

## EXERCISE 13.2

Express the following complex numbers in the standard form  $a + ib$ :

(i)  $(1 + i)(1 + 2i)$

(ii)  $(3 + 2i) / (-2 + i)$

(iii)  $1/(2 + i)^2$

(iv)  $(1 - i) / (1 + i)$

(v)  $(2 + i)^3 / (2 + 3i)$

(vi)  $[(1 + i)(1 + \sqrt{3}i)] / (1 - i)$

(vii)  $(2 + 3i) / (4 + 5i)$

(viii)  $(1 - i)^3 / (1 - i^3)$

(ix)  $(1 + 2i)^{-3}$

(x)  $(3 - 4i) / [(4 - 2i)(1 + i)]$

(xi)

$$\left( \frac{1}{1 - 4i} - \frac{2}{1 + i} \right) \left( \frac{3 - 4i}{5 + i} \right)$$

(xii)  $(5 + \sqrt{2}i) / (1 - \sqrt{2}i)$

**Solution:**

(i)  $(1 + i)(1 + 2i)$

Let us simplify and express in the standard form of  $(a + ib)$ ,

$$\begin{aligned} (1 + i)(1 + 2i) &= (1+i)(1+2i) \\ &= 1(1+2i) + i(1+2i) \\ &= 1 + 2i + i + 2i^2 \\ &= 1 + 3i + 2(-1) \text{ [since, } i^2 = -1] \\ &= 1 + 3i - 2 \\ &= -1 + 3i \end{aligned}$$

$\therefore$  The values of  $a, b$  are  $-1, 3$ .

(ii)  $(3 + 2i) / (-2 + i)$

Let us simplify and express in the standard form of  $(a + ib)$ ,

$$\begin{aligned} (3 + 2i) / (-2 + i) &= [(3 + 2i) / (-2 + i)] \times (-2 - i) / (-2 - i) \text{ [multiply and divide with } (-2 - i)] \\ &= [3(-2 - i) + 2i(-2 - i)] / [(-2)^2 - (i)^2] \\ &= [-6 - 3i - 4i - 2i^2] / (4 - i^2) \\ &= [-6 - 7i - 2(-1)] / (4 - (-1)) \text{ [since, } i^2 = -1] \\ &= [-4 - 7i] / 5 \end{aligned}$$

$\therefore$  The values of  $a, b$  are  $-4/5, -7/5$

(iii)  $1/(2 + i)^2$

Let us simplify and express in the standard form of  $(a + ib)$ ,

$$\begin{aligned}
 1/(2+i)^2 &= 1/(2^2 + i^2 + 2(2)(i)) \\
 &= 1/(4 - 1 + 4i) \text{ [since, } i^2 = -1] \\
 &= 1/(3 + 4i) \text{ [multiply and divide with } (3 - 4i)] \\
 &= 1/(3 + 4i) \times (3 - 4i)/(3 - 4i) \\
 &= (3-4i)/(3^2 - (4i)^2) \\
 &= (3-4i)/(9 - 16i^2) \\
 &= (3-4i)/(9 - 16(-1)) \text{ [since, } i^2 = -1] \\
 &= (3-4i)/25
 \end{aligned}$$

∴ The values of a, b are 3/25, -4/25

**(iv)**  $(1 - i) / (1 + i)$

Let us simplify and express in the standard form of  $(a + ib)$ ,

$$\begin{aligned}
 (1 - i) / (1 + i) &= (1 - i) / (1 + i) \times (1-i)/(1-i) \text{ [multiply and divide with } (1-i)] \\
 &= (1^2 + i^2 - 2(1)(i)) / (1^2 - i^2) \\
 &= (1 + (-1) - 2i) / (1 - (-1)) \\
 &= -2i/2 \\
 &= -i
 \end{aligned}$$

