = nx^{n-1}$$

Let us take  $u = (2 \cot x)$  and  $v =$

$$(\sqrt{x})$$

$$u' = \frac{du}{dx} = \frac{d(2 \cot x)}{dx} = -2 \operatorname{cosec}^2 x$$

$$v' = \frac{dv}{dx} = \frac{d(\sqrt{x})}{dx} = \frac{1}{2\sqrt{x}}$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{2 \cot x}{\sqrt{x}}\right]' = \frac{-2 \operatorname{cosec}^2 x \times (\sqrt{x}) - (2 \cot x) \times \left(\frac{1}{2\sqrt{x}}\right)}{(\sqrt{x})^2}$$

$$= \frac{-2x \operatorname{cosec}^2 x - (\cot x)}{\sqrt{x}(\sqrt{x})^2}$$

$$\text{Ans) } = \frac{-2x \operatorname{cosec}^2 x - \cot x}{x^{3/2}}$$

**Q. 14. Differentiate**

$$\frac{\sin x}{(1 + \cos x)}$$

Answer :

To find: Differentiation of  $\frac{\sin x}{(1 + \cos x)}$

Formula used: (i)  $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)

$$(ii) \frac{d\cos x}{dx} = -\sin x$$

$$(iii) \frac{d\sin x}{dx} = \cos x$$

Let us take  $u = (\sin x)$  and  $v = (1 + \cos x)$

$$u' = \frac{du}{dx} = \frac{d(\sin x)}{dx} = \cos x$$

$$v' = \frac{dv}{dx} = \frac{d(1 + \cos x)}{dx} = -\sin x$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{\sin x}{(1 + \cos x)}\right]' = \frac{\cos x \times (1 + \cos x) - (\sin x) \times (-\sin x)}{(1 + \cos x)^2}$$

$$= \frac{\cos x + \cos^2 x + \sin^2 x}{(1 + \cos x)^2}$$

$$= \frac{\cos x + 1}{(1 + \cos x)^2}$$

$$= \frac{1}{(1 + \cos x)}$$

$$\text{Ans) } = \frac{1}{1 + \cos x}$$

**Q. 15. Differentiate**

$$\left( \frac{1 + \sin x}{1 - \sin x} \right)$$

**Answer :**

To find: Differentiation of  $\left( \frac{1 + \sin x}{1 - \sin x} \right)$

Formula used: (i)  $\left( \frac{u}{v} \right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)

$$(ii) \frac{d \sin x}{dx} = \cos x$$

Let us take  $u = (1 + \sin x)$  and  $v = (1 - \sin x)$

$$u' = \frac{du}{dx} = \frac{d(1 + \sin x)}{dx} = \cos x$$

$$v' = \frac{dv}{dx} = \frac{d(1 - \sin x)}{dx} = -\cos x$$

Putting the above obtained values in the formula:-

$$\left( \frac{u}{v} \right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[ \frac{1 + \sin x}{1 - \sin x} \right]' = \frac{\cos x \times (1 - \sin x) - (1 + \sin x) \times (-\cos x)}{(1 - \sin x)^2}$$

$$= \frac{\cos x - \cos x \sin x + \cos x + \cos x \sin x}{(1 - \sin x)^2}$$

$$= \frac{2 \cos x}{(1 - \sin x)^2}$$

$$\text{Ans) } = \frac{2 \cos x}{(1 - \sin x)^2}$$

**Q. 16. Differentiate**

$$\left( \frac{1 - \cos x}{1 + \cos x} \right)$$

**Answer :**

To find: Differentiation of  $\left( \frac{1 - \cos x}{1 + \cos x} \right)$

Formula used: (i)  $\left( \frac{u}{v} \right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)

$$(ii) \frac{d \cos x}{dx} = -\sin x$$

Let us take  $u = (1 - \cos x)$  and  $v = (1 + \cos x)$

$$u' = \frac{du}{dx} = \frac{d(1 - \cos x)}{dx} = \sin x$$

$$v' = \frac{dv}{dx} = \frac{d(1 + \cos x)}{dx} = -\sin x$$

Putting the above obtained values in the formula:-

$$\left( \frac{u}{v} \right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[ \frac{1 - \cos x}{1 + \cos x} \right]' = \frac{\sin x \times (1 + \cos x) - (1 - \cos x) \times (-\sin x)}{(1 + \cos x)^2}$$

$$= \frac{\sin x + \sin x \cos x + \sin x - \sin x \cos x}{(1 + \cos x)^2}$$

$$= \frac{2 \sin x}{(1 + \cos x)^2}$$

$$\text{Ans) } = \frac{2 \sin x}{(1 + \cos x)^2}$$

**Q. 17. Differentiate**

$$\left( \frac{\sin x - \cos x}{\sin x + \cos x} \right)$$

**Answer :**

To find: Differentiation of  $\left( \frac{\sin x - \cos x}{\sin x + \cos x} \right)$

Formula used: (i)  $\left( \frac{u}{v} \right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)

$$(ii) \frac{d \sin x}{dx} = \cos x$$

$$(iii) \frac{d \cos x}{dx} = -\sin x$$

Let us take  $u = (\sin x - \cos x)$  and  $v = (\sin x + \cos x)$

$$u' = \frac{du}{dx} = \frac{d(\sin x - \cos x)}{dx} = (\cos x + \sin x)$$

$$v' = \frac{dv}{dx} = \frac{d(\sin x + \cos x)}{dx} = (\cos x - \sin x)$$

Putting the above obtained values in the formula:-

$$\left( \frac{u}{v} \right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[ \frac{\sin x - \cos x}{\sin x + \cos x} \right]' = \frac{(\cos x + \sin x) \times (\sin x + \cos x) - (\sin x - \cos x) \times (\cos x - \sin x)}{(\sin x + \cos x)^2}$$

$$= \frac{\sin^2 x + \cos^2 x + 2\sin x \cos x - (\sin x - \cos x) \times -(\sin x - \cos x)}{(\sin x + \cos x)^2}$$

$$= \frac{\sin^2 x + \cos^2 x + 2\sin x \cos x + \sin^2 x + \cos^2 x - 2\sin x \cos x}{(\sin x + \cos x)^2}$$

$$= \frac{2(\sin^2 x + \cos^2 x)}{\sin^2 x + \cos^2 x + 2\sin x \cos x}$$

$$= \frac{2}{1 + \sin 2x}$$

$$\text{Ans) } = \frac{2}{1 + \sin 2x}$$

**Q. 18. Differentiate**

$$\left( \frac{\sec x - \tan x}{\sec x + \tan x} \right)$$

**Answer :**

To find: Differentiation of  $\left( \frac{\sec x - \tan x}{\sec x + \tan x} \right)$

Formula used: (i)  $\left( \frac{u}{v} \right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)

$$(ii) \frac{d\sec x}{dx} = \sec x \tan x$$

$$(iii) \frac{d\tan x}{dx} = \sec^2 x$$

Let us take  $u = (\sec x - \tan x)$  and  $v = (\sec x + \tan x)$

$$u' = \frac{du}{dx} = \frac{d(\sec x - \tan x)}{dx} = (\sec x \tan x - \sec^2 x)$$

$$v' = \frac{dv}{dx} = \frac{d(\sec x + \tan x)}{dx} = (\sec x \tan x + \sec^2 x)$$

Putting the above obtained values in the formula:-

$$\left( \frac{u}{v} \right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[ \frac{\sec x - \tan x}{\sec x + \tan x} \right]' = \frac{(\sec x \tan x - \sec^2 x)(\sec x + \tan x) - (\sec x - \tan x)(\sec x \tan x + \sec^2 x)}{(\sec x + \tan x)^2}$$

$$= \frac{(\sec x \tan x - \sec^2 x)(\sec x + \tan x) - (\sec x - \tan x)(\sec x)(\tan x + \sec x)}{(\sec x + \tan x)^2}$$

$$= \frac{(\sec x + \tan x)[(\sec x \tan x - \sec^2 x) - (\sec x - \tan x)(\sec x)]}{(\sec x + \tan x)^2}$$

$$= \frac{(\sec x + \tan x)[(\sec x \tan x - \sec^2 x) - (\sec^2 x - \sec x \tan x)]}{(\sec x + \tan x)^2}$$

$$= \frac{(\sec x + \tan x)[2\sec x \tan x - 2\sec^2 x]}{(\sec x + \tan x)^2}$$

$$= \frac{2\sec x[\tan x - \sec x]}{(\sec x + \tan x)}$$

$$\text{Ans) } = \frac{2\sec x[\tan x - \sec x]}{(\sec x + \tan x)}$$

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**Q. 19. Differentiate**

$$\left( \frac{e^x + \sin x}{1 + \log x} \right)$$

**Answer :**

To find: Differentiation of  $\left( \frac{e^x + \sin x}{1 + \log x} \right)$

Formula used: (i)  $\left( \frac{u}{v} \right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)

$$(ii) \frac{d \sin x}{dx} = \cos x$$

$$(iii) \frac{d \log x}{dx} = \frac{1}{x}$$

$$(iv) \frac{de^x}{dx} = e^x$$

Let us take  $u = (e^x + \sin x)$  and  $v = (1 + \log x)$

$$u' = \frac{du}{dx} = \frac{d(e^x + \sin x)}{dx} = (e^x + \cos x)$$

$$v' = \frac{dv}{dx} = \frac{d(1 + \log x)}{dx} = \frac{1}{x}$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\begin{aligned} \left[\frac{e^x + \sin x}{1 + \log x}\right]' &= \frac{(e^x + \cos x) \times (1 + \log x) - (e^x + \sin x) \times \left(\frac{1}{x}\right)}{(1 + \log x)^2} \\ &= \frac{x(e^x + \cos x)(1 + \log x) - (e^x + \sin x)}{x(1 + \log x)^2} \end{aligned}$$

$$\text{Ans) } = \frac{x(e^x + \cos x)(1 + \log x) - (e^x + \sin x)}{x(1 + \log x)^2}$$

