EXERCISE 19.3

The Sum of the three terms of an A.P. is 21 and the product of the first, and the third terms exceed the second term by 6, find three terms.

Solution:

Given:

The sum of first three terms is 21

Let us assume the first three terms as a - d, a, a + d [where a is the first term and d is the common difference]

So, sum of first three terms is

$a - d + a + a + d = 21$

$3a = 21$

$a = 7$

It is also given that product of first and third term exceeds the second by 6

So, $(a - d)(a + d) - a = 6$

$a^2 - d^2 - a = 6$

Substituting the value of a = 7, we get

$7^2 - d^2 - 7 = 6$

$d^2 = 36$

$d = 6$ or $d = -6$

Hence, the terms of AP are a - d, a, a + d which is 1, 7, 13.

1. Three numbers are in A.P. If the sum of these numbers be 27 and the product 648, find the numbers

Solution:

Given:

Sum of first three terms is 27

Let us assume the first three terms as a - d, a, a + d [where a is the first term and d is the common difference]

So, sum of first three terms is

$a - d + a + a + d = 27$

$3a = 27$

$a = 9$

It is given that the product of three terms is 648

So, $a^3 - ad^2 = 648$

Substituting the value of a = 9, we get

$9^3 - 9d^2 = 648$

$729 - 9d^2 = 648$

$81 = 9d^2$

d = 3 or d = - 3
Hence, the given terms are a - d, a, a + d which is 6, 9, 12.

2. Find the four numbers in A.P., whose sum is 50 and in which the greatest number is 4 times the least.
Solution:
Given:
Sum of four terms is 50.
Let us assume these four terms as a – 3d, a – d, a + d, a + 3d
It is given that, sum of these terms is 4a = 50
So, a = 50/4
= 25/2 … (i)
It is also given that the greatest number is 4 time the least
a + 3d = 4(a - 3d)
Substitute the value of a = 25/2, we get
(25+6d)/2 = 50 – 12d
30d = 75
d = 75/30
= 25/10
= 5/2 … (ii)
Hence, the terms of AP are a – 3d, a – d, a + d, a + 3d which is 5, 10, 15, 20

3. The sum of three numbers in A.P. is 12, and the sum of their cubes is 288. Find the numbers.
Solution:

Given:
The sum of three numbers is 12
Let us assume the numbers in AP are a - d, a, a + d
So,
3a = 12
a = 4
It is also given that the sum of their cube is 288
$(a - d)^3 + a^3 + (a + d)^3 = 288$
$a^3 - d^3 - 3ad(a - d) + a^3 + a^3 + d^3 + 3ad(a + d) = 288$
Substitute the value of a = 4, we get
$64 - d^3 - 12d(4 - d) + 64 + 64 + d^3 + 12d(4 + d) = 288$
$192 + 24d^2 = 288$
d = 2 or d = - 2

Hence, the numbers are a - d, a, a + d which is 2, 4, 6 or 6, 4, 2

4. If the sum of three numbers in A.P. is 24 and their product is 440, find the numbers.

Solution:

Given:

Sum of first three terms is 24

Let us assume the first three terms are a - d, a, a + d [where a is the first term and d is the common difference]

So, sum of first three terms is a - d + a + a + d = 24

3a = 24

a = 8

It is given that the product of three terms is 440

So $a^3 - ad^2 = 440$

Substitute the value of a = 8, we get

$8^3 - 8d^2 = 440$

$512 - 8d^2 = 440$

$72 = 8d^2$

d = 3 or d = - 3

Hence, the given terms are a - d, a, a + d which is 5, 8, 11

5. The angles of a quadrilateral are in A.P. whose common difference is 10. Find the angles

Solution:

Given: d = 10

We know that the sum of all angles in a quadrilateral is 360^o

Let us assume the angles are a - 3d, a - d, a + d, a + 3d

So, a - 3d + a - d + a + d + a + 3d = 360^o

4a = 360^o

a = 90… (i)

And,

(a - d) – (a – 3d) = 10

2d = 10

d = 10/2

= 5

Hence, the angles are a - 3d, a - d, a + d, a + 3d which is 75^o, 85^o, 95^o, 105^o