∴ The values of a, b are 0, -1

**(v)**  $(2 + i)^3 / (2 + 3i)$

Let us simplify and express in the standard form of  $(a + ib)$ ,

$$\begin{aligned}
 (2 + i)^3 / (2 + 3i) &= (2^3 + i^3 + 3(2)^2(i) + 3(i)^2(2)) / (2 + 3i) \\
 &= (8 + (i^2 \cdot i) + 3(4)(i) + 6i^2) / (2 + 3i) \\
 &= (8 + (-1)i + 12i + 6(-1)) / (2 + 3i) \\
 &= (2 + 11i) / (2 + 3i) \\
 &\text{ [multiply and divide with } (2-3i)] \\
 &= (2 + 11i)/(2 + 3i) \times (2-3i)/(2-3i) \\
 &= [2(2-3i) + 11i(2-3i)] / (2^2 - (3i)^2) \\
 &= (4 - 6i + 22i - 33i^2) / (4 - 9i^2) \\
 &= (4 + 16i - 33(-1)) / (4 - 9(-1)) \text{ [since, } i^2 = -1] \\
 &= (37 + 16i) / 13
 \end{aligned}$$

∴ The values of a, b are 37/13, 16/13

**(vi)**  $[(1 + i) (1 + \sqrt{3}i)] / (1 - i)$

Let us simplify and express in the standard form of  $(a + ib)$ ,

$$\begin{aligned}
 [(1 + i) (1 + \sqrt{3}i)] / (1 - i) &= [1(1 + \sqrt{3}i) + i(1 + \sqrt{3}i)] / (1 - i) \\
 &= (1 + \sqrt{3}i + i + \sqrt{3}i^2) / (1 - i) \\
 &= (1 + (\sqrt{3}+1)i + \sqrt{3}(-1)) / (1 - i) \text{ [since, } i^2 = -1] \\
 &= [(1 - \sqrt{3}) + (1 + \sqrt{3})i] / (1 - i)
 \end{aligned}$$

$$\begin{aligned}
 & \text{[multiply and divide with } (1+i)\text{]} \\
 & = [(1-\sqrt{3}) + (1+\sqrt{3})i] / (1-i) \times (1+i)/(1+i) \\
 & = [(1-\sqrt{3})(1+i) + (1+\sqrt{3})i(1+i)] / (1^2 - i^2) \\
 & = [1-\sqrt{3} + (1-\sqrt{3})i + (1+\sqrt{3})i + (1+\sqrt{3})i^2] / (1-(-1)) \text{ [since, } i^2 = -1\text{]} \\
 & = [(1-\sqrt{3})+(1-\sqrt{3}+1+\sqrt{3})i+(1+\sqrt{3})(-1)] / 2 \\
 & = (-2\sqrt{3} + 2i) / 2 \\
 & = -\sqrt{3} + i
 \end{aligned}$$

∴ The values of a, b are  $-\sqrt{3}, 1$

**(vii)**  $(2 + 3i) / (4 + 5i)$

Let us simplify and express in the standard form of  $(a + ib)$ ,

$$\begin{aligned}
 (2 + 3i) / (4 + 5i) &= \text{[multiply and divide with } (4-5i)\text{]} \\
 &= (2 + 3i) / (4 + 5i) \times (4-5i)/(4-5i) \\
 &= [2(4-5i) + 3i(4-5i)] / (4^2 - (5i)^2) \\
 &= [8 - 10i + 12i - 15i^2] / (16 - 25i^2) \\
 &= [8+2i-15(-1)] / (16 - 25(-1)) \text{ [since, } i^2 = -1\text{]} \\
 &= (23 + 2i) / 41
 \end{aligned}$$