**Q. 20. Differentiate**

$$\frac{e^x \sin x}{\sec x}$$

**Answer :**

To find: Differentiation of  $\left(\frac{e^x \sin x}{\sec x}\right)$

Formula used: (i)  $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)

$$(ii) \frac{d \sin x}{dx} = \cos x$$

$$(iii) \frac{d \sec x}{dx} = \sec x \tan x$$

$$(iv) \frac{d e^x}{dx} = e^x$$

(v)  $(uv)' = u'v + uv'$  (Leibnitz or product rule)

Let us take  $u = (e^x \sin x)$  and  $v = (\sec x)$

$$u' = \frac{du}{dx} = \frac{d(e^x \sin x)}{dx}$$

Applying Product rule

$$(gh)' = g'h + gh'$$

Taking  $g = e^x$  and  $h = \sin x$

$$= e^x \sin x + e^x \cos x$$

$$u' = e^x \sin x + e^x \cos x$$

$$v' = \frac{dv}{dx} = \frac{d(\sec x)}{dx} = \sec x \tan x$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\begin{aligned} \left[\frac{e^x \sin x}{\sec x}\right]' &= \frac{(e^x \sin x + e^x \cos x) \times (\sec x) - (e^x \sin x) \times (\sec x \tan x)}{(\sec x)^2} \\ &= \frac{(e^x \sin x + e^x \cos x) - (e^x \sin x) \times (\tan x)}{(\sec x)} \end{aligned}$$

$$\begin{aligned}
&= \cos x [(e^x \sin x + e^x \cos x) - (e^x \sin x) \times (\tan x)] \\
&= [(e^x \sin x \cos x + e^x \cos^2 x) - (e^x \sin x \cos x) \times (\tan x)] \\
&= [(e^x \sin x \cos x + e^x \cos^2 x) - (e^x \sin^2 x)] \\
&= (e^x \sin x \cos x + e^x \cos^2 x - e^x \sin^2 x) \\
&= (e^x \sin x \cos x + e^x \cos^2 x - e^x \sin^2 x) \\
&= e^x \sin x \cos x + e^x \cos 2x \\
&= e^x (\sin x \cos x + \cos 2x)
\end{aligned}$$

$$\text{Ans) } = e^x (\sin x \cos x + \cos 2x)$$

**Q. 21. Differentiate**

$$\frac{2^x \cot x}{\sqrt{x}}$$



**Answer :**

To find: Differentiation of  $\left(\frac{2^x \cot x}{\sqrt{x}}\right)$

Formula used: (i)  $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)

$$(ii) \frac{d \cot x}{dx} = -\operatorname{cosec}^2 x$$

$$(iii) \frac{dx^n}{dx} = nx^{n-1}$$

$$(iv) \frac{da^x}{dx} = a^x \log a$$

(v)  $(uv)' = u'v + uv'$  (Leibnitz or product rule)

Let us take  $u = (2^x \cot x)$  and  $v = (\sqrt{x})$

$$u' = \frac{du}{dx} = \frac{d(2^x \cot x)}{dx}$$

Applying Product rule

$$(gh)' = g'h + gh'$$

Taking  $g = 2^x$  and  $h = \cot x$

$$= (2^x \log 2) \cot x + 2^x (-\operatorname{cosec}^2 x)$$

$$u' = (2^x \log 2) \cot x - 2^x (\operatorname{cosec}^2 x)$$

$$u' = 2^x [\log 2 \cot x - \operatorname{cosec}^2 x]$$

$$v' = \frac{dv}{dx} = \frac{d(\sqrt{x})}{dx} = \frac{1}{2\sqrt{x}}$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{2^x \cot x}{\sqrt{x}}\right]' = \frac{\{2^x [\log 2 \cot x - \operatorname{cosec}^2 x] \times \sqrt{x}\} - \{(2^x \cot x) \times \left(\frac{1}{2\sqrt{x}}\right)\}}{(\sqrt{x})^2}$$

$$= \frac{\{2^x [\log 2 \cot x - \operatorname{cosec}^2 x] \times \sqrt{x}\} - \{(2^x \cot x) \times \left(\frac{1}{2\sqrt{x}}\right)\}}{x}$$



$$= \frac{\{2^x [\log 2 \cot x - \operatorname{cosec}^2 x] \times \sqrt{x}\} - \{(2^{x-1} \cot x) \times (\frac{1}{\sqrt{x}})\}}{x}$$

$$= \frac{\{x2^x [\log 2 \cot x - \operatorname{cosec}^2 x]\} - \{(2^{x-1} \cot x)\}}{x\sqrt{x}}$$

$$= \frac{\{2^x [x \log 2 \cot x - x \operatorname{cosec}^2 x]\} - \{(2^{x-1} \cot x)\}}{x^{\frac{3}{2}}}$$

$$\text{Ans) } = \frac{\{2^x [x \log 2 \cot x - x \operatorname{cosec}^2 x]\} - \{(2^{x-1} \cot x)\}}{x^{\frac{3}{2}}}$$

**Q. 22. Differentiate**

$$\frac{e^x(x-1)}{(x+1)}$$



**Answer :**

To find: Differentiation of  $\frac{e^x(x-1)}{(x+1)}$

Formula used: (i)  $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)

$$(ii) \frac{de^x}{dx} = e^x$$

$$(iii) \frac{dx^n}{dx} = nx^{n-1}$$

(iv)  $(uv)' = u'v + uv'$  (Leibnitz or product rule)

Let us take  $u = e^x(x-1)$  and  $v = (x+1)$

$$u' = \frac{du}{dx} = \frac{d[e^x(x-1)]}{dx}$$

Applying Product rule

$$(gh)' = g'h + gh'$$

Taking  $g = e^x$  and  $h = x - 1$

$$[e^x(x-1)]' = e^x(x-1) + e^x(1)$$

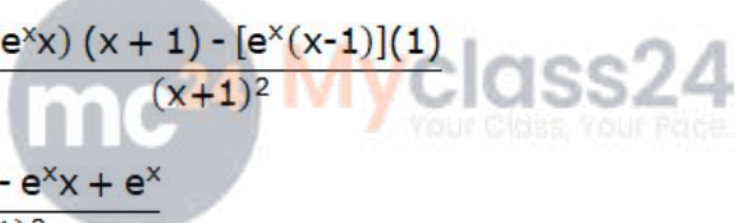
$$= e^x(x-1) + e^x$$

$$u' = e^x x$$

$$v' = \frac{dv}{dx} = \frac{d(x+1)}{dx} = 1$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{e^x(x-1)}{(x+1)}\right]' = \frac{(e^x x)(x+1) - [e^x(x-1)](1)}{(x+1)^2}$$


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$$= \frac{e^x x^2 + e^x x - e^x x + e^x}{(x+1)^2}$$

$$= \frac{e^x x^2 + e^x}{(x+1)^2}$$

$$= \frac{e^x(x^2 + 1)}{(x+1)^2}$$

$$\text{Ans) } = \frac{e^x(x^2 + 1)}{(x+1)^2}$$

**Q. 23. Differentiate**

$$\frac{x \tan x}{(\sec x + \tan x)}$$

**Answer :**

To find: Differentiation of  $\frac{x \tan x}{(\sec x + \tan x)}$

Formula used: (i)  $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)

$$(ii) \frac{d \sec x}{dx} = \sec x \tan x$$

$$(iii) \frac{d \tan x}{dx} = \sec^2 x$$

$$(iii) \frac{dx^n}{dx} = nx^{n-1}$$

(iv)  $(uv)' = u'v + uv'$  (Leibnitz or product rule)

Let us take  $u = (x \tan x)$  and  $v = (\sec x + \tan x)$

$$u' = \frac{du}{dx} = \frac{d[x \tan x]}{dx}$$

Applying Product rule for finding  $u'$

$$(gh)' = g'h + gh'$$

Taking  $g = x$  and  $h = \tan x$

$$[x \tan x]' = (1) (\tan x) + x (\sec^2 x)$$

$$= \tan x + x \sec^2 x$$

$$u' = \tan x + x \sec^2 x$$

$$v' = \frac{dv}{dx} = \frac{d(\sec x + \tan x)}{dx} = \sec x \tan x + \sec^2 x$$

$$v' = \sec x (\tan x + \sec x)$$

**Putting the above obtained values in the formula:-**

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\begin{aligned} \left[\frac{x \tan x}{(\sec x + \tan x)}\right]' &= \frac{(\tan x + x \sec^2 x)(\sec x + \tan x) - [x \tan x][\sec x(\tan x + \sec x)]}{(\sec x + \tan x)^2} \\ &= \frac{(\sec x + \tan x)[(\tan x + x \sec^2 x) - (x \tan x)(\sec x)]}{(\sec x + \tan x)^2} \\ &= \frac{[\tan x + x \sec^2 x - x \tan x \sec x]}{(\sec x + \tan x)} \\ &= \frac{\tan x + x \sec x (\sec x - \tan x)}{(\sec x + \tan x)} \end{aligned}$$

$$\text{Ans) } = \frac{\tan x + x \sec x (\sec x - \tan x)}{(\sec x + \tan x)}$$

Q. 24. Differentiate

$$\left(\frac{ax^2 + bx + c}{px^2 + qx + r}\right)$$



Answer :

To find: Differentiation of  $\frac{ax^2 + bx + c}{px^2 + qx + r}$

Formula used: (i)  $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)

$$(ii) \frac{dx^n}{dx} = nx^{n-1}$$

Let us take  $u = (ax^2 + bx + c)$  and  $v = (px^2 + qx + r)$

$$u' = \frac{du}{dx} = \frac{d[ax^2 + bx + c]}{dx} = 2ax + b$$

$$v' = \frac{dv}{dx} = \frac{d(px^2+qx+r)}{dx} = 2px + q$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{ax^2+bx+c}{px^2+qx+r}\right]' = \frac{(2ax+b)(px^2+qx+r) - [ax^2+bx+c][2px+q]}{(px^2+qx+r)^2}$$

$$= \frac{2apx^3+2aqx^2+2axr+bp^2x^2+bqx+br-[2apx^3+qax^2+2bpx^2+bqx+2pcx+cq]}{(px^2+qx+r)^2}$$

$$= \frac{(aq-bp)x^2+2(ra-pc)x+br-cp}{(px^2+qx+r)^2}$$

$$\text{Ans) } = \frac{(aq-bp)x^2+2(ra-pc)x+br-cp}{(px^2+qx+r)^2}$$

Q. 25. Differentiate

$$\left(\frac{\sin x - x \cos x}{x \sin x + \cos x}\right)$$

Answer :

To find: Differentiation of  $\frac{(\sin x - x \cos x)}{(x \sin x + \cos x)}$

Formula used: (i)  $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)

$$(ii) \frac{d \sin x}{dx} = \cos x$$

$$(iii) \frac{d \cos x}{dx} = -\sin x$$

(iv)  $(uv)' = u'v + uv'$  (Leibnitz or product rule)

Let us take  $u = (\sin x - x \cos x)$  and  $v = (x \sin x + \cos x)$

$$u' = \frac{du}{dx} = \frac{d[\sin x - x \cos x]}{dx}$$

Applying Product rule for finding the term  $x \cos x$  in  $u'$

$$(gh)' = g'h + gh'$$

Taking  $g = x$  and  $h = \cos x$

$$[x \cos x]' = (1)(\cos x) + x(-\sin x)$$

$$[x \cos x]' = \cos x - x \sin x$$

Applying the above obtained value for finding  $u'$

$$u' = \cos x - (\cos x - x \sin x)$$

$$u' = x \sin x$$

$$v' = \frac{dv}{dx} = \frac{d(x \sin x + \cos x)}{dx}$$

Applying Product rule for finding the term  $x \sin x$  in  $v'$

$$(gh)' = g'h + gh'$$

Taking  $g = x$  and  $h = \sin x$

$$[x \sin x]' = (1)(\sin x) + x(\cos x)$$

$$[x \sin x]' = \sin x + x \cos x$$

Applying the above obtained value for finding  $v'$

$$v' = \sin x + x \cos x - \sin x$$

$$v' = x \cos x$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{(\sin x - x \cos x)}{(x \sin x + \cos x)}\right]' = \frac{(x \sin x)(x \sin x + \cos x) - (\sin x - x \cos x)(x \cos x)}{(x \sin x + \cos x)^2}$$

$$= \frac{(x^2 \sin^2 x + x \sin x \cos x) - (x \sin x \cos x - x^2 \cos^2 x)}{(x \sin x + \cos x)^2}$$

$$= \frac{x^2 \sin^2 x + x \sin x \cos x - x \sin x \cos x + x^2 \cos^2 x}{(x \sin x + \cos x)^2}$$

$$= \frac{x^2(\sin^2 x + \cos^2 x)}{(x \sin x + \cos x)^2}$$

$$= \frac{x^2}{(x \sin x + \cos x)^2}$$

$$\text{Ans) } = \frac{x^2}{(x \sin x + \cos x)^2}$$

**Q. 26**

(i)  $\cot x$

(ii)  $\sec x$

**Answer :**

To find: Differentiation of  $\cot x$

Formula used: (i)  $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)

$$(ii) \frac{d \sin x}{dx} = \cos x$$

$$(iii) \frac{d \cos x}{dx} = -\sin x$$

We can write  $\cot x$  as  $\frac{\cos x}{\sin x}$

Let us take  $u = \cos x$  and  $v = \sin x$

$$u' = (\cos x)' = -\sin x$$

$$v' = (\sin x)' = \cos x$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left(\frac{\cos x}{\sin x}\right)' = \frac{(-\sin x)(\sin x) - (\cos x)(\cos x)}{(\sin x)^2}$$

$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$$

$$= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$$

$$= \frac{-1}{\sin^2 x}$$

$$= -\operatorname{cosec}^2 x$$

Ans).

$$-\operatorname{cosec}^2 x$$

(ii)

To find: Differentiation of  $\sec x$

$$\text{Formula used: (i) } \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$(ii) \frac{d \cos x}{dx} = -\sin x$$

We can write  $\sec x$  as  $\frac{1}{\cos x}$

Let us take  $u = 1$  and  $v = \cos x$

$$u' = (1)' = 0$$

$$v' = (\cos x)' = -\sin x$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left(\frac{1}{\cos x}\right)' = \frac{(0)(\cos x) - (1)(-\sin x)}{(\cos x)^2}$$

$$= \frac{\sin x}{\cos^2 x}$$
$$= \sec x \tan x$$



Ans).

$$-\operatorname{cosec}^2 x$$

### Exercise 28E

Q. 1. Differentiate the following with respect to  $x$ :

$$\sin 4x$$

**Answer :** To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e

$$\frac{d(u.v)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$

Formula used:  $\frac{d}{dx}(\sin nu) = \cos (nu) \frac{d}{dx}(nu)$

Let us take  $y = \sin 4x$ .

So, by using the above formula, we have

$$\frac{d}{dx}(\sin 4x) = \cos (4x) \times \frac{d}{dx} (4x) = 4\cos 4x.$$

Differentiation of  $y = \sin 4x$  is  $4\cos 4x$

**Q. 2. Differentiate the following with respect to x:**

**cos 5x**

**Answer :** To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e

$$\frac{d(u.v)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$

Formula used:  $\frac{d}{dx}(\cos nu) = -\sin (nu) \frac{d}{dx}(nu)$ .

Let us take  $y = \cos 5x$ .

So, by using the above formula, we have

$$\frac{d}{dx}(\cos 5x) = -\sin(5x) \times \frac{d}{dx} (5x) = -5\sin 5x.$$

Differentiation of  $y = \cos 5x$  is  $-5\sin 5x$

**Q. 3. Differentiate the following with respect to x:**

**tan 3x**

**Answer :** To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e

$$\frac{d(u.v)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$

Formula used:  $\frac{d}{dx}(\tan nu) = \sec^2 (nu) \cdot \frac{d}{dx}(nu)$ .