∴ The values of a, b are  $23/41, 2/41$

**(viii)**  $(1 - i)^3 / (1 - i^3)$

Let us simplify and express in the standard form of  $(a + ib)$ ,

$$\begin{aligned}
 (1 - i)^3 / (1 - i^3) &= [1^3 - 3(1)^2i + 3(1)(i)^2 - i^3] / (1 - i^2 \cdot i) \\
 &= [1 - 3i + 3(-1) - i^2 \cdot i] / (1 - (-1)i) \text{ [since, } i^2 = -1\text{]} \\
 &= [-2 - 3i - (-1)i] / (1+i) \\
 &= [-2-4i] / (1+i) \\
 & \text{[Multiply and divide with } (1-i)\text{]} \\
 &= [-2-4i] / (1+i) \times (1-i)/(1-i) \\
 &= [-2(1-i)-4i(1-i)] / (1^2 - i^2) \\
 &= [-2+2i-4i+4i^2] / (1 - (-1)) \\
 &= [-2-2i+4(-1)] / 2 \\
 &= (-6-2i)/2 \\
 &= -3 - i
 \end{aligned}$$

∴ The values of a, b are  $-3, -1$

**(ix)**  $(1 + 2i)^{-3}$

Let us simplify and express in the standard form of  $(a + ib)$ ,

$$\begin{aligned}
 (1 + 2i)^{-3} &= 1/(1 + 2i)^3 \\
 &= 1/(1^3+3(1)^2(2i)+2(1)(2i)^2 + (2i)^3) \\
 &= 1/(1+6i+4i^2+8i^3)
 \end{aligned}$$

$$\begin{aligned}
 &= 1/(1+6i+4(-1)+8i^2.i) \text{ [since, } i^2 = -1] \\
 &= 1/(-3+6i+8(-1)i) \text{ [since, } i^2 = -1] \\
 &= 1/(-3-2i) \\
 &= -1/(3+2i) \\
 &\text{[Multiply and divide with } (3-2i)] \\
 &= -1/(3+2i) \times (3-2i)/(3-2i) \\
 &= (-3+2i)/(3^2 - (2i)^2) \\
 &= (-3+2i) / (9-4i^2) \\
 &= (-3+2i) / (9-4(-1)) \\
 &= (-3+2i) / 13
 \end{aligned}$$

∴ The values of a, b are  $-3/13, 2/13$

**(x)**  $(3 - 4i) / [(4 - 2i)(1 + i)]$

Let us simplify and express in the standard form of  $(a + ib)$ ,

$$\begin{aligned}
 (3 - 4i) / [(4 - 2i)(1 + i)] &= (3-4i) / [4(1+i)-2i(1+i)] \\
 &= (3-4i) / [4+4i-2i-2i^2] \\
 &= (3-4i) / [4+2i-2(-1)] \text{ [since, } i^2 = -1] \\
 &= (3-4i) / (6+2i) \\
 &\text{[Multiply and divide with } (6-2i)] \\
 &= (3-4i) / (6+2i) \times (6-2i)/(6-2i) \\
 &= [3(6-2i)-4i(6-2i)] / (6^2 - (2i)^2) \\
 &= [18 - 6i - 24i + 8i^2] / (36 - 4i^2) \\
 &= [18 - 30i + 8(-1)] / (36 - 4(-1)) \text{ [since, } i^2 = -1] \\
 &= [10-30i] / 40 \\
 &= (1 - 3i) / 4
 \end{aligned}$$

∴ The values of a, b are  $1/4, -3/4$

**(xi)**

$$\left( \frac{1}{1-4i} - \frac{2}{1+i} \right) \left( \frac{3-4i}{5+i} \right)$$