Let us take  $y = \tan 3x$

So, by using the above formula, we have

$$\frac{d}{dx}\tan 3x = \sec^2(3x) \times \frac{d}{dx}(3x) = 3\sec^2(3x)$$

Differentiation of  $y = \tan 3x$  is  $3\sec^2(3x)$

**Q. 4. Differentiate the following with respect to x:**

**$\cos x^3$**

**Answer :** To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e

$$\frac{d(u.v)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$

Formula used:  $\frac{d}{dx}(\cos nu) = -\sin nu \frac{d}{dx}(nu)$  and  $\frac{d x^n}{dx} = nx^{n-1}$

Let us take  $y = \cos x^3$

So, by using the above formula, we have

$$\frac{d}{dx} \cos x^3 = -\sin(x^3) \times \frac{d}{dx}(x^3) = -3x^2 \sin(x^3)$$

Differentiation of  $y = \cos x^3$  is  $-3x^2 \sin(x^3)$

**Q. 5. Differentiate the following with respect to x:**

**cot<sup>2</sup>x**

**Answer :** To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e

$$\frac{d(u.v)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$

Formula used:  $\frac{d}{dx}(\cot^a nu) = a \cot^{a-1}(nu) \times \frac{d}{dx}(\cot nu) \times \frac{d}{dx}(nu)$  and  $\frac{d}{dx} x^n = nx^{n-1}$

Let us take  $y = \cot^2 x$

So, by using the above formula, we have

$$\frac{d}{dx} \cot^2 x = 2 \cot(x) \times \frac{d \cot x}{dx} \times \frac{dx}{dx} = -2 \cot x (\operatorname{cosec}^2 x).$$

Differentiation of  $y = \cot^2 x$  is  $-2 \cot x (\operatorname{cosec}^2 x)$

**Q. 6. Differentiate the following with respect to x:**

**tan<sup>3</sup>x**

**Answer :** To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e

$$\frac{d(u.v)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$

**Formula used:**

$\frac{d}{dx}(\tan^a nu) = a \tan^{a-1} nu \times \frac{d(\tan nu)}{dx} \times \frac{d(nu)}{dx}$  and  $\frac{d}{dx} x^n = nx^{n-1}$

Let us take  $y = \tan^3 x$

So, by using the above formula, we have

$$\frac{d}{dx} \tan^3 x = 3 \tan^2(x) \times \frac{d(\tan x)}{dx} \times \frac{dx}{dx} = 3 \tan^2 x \times (\sec^2 x).$$

Differentiation of  $y = \tan^3 x$  is  $3\tan^2 x \times (\sec^2 x)$

**Q. 8. Differentiate the following with respect to x:**

$$e^{x^2}$$

**Answer :** To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e

$$\frac{d(u.v)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$

Formula used:  $\frac{d}{dx}(e^{a^t}) = e^{a^t} \times \frac{d}{dx}(a^t)$  and  $\frac{d}{dx} x^n = nx^{n-1}$

Let us take  $y = e^{x^2}$

So, by using the above formula, we have

$$\frac{d}{dx} e^{x^2} = e^{x^2} \times \frac{d}{dx}(x^2) = 2xe^{x^2}$$

Differentiation of  $y = e^{x^2}$  is  $2xe^{x^2}$

**Q. 9. Differentiate the following with respect to x:**

$$e^{\cot x}$$

**Answer :** To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e

$$\frac{d(u.v)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$

Formula used:  $\frac{d}{dx}(e^a) = e^a \times \frac{da}{dx}$  and  $\frac{d x^n}{dx} = nx^{n-1}$

Let us take  $y = e^{\cot x}$

So, by using the above formula, we have

$$\frac{d}{dx} e^{\cot x} = e^{\cot x} \times \frac{d \cot x}{dx} = -e^{\cot x} \operatorname{cosec}^2 x.$$

Differentiation of  $y = e^{\cot x}$  is  $-e^{\cot x} \operatorname{cosec}^2 x$

**Q. 10. Differentiate the following with respect to x:**

$$\sqrt{\sin x}$$

**Answer :** To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e

$$\frac{d(u.v)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$

Formula used:  $\frac{d}{dx}(\sqrt{\sin u}) = \frac{1}{2\sqrt{\sin u}} \times \frac{d}{dx}(\sin u) \times \frac{d}{dx}(u)$  and  $\frac{d x^n}{dx} = nx^{n-1}$

Let us take  $y = \sqrt{\sin x}$

So, by using the above formula, we have

$$\frac{d}{dx} \sqrt{\sin x} = \frac{1}{2\sqrt{\sin x}} \times \frac{d}{dx}(\sin x) \frac{d}{dx}(x) = \frac{1}{2\sqrt{\sin x}} \cos x$$

Differentiation of  $y = \sqrt{\sin x}$  is  $\frac{1}{2\sqrt{\sin x}} \cos x$

**Q. 11. Differentiate the following with respect to x:**

$$(5 + 7x)^6$$

**Answer :** To Find: Differentiation

**NOTE :** When 2 functions are in the product then we used product rule i.e

$$\frac{d(u.v)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$

$$\text{Formula used: } \frac{d}{dx}(y^n) = ny^{n-1} \times \frac{dy}{dx}$$

$$\text{Let us take } y = (5 + 7x)^6$$

So, by using the above formula, we have

$$\frac{d}{dx}(5 + 7x)^6 = 6(5 + 7x)^5 \times \frac{d}{dx}(5 + 7x) = 6(5 + 7x)^5 \times 7 = 42(5 + 7x)^5$$

Differentiation of  $y = (5 + 7x)^6$  is  $42(5 + 7x)^5$

**Q. 12. Differentiate the following with respect to x:**

$$(3 - 4x)^5$$

**Answer :** To Find: Differentiation

**NOTE :** When 2 functions are in the product then we used product rule i.e

$$\frac{d(u.v)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$

$$\text{Formula used: } \frac{d}{dx}(y^n) = ny^{n-1} \times \frac{dy}{dx}$$

$$\text{Let us take } y = (3 - 4x)^5$$

So, by using the above formula, we have

$$\frac{d}{dx}(3 - 4x)^5 = 4(3 - 4x)^5 \times \frac{d}{dx}(3 - 4x) = 4(3 - 4x)^5 \times (-4) = -16(3 - 4x)^5$$

Differentiation of  $y = (3 - 4x)^5$  is  $-16(3 - 4x)^5$

**Q. 13. Differentiate the following with respect to x:**

$$(3x^2 - x + 1)^4$$

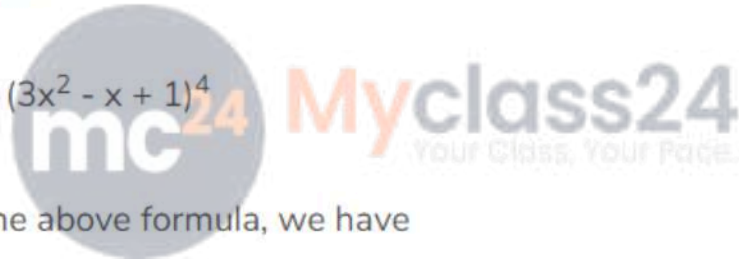
**Answer :** To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e

$$\frac{d(u.v)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$

$$\text{Formula used: } \frac{d}{dx}(y^n) = ny^{n-1} \times \frac{dy}{dx}$$

Let us take  $y = (3x^2 - x + 1)^4$



So, by using the above formula, we have

$$\begin{aligned} \frac{d}{dx}(3x^2 - x + 1)^4 &= 4(3x^2 - x + 1)^3 \times \frac{d}{dx}(3x^2 - x + 1) = 4(3x^2 - x + 1)^3 \times (3 \times 6x - 1) \\ &= 4(3x^2 - x + 1)^3(6x - 1) \end{aligned}$$

Differentiation of  $y = (3x^2 - x + 1)^4$  is  $4(3x^2 - x + 1)^3(6x - 1)$

**Q. 14. Differentiate the following with respect to x:**

$$(ax^2 + bx + c)$$

**Answer :** To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e

$$\frac{d(u.v)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$

Formula used:  $\frac{d}{dx}(y^n) = ny^{n-1} \times \frac{dy}{dx}$

Let us take  $y = (ax^2 + bx + c)$

So, by using the above formula, we have

$$\frac{d}{dx}(ax^2 + bx + c) = 2ax + b$$

Differentiation of  $y = (ax^2 + bx + c)$  is  $2ax + b$

**Q. 15. Differentiate the following with respect to x:**

$$\frac{1}{(x^2 - x + 3)^3}$$

**Answer :** To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e

$$\frac{d(u.v)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$

Formula used:  $\frac{d}{dx}(y^n) = ny^{n-1} \times \frac{dy}{dx}$

$$\text{Let us take } y = \frac{1}{(x^2 - x + 3)^3} = (x^2 - x + 3)^{-3}$$

So, by using the above formula, we have

$$\frac{d}{dx}(x^2 - x + 3)^{-3} = -3(x^2 - x + 3)^{-4} \times (2x - 1) = -3 \frac{1}{(x^2 - x + 3)^4} (2x - 1)$$

Differentiation of  $y = (x^2 - x + 3)^{-3}$  is  $\frac{-3(2x-1)}{(x^2 - x + 3)^4}$

**Q. 16. Differentiate the following with respect to x:**

$$\sin^2 (2x + 3)$$

**Answer :** To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e

$$\frac{d(u.v)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$

$$\text{Formula used: } \frac{d}{dx} \sin^2 (ax + b) = 2 \sin (ax + b) \frac{d}{dx} \sin(ax + b) \frac{d}{dx}(ax + b)$$

$$\text{Let us take } y = \sin^2 (2x + 3)$$

So, by using above formula, we have

$$\frac{d}{dx} \sin^2 (2x + 3) = 2 \sin (2x + 3) \frac{d}{dx} \sin(2x + 3) \frac{d}{dx}(2x + 3) = 4 \sin(2x + 3) \cos(2x + 3)$$

$$\text{Differentiation of } y = \sin^2 (2x + 3) \text{ is } 4 \sin(2x + 3) \cos(2x + 3)$$

**Q. 17. Differentiate the following with respect to x:**

$$\cos^2(x^3)$$

**Answer :** To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e

$$\frac{d(u.v)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$

Formula used:  $\frac{d}{dx}(\cos^a nu) = a \cos^{a-1} nu \frac{d}{dx}(\cos nu) \frac{d}{dx}(nu)$

Let us take  $y = \cos^2(x^3)$

So, by using the above formula, we have

$$\frac{d}{dx} \cos^2(x^3) = 2 \cos x^3 (-\sin(x^3)) 3x^2 = -6x^2 \cos(x^3) \sin x^3$$

Differentiation of  $y = \cos^2(x^3)$  is  $-6x^2 \cos(x^3) \sin x^3$

**Q. 18. Differentiate the following with respect to x:**

$$\sqrt{\sin x^3}$$

**Answer :** To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e

$$\frac{d(u.v)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$

Formula used:  $\frac{d}{dx}(\sqrt{\sin u^a}) = \frac{1}{2\sqrt{\sin u^a}} \times \frac{d}{dx}(\sin u^a) \times \frac{d}{dx}(u^a)$

Let us take  $y = \sqrt{\sin x^3}$

So, by using the above formula, we have

$$\frac{d}{dx} \sqrt{\sin x^3} = \frac{1}{2\sqrt{\sin x^3}} \times \frac{d}{dx}(\sin x^3) \times \frac{d}{dx}(x^3) = \frac{1}{2\sqrt{\sin x^3}} \times (\cos x^3) \times 3x^2 = \frac{3x^2(\cos x^3)}{2\sqrt{\sin x^3}}$$

Differentiation of  $y = \sqrt{\sin x^3}$  is  $\frac{3x^2(\cos x^3)}{2\sqrt{\sin x^3}}$

**Q. 19. Differentiate the following with respect to x:**

$$\sqrt{x \sin x}$$

**Answer :** To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e

$$\frac{d(u.v)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$



Formula used:  $\frac{d}{dx}(\sqrt{u \sin u}) = \frac{1}{2\sqrt{u \sin u}} \times \frac{d}{dx}(u \sin u)$

Let us take  $y = \sqrt{x \sin x}$

So, by using the above formula, we have

$$\frac{d}{dx} \sqrt{x \sin x} = \frac{1}{2\sqrt{x \sin x}} \times \frac{d}{dx}(x \sin x) = \frac{1}{2\sqrt{x \sin x}} \times (\sin x + x \cos x) = \frac{(\sin x + x \cos x)}{2\sqrt{x \sin x}}$$

Differentiation of  $y = \sqrt{x \sin x}$  is  $\frac{(\sin x + x \cos x)}{2\sqrt{x \sin x}}$

**Q. 20. Differentiate the following with respect to x:**

$$\sqrt{\cot \sqrt{x}}$$

**Answer :** To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e

$$\frac{d(u.v)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$

Formula used:  $\frac{d}{dx}(\sqrt{\cot \sqrt{x}}) = \frac{1}{2\sqrt{\cot \sqrt{x}}} \times \frac{d}{dx}(\cot \sqrt{x}) \cdot \frac{d}{dx}(\sqrt{x})$

Let us take  $y = \sqrt{\cot \sqrt{x}}$

So, by using the above formula, we have

$$\frac{d}{dx} \sqrt{\cot \sqrt{x}} = \frac{1}{2\sqrt{\cot \sqrt{x}}} \times \frac{d}{dx} \cot$$

$$\sqrt{x} \times \frac{d}{dx} \sqrt{x} = \frac{1}{2\sqrt{\cot \sqrt{x}}} \times (-\sec^2 \sqrt{x}) \times \frac{1}{2\sqrt{x}} = \frac{-\sec^2 \sqrt{x}}{4\sqrt{x} \sqrt{\cot \sqrt{x}}}$$

Differentiation of  $y = \sqrt{\cot \sqrt{x}}$  is  $\frac{-\sec^2 \sqrt{x}}{4\sqrt{x}\sqrt{\cot \sqrt{x}}}$

**Q. 21. Differentiate the following with respect to x:**

**cos 3x sin 5x**

**Answer :** To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e

$$\frac{d(u.v)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$

Let us take  $y = \cos 3x \sin 5x$

So, by using the above formula, we have

$$\frac{d}{dx} (\cos 3x \sin 5x) = \sin 5x \frac{d(\cos 3x)}{dx} + \cos 3x \frac{d(\sin 5x)}{dx} =$$

$$\sin 5x (-3 \sin 3x) + \cos 3x (5 \cos 5x) = 5 \cos (3x) \cos (5x) - 3 \sin (5x) 3 \sin (3x)$$

Differentiation of  $y = \cos 3x \sin 5x$  is  $5 \cos (3x) \cos (5x) - 3 \sin (5x) 3 \sin (3x)$

**Q. 22. Differentiate the following with respect to x:**

**sin x sin 2x**

**Answer :** To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e

$$\frac{d(u.v)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$

Let us take  $y = \sin x \sin 2x$

So, by using the above formula, we have

$$\frac{d}{dx} (\sin x \sin 2x) = \sin x \frac{d(\sin 2x)}{dx} + \sin 2x \frac{d(\sin x)}{dx} = \sin x (2 \cos 2x) + \sin 2x (\sin x) = 2 \sin(x) \cos (2x) + \sin 2x (\sin x)$$

Differentiation of  $y = \sin x \sin 2x$  is  $2 \sin(x) \cos (2x) + \sin 2x (\sin x)$

**Q. 23. Differentiate w.r.t x:**

$$\cos(\sin \sqrt{ax + b})$$

**Answer :**

$$\text{Let } y = \cos(\sin \sqrt{ax + b}), z = \sin \sqrt{ax + b} \text{ and } w = \sqrt{ax + b}$$

**Formula :**

$$\frac{d(\cos x)}{dx} = -\sin x \text{ and } \frac{d(\sin x)}{dx} = \cos x$$

$$\frac{d(\sqrt{ax+b})}{dx} = \frac{1}{2} \times (ax + b)^{\frac{1}{2}-1} \times a$$

According to the chain rule of differentiation

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{dz} \times \frac{dz}{dw} \times \frac{dw}{dx} \\ &= -\sin(\sin \sqrt{ax + b}) \times \cos \sqrt{ax + b} \times \frac{1}{2} \times (ax + b)^{-\frac{1}{2}} \times a \\ &= -\frac{a}{2} \sin(\sin \sqrt{ax + b}) \times \cos \sqrt{ax + b} \times (ax + b)^{-\frac{1}{2}} \end{aligned}$$

**Q. 24. Differentiate w.r.t x:  $e^{2x} \sin 3x$**

**Answer :** Let  $y = e^{2x} \sin 3x$ ,  $z = e^{2x}$  and  $w = \sin 3x$

**Formula :**

$$\frac{d(e^x)}{dx} = e^x \text{ and } \frac{d(\sin x)}{dx} = \cos x$$

According to product rule of differentiation

$$\begin{aligned} \frac{dy}{dx} &= w \times \frac{dz}{dx} + z \times \frac{dw}{dx} \\ &= [\sin 3x \times (2 \times e^{2x})] + [e^{2x} \times 3 \cos 3x] \end{aligned}$$

$$= e^{2x} \times [2 \sin 3x + 3 \cos 3x]$$

**Q. 25. Differentiate w.r.t x:  $e^{3x} \cos 2x$**

**Answer :** Let  $y = e^{3x} \cos 2x$ ,  $z = e^{3x}$  and  $w = \cos 2x$

**Formula :**

$$\frac{d(e^x)}{dx} = e^x \text{ and } \frac{d(\cos x)}{dx} = -\sin x$$

According to the product rule of differentiation

$$\begin{aligned} \frac{dy}{dx} &= w \times \frac{dz}{dx} + z \times \frac{dw}{dx} \\ &= [\cos 2x \times (3 \times e^{3x})] + [e^{3x} \times (-2 \sin 2x)] \\ &= e^{3x} \times [3 \cos 2x - 2 \sin 2x] \end{aligned}$$

**Q. 26. Differentiate w.r.t x:  $e^{-5x} \cot 4x$**

**Answer :** Let  $y = e^{-5x} \cot 4x$ ,  $z = e^{-5x}$  and  $w = \cot 4x$

**Formula :**

$$\frac{d(e^x)}{dx} = e^x \text{ and } \frac{d(\cot x)}{dx} = -\operatorname{cosec}^2 x$$

According to the product rule of differentiation

$$\begin{aligned} \frac{dy}{dx} &= w \times \frac{dz}{dx} + z \times \frac{dw}{dx} \\ &= [\cot 4x \times (-5e^{-5x})] + [e^{-5x} \times (-4 \operatorname{cosec}^2 4x)] \\ &= -e^{-5x} \times [5 \cot 4x + 4 \operatorname{cosec}^2 4x] \end{aligned}$$

**Q. 27. Differentiate w.r.t x:  $\cos (x^3 \cdot e^x)$**

**Answer :** Let  $y = \cos (x^3 \cdot e^x)$ ,  $z = x^3 \cdot e^x$ ,  $m = e^x$  and  $w = x^3$

**Formula :**

$$\frac{d(e^x)}{dx} = e^x, \frac{d(x^n)}{dx} = n \times x^{n-1} \text{ and } \frac{d(\cos x)}{dx} = -\sin x$$

According to the product rule of differentiation

$$\begin{aligned} \frac{dz}{dx} &= w \times \frac{dm}{dx} + m \times \frac{dw}{dx} \\ &= [x^3 \times (e^x)] + [e^x \times (3x^2)] \\ &= e^x \times [x^3 + 3x^2] \end{aligned}$$

According to the chain rule of differentiation

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{dz} \times \frac{dz}{dx} \\ &= -\sin(x^3 \times e^x) \times \{e^x \times [x^3 + 3x^2]\} \end{aligned}$$

**Q. 28. Differentiate w.r.t x:  $e^{(x \sin x + \cos x)}$**

**Answer :** Let  $y = e^{(x \sin x + \cos x)}$ ,  $z = x \sin x + \cos x$ ,  $m = x$  and  $w = \sin x$

**Formula :**

$$\frac{d(e^x)}{dx} = e^x, \frac{d(\sin x)}{dx} = \cos x \text{ and } \frac{d(\cos x)}{dx} = -\sin x$$

According to the product rule of differentiation

$$\begin{aligned} \frac{dz}{dx} &= w \times \frac{dm}{dx} + m \times \frac{dw}{dx} + \frac{d(\cos x)}{dx} \\ &= [\sin x \times (1)] + [x \times (\cos x)] - \sin x \\ &= x \cos x \end{aligned}$$

According to the chain rule of differentiation

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{dz} \times \frac{dz}{dx} \\ &= e^{(x \sin x + \cos x)} \times (x \cos x) \end{aligned}$$

**Q. 29. Differentiate w.r.t x:**

$$\frac{e^x + e^{-x}}{e^x - e^{-x}}$$

**Answer :**

$$\text{Let } y = \frac{e^x + e^{-x}}{e^x - e^{-x}}, u = e^x + e^{-x}, v = e^x - e^{-x}$$

**Formula :**

$$\frac{d(e^x)}{dx} = e^x$$

According to the quotient rule of differentiation

$$\text{If } y = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(e^x - e^{-x}) \times (e^x - e^{-x}) - (e^x + e^{-x}) \times (e^x + e^{-x})}{(e^x - e^{-x})^2}$$

$$= \frac{(e^x - e^{-x})^2 - (e^x + e^{-x})^2}{(e^x - e^{-x})^2}$$

$$= \frac{(e^x - e^{-x} + e^x + e^{-x})(e^x - e^{-x} - e^x - e^{-x})}{(e^x - e^{-x})^2}$$

$$(a^2 - b^2 = (a - b)(a + b))$$

$$= \frac{(2e^x)(-2e^{-x})}{(e^x - e^{-x})^2}$$

$$= \frac{-4}{(e^x - e^{-x})^2}$$

**Q. 30. Differentiate w.r.t x:**

$$\frac{e^{2x} + e^{-2x}}{e^{2x} - e^{-2x}}$$

**Answer :**

$$\text{Let } y = \frac{e^{2x} + e^{-2x}}{e^{2x} - e^{-2x}}, u = e^{2x} + e^{-2x}, v = e^{2x} - e^{-2x}$$

**Formula :**

$$\frac{d(e^x)}{dx} = e^x$$

According to the quotient rule of differentiation

$$\text{If } y = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(e^{2x} - e^{-2x}) \times (2e^{2x} - 2e^{-2x}) - (e^{2x} + e^{-2x}) \times (2e^{2x} + 2e^{-2x})}{(e^{2x} - e^{-2x})^2}$$

$$= \frac{2(e^{2x} - e^{-2x})^2 - 2(e^{2x} + e^{-2x})^2}{(e^{2x} - e^{-2x})^2}$$

$$= \frac{2(e^{2x} - e^{-2x} + e^{2x} + e^{-2x})(e^{2x} - e^{-2x} - e^{2x} - e^{-2x})}{(e^{2x} - e^{-2x})^2}$$

$$(a^2 - b^2) = (a - b)(a + b)$$

$$= \frac{2(2e^{2x})(-2e^{-2x})}{(e^{2x} - e^{-2x})^2}$$

$$= \frac{-8}{(e^{2x} - e^{-2x})^2}$$

**Q. 31. Differentiate w.r.t x:**

$$\sqrt{\frac{1-x^2}{1+x^2}}$$

**Answer :**

$$\text{Let } y = \sqrt{\frac{1-x^2}{1+x^2}}, u = 1 - x^2, v = 1 + x^2, z = \frac{1-x^2}{1+x^2}$$

**Formula :**

$$\frac{d(x^2)}{dx} = 2x$$



According to the quotient rule of differentiation

$$\text{If } z = \frac{u}{v}$$

$$\frac{dz}{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(1+x^2) \times (-2x) - (1-x^2) \times (2x)}{(1+x^2)^2}$$

$$= \frac{-2x - 2x^3 - 2x + 2x^3}{(1+x^2)^2}$$

$$= \frac{-4x}{(1+x^2)^2}$$

According to chain rule of differentiation

$$\frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left[ \frac{1}{2} \times \left( \frac{1-x^2}{1+x^2} \right)^{\frac{1}{2}-1} \right] \times \left[ \frac{-4x}{(1+x^2)^2} \right]$$

$$= \left[ \frac{-2x}{1} \times \left( \frac{1-x^2}{1} \right)^{-\frac{1}{2}} \right] \times \left[ \frac{1}{(1+x^2)^2-\frac{1}{2}} \right]$$

$$= \left[ -2x \times (1-x^2)^{-\frac{1}{2}} \right] \times (1+x^2)^{-\frac{3}{2}}$$

**Q. 32. Differentiate w.r.t x:**

$$\sqrt{\frac{a^2 - x^2}{a^2 + x^2}}$$

**Answer :**



$$\text{Let } y = \sqrt{\frac{a^2 - x^2}{a^2 + x^2}}, u = a^2 - x^2, v = a^2 + x^2, z = \frac{a^2 - x^2}{a^2 + x^2}$$

**Formula :**

$$\frac{d(x^2)}{dx} = 2x$$

According to the quotient rule of differentiation

$$\text{If } z = \frac{u}{v}$$

$$\frac{dz}{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$\begin{aligned}
&= \frac{(a^2 + x^2) \times (-2x) - (a^2 - x^2) \times (2x)}{(a^2 + x^2)^2} \\
&= \frac{-2xa^2 - 2x^3 - 2xa^2 + 2x^3}{(1 + x^2)^2} \\
&= \frac{-4x a^2}{(1 + x^2)^2}
\end{aligned}$$

According to the chain rule of differentiation

$$\frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left[ \frac{1}{2} \times \left( \frac{a^2 - x^2}{a^2 + x^2} \right)^{\frac{1}{2} - 1} \right] \times \left[ \frac{-4x a^2}{(a^2 + x^2)^2} \right]$$

$$= \left[ \frac{-2xa^2}{1} \times \left( \frac{a^2 - x^2}{1} \right)^{-\frac{1}{2}} \right] \times \left[ \frac{1}{(a^2 + x^2)^2 \cdot \frac{1}{2}} \right]$$

$$= \left[ -2xa^2 \times (a^2 - x^2)^{-\frac{1}{2}} \right] \times (a^2 + x^2)^{-\frac{3}{2}}$$

**Q. 33. Differentiate w.r.t x:**

$$\sqrt{\frac{1 + \sin x}{1 - \sin x}}$$

**Answer :**

$$\text{Let } y = \sqrt{\frac{1 + \sin x}{1 - \sin x}}, u = 1 + \sin x, v = 1 - \sin x, z = \frac{1 + \sin x}{1 - \sin x}$$

**Formula :**

$$\frac{d(\sin x)}{dx} = \cos x$$

According to the quotient rule of differentiation

$$\text{If } z = \frac{u}{v}$$

$$\frac{dz}{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(1 - \sin x) \times (\cos x) - (1 + \sin x) \times (-\cos x)}{(1 - \sin x)^2}$$

$$= \frac{\cos x - \sin x \cos x + \cos x + \sin x \cos x}{(1 - \sin x)^2}$$

$$= \frac{2 \cos x}{(1 - \sin x)^2}$$

According to the chain rule of differentiation

$$\frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left[ \frac{1}{2} \times \left( \frac{1 + \sin x}{1 - \sin x} \right)^{\frac{1}{2} - 1} \right] \times \left[ \frac{2 \cos x}{(1 - \sin x)^2} \right]$$

$$= \left[ \frac{\cos x}{1} \times \left( \frac{1 + \sin x}{1} \right)^{-\frac{1}{2}} \right] \times \left[ \frac{1}{(1 - \sin x)^{2 - \frac{1}{2}}} \right]$$

$$= \left[ \cos x \times (1 + \sin x)^{-\frac{1}{2}} \right] \times (1 - \sin x)^{-\frac{3}{2}}$$

**Q. 34. Differentiate w.r.t x:**

$$\sqrt{\frac{1 + e^x}{1 - e^x}}$$

**Answer :**

$$\text{Let } y = \sqrt{\frac{1+e^x}{1-e^x}}, u = 1 + e^x, v = 1 - e^x, z = \frac{1+e^x}{1-e^x}$$

**Formula :**

$$\frac{d(e^x)}{dx} = e^x$$

According to the quotient rule of differentiation

$$\text{If } z = \frac{u}{v}$$

$$\frac{dz}{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(1 - e^x) \times (e^x) - (1 + e^x) \times (-e^x)}{(1 - e^x)^2}$$

$$= \frac{e^x - e^{2x} + e^x + e^{2x}}{(1 - e^x)^2}$$

$$= \frac{2e^x}{(1 - e^x)^2}$$



According to chain rule of differentiation

$$\frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left[ \frac{1}{2} \times \left( \frac{1+e^x}{1-e^x} \right)^{\frac{1}{2}-1} \right] \times \left[ \frac{2e^x}{(1-e^x)^2} \right]$$

$$= \left[ \frac{e^x}{1} \times \left( \frac{1+e^x}{1} \right)^{-\frac{1}{2}} \right] \times \left[ \frac{1}{(1-e^x)^{2-\frac{1}{2}}} \right]$$

$$= \left[ e^x \times (1 + e^x)^{-\frac{1}{2}} \right] \times (1 - e^x)^{-\frac{3}{2}}$$

**Q. 35. Differentiate w.r.t x:**

$$\frac{e^{2x} + x^3}{\operatorname{cosec} 2x}$$

**Answer :**

**Formula:**

$$\frac{d(e^x)}{dx} = e^x, \frac{d(x^n)}{dx} = n \times x^{n-1} \text{ and } \frac{d(\operatorname{cosec} x)}{dx} = -\operatorname{cosec} x \cot x$$

According to the quotient rule of differentiation

$$\text{if } y = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(\operatorname{cosec} 2x) \times (2e^{2x} + 3x^2) - (e^{2x} + x^3) \times (-2 \operatorname{cosec} 2x \cot 2x)}{(\operatorname{cosec} 2x)^2}$$

$$= \frac{2e^{2x} \operatorname{cosec} 2x + 3x^2 \operatorname{cosec} 2x + 2e^{2x} \operatorname{cosec} 2x \cot 2x + 2x^3 \operatorname{cosec} 2x \cot 2x}{(\operatorname{cosec} 2x)^2}$$

$$= \frac{2e^{2x} \operatorname{cosec} 2x (1 + \cot 2x) + 3x^2 \operatorname{cosec} 2x (1 + \cot 2x)}{(\operatorname{cosec} 2x)^2}$$

$$= \frac{(1 + \cot 2x)(2e^{2x} \operatorname{cosec} 2x + 3x^2 \operatorname{cosec} 2x)}{(\operatorname{cosec} 2x)^2}$$

$$= \frac{(1 + \cot 2x)(2e^{2x} + 3x^2)(\operatorname{cosec} 2x)}{(\operatorname{cosec} 2x)^2}$$

$$= \frac{(1 + \cot 2x)(2e^{2x} + 3x^2)}{(\operatorname{cosec} 2x)^1}$$

$$= (1 + \cot 2x)(2e^x + 3x^2)(\sin 2x)$$

**Q. 36**

Find  $\frac{dy}{dx}$ , When  $y = \sin \sqrt{\sin x + \cos x}$

**Answer :**

$$\text{Let } y = \sin(\sqrt{\sin x + \cos x}), z = \sqrt{\sin x + \cos x}$$

$$\text{Formula : } \frac{d(\cos x)}{dx} = -\sin x \text{ and } \frac{d(\sin x)}{dx} = \cos x$$

$$\frac{d(\sqrt{\sin x + \cos x})}{dx} = \frac{1}{2} \times (\sin x + \cos x)^{\frac{1}{2}-1} \times (\cos x - \sin x)$$

According to the chain rule of differentiation

$$\frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \cos(\sin \sqrt{\sin x + \cos x}) \times \frac{1}{2} \times (\sin x + \cos x)^{\frac{1}{2}-1} \times (\cos x - \sin x)$$

$$= \cos(\sin \sqrt{\sin x + \cos x}) \times \frac{1}{2} \times (\sin x + \cos x)^{-\frac{1}{2}} \times (\cos x - \sin x)$$

**Q. 37.**

Find  $\frac{dy}{dx}$ , When  $y = e^x \log(\sin 2x)$

**Answer :**

$$\text{Let } y = e^x \log(\sin 2x), z = e^x \text{ and } w = \log(\sin 2x)$$

**Formula :**

$$\frac{d(e^x)}{dx} = e^x, \frac{d(\log x)}{dx} = \frac{1}{x} \text{ and } \frac{d(\sin x)}{dx} = \cos x$$

According to the product rule of differentiation

$$\begin{aligned} \frac{dy}{dx} &= w \times \frac{dz}{dx} + z \times \frac{dw}{dx} \\ &= [\log(\sin 2x) \times (e^x)] + [e^x \times \frac{1}{\sin 2x} \times 2 \cos 2x] \\ &= e^x \times [\log(\sin 2x) + \frac{2 \cos 2x}{\sin 2x}] \\ &= e^x \times [\log(\sin 2x) + 2 \cot 2x] \end{aligned}$$

**Q. 38.**

Find  $\frac{dy}{dx}$ , When  $y = \cos\left(\frac{1-x^2}{1+x^2}\right)$

**Answer :**

Let  $y = \cos\left(\frac{1-x^2}{1+x^2}\right)$ ,  $u = 1 - x^2$ ,  $v = 1 + x^2$ ,  $z = \frac{1-x^2}{1+x^2}$

**Formula :**

$$\frac{d(x^2)}{dx} = 2x \text{ and } \frac{d(\cos x)}{dx} = -\sin x$$

According to the quotient rule of differentiation

$$\text{If } z = \frac{u}{v}$$

$$\begin{aligned} \frac{dz}{dx} &= \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2} \\ &= \frac{(1+x^2) \times (-2x) - (1-x^2) \times (2x)}{(1+x^2)^2} \end{aligned}$$

$$= \frac{-2x - 2x^3 - 2x + 2x^3}{(1+x^2)^2}$$

$$= \frac{-4x}{(1+x^2)^2}$$

According to the chain rule of differentiation

$$\frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left[ -\sin \frac{1-x^2}{1+x^2} \right] \times \left[ \frac{-4x}{(1+x^2)^2} \right]$$

$$= \left[ \sin \frac{1-x^2}{1+x^2} \right] \times \left[ \frac{4x}{(1+x^2)^2} \right]$$

Q. 39. Find  $\frac{dy}{dx}$ , When  $y = \sin \left( \frac{1+x^2}{1-x^2} \right)$

Answer :

Let  $y = \sin \left( \frac{1+x^2}{1-x^2} \right)$ ,  $u = 1+x^2$ ,  $v = 1-x^2$ ,  $z = \frac{1+x^2}{1-x^2}$

Formula :  $\frac{d(x^2)}{dx} = 2x$  and  $\frac{d(\sin x)}{dx} = \cos x$

According to the quotient rule of differentiation

If  $z = \frac{u}{v}$

$$\frac{dz}{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(1 - x^2) \times (2x) - (1 + x^2) \times (-2x)}{(1 - x^2)^2}$$

$$= \frac{2x - 2x^3 + 2x + 2x^3}{(1 + x^2)^2}$$

$$= \frac{4x}{(1 + x^2)^2}$$

According to the chain rule of differentiation

$$\frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left[ \cos \frac{1+x^2}{1-x^2} \right] \times \left[ \frac{4x}{(1+x^2)^2} \right]$$

Q. 40. Find  $\frac{dy}{dx}$ , When  $y = \frac{\sin x + x^2}{\cot 2x}$

Answer :

Let  $y = \frac{\sin x + x^2}{\cot 2x}$ ,  $u = \sin x + x^2$ ,  $v = \cot 2x$

Formula:

$$\frac{d(\sin x)}{dx} = \cos x, \frac{d(x^n)}{dx} = n \times x^{n-1} \text{ and } \frac{d(\cot x)}{dx} = -\operatorname{cosec}^2 x$$

According to the quotient rule of differentiation

If  $y = \frac{u}{v}$

$$\frac{dy}{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$\begin{aligned}
&= \frac{(\cot 2x) \times (\cos x + 2x) - (\sin x + x^2) \times (-2 \operatorname{cosec}^2 2x)}{(\cot 2x)^2} \\
&= \frac{\cot 2x \cos x + 2x \cot 2x + 2 \operatorname{cosec}^2 2x \sin x + 2x^2 \operatorname{cosec}^2 2x}{(\operatorname{cosec} 2x)^2} \\
&= \frac{\cot 2x (\cos x + 2x) + 2 \operatorname{cosec}^2 2x (\sin x + x^2)}{(\operatorname{cosec} 2x)^2} \\
&= \frac{2 \operatorname{cosec}^2 2x (\sin x + x^2)}{(\operatorname{cosec} 2x)^2} + \frac{\cot 2x (\cos x + 2x)}{(\operatorname{cosec} 2x)^2} \\
&= \frac{2 (\sin x + x^2)}{1} + \frac{\cos 2x (\cos x + 2x)}{\sin 2x \frac{1}{\sin^2 2x}} \\
&= 2(\sin x + x^2) + \cos 2x \sin 2x (\cos x + 2x)
\end{aligned}$$

Q. 41.

If  $y = \frac{\cos x - \sin x}{\cos x + \sin x}$ , show that  $\frac{dy}{dx} + y^2 + 1 = 0$

**Answer :**

$$= -\frac{1}{1} - y^2 \left( y = \frac{\cos x - \sin x}{\cos x + \sin x} \right)$$

**Formula:**

$$\frac{d(\sin x)}{dx} = \cos x \text{ and } \frac{d(\cos x)}{dx} = -\sin x$$

According to the quotient rule of differentiation

If  $y = u/v$

$$\begin{aligned} \frac{dy}{dx} &= \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2} \\ &= \frac{(\cos x + \sin x) \times (-\sin x - \cos x) - (\cos x - \sin x) \times (-\sin x + \cos x)}{(\cos x + \sin x)^2} \\ &= \frac{-(\cos x + \sin x)^2 - (\cos x - \sin x)^2}{(\cos x + \sin x)^2} \\ &= -\frac{(\cos x + \sin x)^2}{(\cos x + \sin x)^2} - \frac{(\cos x - \sin x)^2}{(\cos x + \sin x)^2} \\ &= -\frac{1}{1} - y^2 \left( y = \frac{\cos x - \sin x}{\cos x + \sin x} \right) \end{aligned}$$

$$\frac{dy}{dx} + y^2 + 1 = 0$$

HENCE PROVED.

**Q. 42.**

If  $y = \frac{\cos x + \sin x}{\cos x - \sin x}$ , show that  $\frac{dy}{dx} = \sec^2 \left( x + \frac{\pi}{4} \right)$ .

**Answer :**

Let  $y = \frac{\cos x + \sin x}{\cos x - \sin x}$ , ,  $u = \cos x + \sin x$ ,  $v = \cos x - \sin x$

**Formula:**

$$\frac{d(\sin x)}{dx} = \cos x \text{ and } \frac{d(\cos x)}{dx} = -\sin x$$

According to the quotient rule of differentiation

$$\text{If } y = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(\cos x - \sin x) \times (-\sin x + \cos x) - (\cos x + \sin x) \times (-\sin x - \cos x)}{(\cos x - \sin x)^2}$$

$$= \frac{(\cos x - \sin x)^2 + (\cos x + \sin x)^2}{(\cos x - \sin x)^2}$$

$$= \frac{(\cos^2 x + \sin^2 x - 2 \cos x \sin x) + (\cos^2 x + \sin^2 x + 2 \cos x \sin x)}{(\cos x - \sin x)^2}$$

$$= \frac{2(\cos^2 x + \sin^2 x)}{(\cos x - \sin x)^2}$$

$$= \frac{(1)}{(\cos x - \sin x)^2 / 2} (\cos^2 x + \sin^2 x) = 1$$

$$= \frac{1}{\left(\frac{\cos x}{\sqrt{2}} - \frac{\sin x}{\sqrt{2}}\right)^2}$$

$$= \frac{1}{\left(\frac{\cos x \cos 45^\circ}{1} - \frac{\sin x \sin 45^\circ}{1}\right)^2}$$

$$= \frac{1}{\cos^2\left(x + \frac{\pi}{4}\right)} [\cos a \cos b - \sin a \sin b = \cos(a + b)]$$

$$= \sec^2\left(x + \frac{\pi}{4}\right)$$

HENCE PROVED.

**Q. 43.**

$$y = \sqrt{\frac{1-x}{1+x}}, \text{ prove that } (1-x^2) \frac{dy}{dx} + y = 0$$

**Answer :**

$$\text{Let } y = \sqrt{\frac{1-x^1}{1+x^1}}, u = 1 - x^1, v = 1 + x^1, z = \frac{1-x^1}{1+x^1}$$

**Formula :**

$$\frac{d(x^1)}{dx} = 1$$

According to quotient rule of differentiation

$$\text{If } z = \frac{u}{v}$$

$$\begin{aligned} \frac{dz}{dx} &= \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2} \\ &= \frac{(1+x^1) \times (-1) - (1-x^1) \times (1)}{(1+x^1)^2} \end{aligned}$$

$$= \frac{-1 - x^1 - 1 + x}{(1+x^1)^2}$$

$$= \frac{-2}{(1+x)^2}$$

According to the chain rule of differentiation

$$\frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left[ \frac{1}{2} \times \left( \frac{1-x^1}{1+x^1} \right)^{\frac{1}{2}-1} \right] \times \left[ \frac{-2}{(1+x^1)^2} \right]$$

$$= \left[ \frac{-1}{1} \times \left( \frac{1-x^1}{1+x} \right)^{-\frac{1}{2}} \right] \times \left[ \frac{1}{(1+x^1)^2} \right]$$

$$= \left[ -1 \times \frac{(1-x^1)^{-\frac{1}{2}}}{(1+x^1)^{1-\frac{1}{2}}} \right] \times \left[ \frac{1}{(1+x^1)^1} \right] \times \frac{1-x}{1-x}$$

(Multiplying and dividing by 1-x )

$$= \left[ -1 \times \frac{(1-x^1)^{1-\frac{1}{2}}}{(1+x^1)^{\frac{1}{2}}} \right] \times \frac{1}{(1-x)(1+x)}$$

$$= \left[ -1 \times \frac{(1-x^1)^{\frac{1}{2}}}{(1+x^1)^{\frac{1}{2}}} \right] \times \frac{1}{(1-x)(1+x)} = -\frac{y}{1-x^2}$$

Therefore

$$(1-x^2) \frac{dy}{dx} = -y$$

$$(1-x^2) \frac{dy}{dx} + y = 0$$

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**Q. 44.**

$$y = \sqrt{\frac{\sec x - \tan x}{\sec x + \tan x}}, \text{ show that } \frac{dy}{dx} = \sec x (\tan x + \sec x)$$

**Answer :**

$$y = \sqrt{\frac{\sec x - \tan x}{\sec x + \tan x}}$$

$$y = \sqrt{\frac{\frac{1}{\cos x} \frac{\sin x}{\cos x}}{\frac{1}{\cos x} + \frac{\sin x}{\cos x}}} = \sqrt{\frac{1 - \sin x}{1 + \sin x}}$$

$$u = 1 - \sin x, v = 1 + \sin x, z = \frac{1 - \sin x}{1 + \sin x}$$

$$\text{Formula : } \frac{d(\sin x)}{dx} = \cos x$$

According to quotient rule of differentiation

$$\text{If } z = \frac{u}{v}$$

$$\frac{dz}{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(1 + \sin x) \times (-\cos x) - (1 - \sin x) \times (\cos x)}{(1 + \sin x)^2}$$

$$= \frac{-\cos x - \sin x \cos x - \cos x + \sin x \cos x}{(1 + \sin x)^2}$$

$$= \frac{-2 \cos x}{(1 + \sin x)^2}$$

According to the chain rule of differentiation

$$\frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left[ \frac{1}{2} \times \left( \frac{1-\sin x}{1+\sin x} \right)^{\frac{1}{2}-1} \right] \times \left[ \frac{-2 \cos x}{(1+\sin x)^2} \right]$$

$$= \left[ -\frac{\cos x}{1} \times \left( \frac{1-\sin x}{1} \right)^{-\frac{1}{2}} \right] \times \left[ \frac{1}{(1+\sin x)^{2-\frac{1}{2}}} \right]$$

$$= \left[ \cos x \times (1 + \sin x)^{-\frac{1}{2}} \right] \times (1 - \sin x)^{-\frac{3}{2}} \times \left( \frac{1+\sin x}{1+\sin x} \right)^{\frac{3}{2}}$$

( Multiplying and dividing by  $(1 + \sin x)^{\frac{3}{2}}$  )

$$= \left[ \cos x \times (1 + \sin x)^{\frac{3}{2}-\frac{1}{2}} \right] \times (1 - \sin x)^{-\frac{3}{2}} \times \left( \frac{1}{1+\sin x} \right)^{\frac{3}{2}}$$

$$= \left[ \cos x \times (1 + \sin x)^{\frac{3}{2}-\frac{1}{2}} \right] \times (1 - \sin x)^{-\frac{3}{2}} \times (1 + \sin x)^{-\frac{3}{2}}$$

$$= \left[ \cos x \times (1 + \sin x)^1 \right] \times (1 - \sin^2 x)^{-\frac{3}{2}}$$

$$= \left[ \cos x \times (1 + \sin x)^1 \right] \times (\cos^2 x)^{-\frac{3}{2}}$$

$$= \left[ \cos x \times (1 + \sin x)^1 \right] \times (\cos x)^{-3}$$

$$= \left[ (1 + \sin x)^1 \right] \times (\cos x)^{-3+1}$$

$$= \frac{1+\sin x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} \times \frac{1 + \sin x}{\cos^2 x}$$

$$= \sec x \left( \frac{1}{\cos x} + \frac{\sin x}{\cos x} \right)$$

$$= \sec x (\sec x + \tan x)$$

HENCE PROVED