Let us simplify and express in the standard form of  $(a + ib)$ ,

$$\begin{aligned}
 \left( \frac{1}{1-4i} - \frac{2}{1+i} \right) \left( \frac{3-4i}{5+i} \right) &= \left( \frac{1+i-2(1-4i)}{(1-4i)(1+i)} \right) \left( \frac{3-4i}{5+i} \right) \\
 &= \left( \frac{1+i-2+8i}{1(1+i)-4i(1+i)} \right) \left( \frac{3-4i}{5+i} \right) \\
 &= \left( \frac{-1+9i}{1+i-4i-4i^2} \right) \left( \frac{3-4i}{5+i} \right) \\
 &= \left( \frac{-1+9i}{1-3i-4(-1)} \right) \left( \frac{3-4i}{5+i} \right)
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{(-1+9i)(3-4i)}{(5-3i)(5+i)} \\
 &= \frac{-1(3-4i)+9i(3-i)}{5(5+i)-3i(5+i)} \\
 &= \frac{-3+4i+27i-9i^2}{25+5i-15i-3i^2} \\
 &= \frac{-3+31i-9(-1)}{25-10i-3(-1)} \\
 &= \frac{6+31i}{28-10i}
 \end{aligned}$$

[Multiply and divide with (28+10i)]

$$\begin{aligned}
 &= \frac{6+31i}{28-10i} \times \frac{28+10i}{28+10i} \\
 &= \frac{6(28+10i)+31i(28+10i)}{28^2-(10i)^2} \\
 &= \frac{168+60i+868i+310i^2}{784-100i^2} \\
 &= \frac{168+928i+310(-1)}{784-100(-1)} \\
 &= \frac{478+928i}{884}
 \end{aligned}$$

Therefore value of a, b are 478/884, 928/884.

(xii)  $(5 + \sqrt{2}i) / (1 - \sqrt{2}i)$

Let us simplify and express in the standard form of (a + ib),

$$\begin{aligned}
 (5 + \sqrt{2}i) / (1 - \sqrt{2}i) &= [\text{Multiply and divide with } (1 + \sqrt{2}i)] \\
 &= (5 + \sqrt{2}i) / (1 - \sqrt{2}i) \times (1 + \sqrt{2}i) / (1 + \sqrt{2}i) \\
 &= [5(1 + \sqrt{2}i) + \sqrt{2}i(1 + \sqrt{2}i)] / (1^2 - (\sqrt{2})^2) \\
 &= [5 + 5\sqrt{2}i + \sqrt{2}i + 2i^2] / (1 - 2i^2) \\
 &= [5 + 6\sqrt{2}i + 2(-1)] / (1 - 2(-1)) \quad [\text{since, } i^2 = -1] \\
 &= [3 + 6\sqrt{2}i] / 3 \\
 &= 1 + 2\sqrt{2}i
 \end{aligned}$$

∴ The values of a, b are 1, 2√2

**2. Find the real values of x and y, if**

(i)  $(x + iy)(2 - 3i) = 4 + i$

(ii)  $(3x - 2iy)(2 + i)^2 = 10(1 + i)$

(iii)  $\frac{(1 + i)x - 2i}{3 + i} + \frac{(2 - 3i)y + i}{3 - i} = i$

**(iv)  $(1 + i)(x + iy) = 2 - 5i$**

**Solution:**

**(i)  $(x + iy)(2 - 3i) = 4 + i$**

Given:

$$(x + iy)(2 - 3i) = 4 + i$$

Let us simplify the expression we get,

$$x(2 - 3i) + iy(2 - 3i) = 4 + i$$

$$2x - 3xi + 2yi - 3yi^2 = 4 + i$$

$$2x + (-3x+2y)i - 3y(-1) = 4 + i \text{ [since, } i^2 = -1]$$

$$2x + (-3x+2y)i + 3y = 4 + i \text{ [since, } i^2 = -1]$$

$$(2x+3y) + i(-3x+2y) = 4 + i$$

Equating Real and Imaginary parts on both sides, we get

$$2x+3y = 4 \dots \text{ (i)}$$

$$\text{And } -3x+2y = 1 \dots \text{ (ii)}$$

Multiply (i) by 3 and (ii) by 2 and add

On solving we get,

$$6x - 6x - 9y + 4y = 12 + 2$$

$$13y = 14$$

$$y = 14/13$$

Substitute the value of y in (i) we get,

$$2x+3y = 4$$

$$2x + 3(14/13) = 4$$

$$2x = 4 - (42/13)$$

$$= (52-42)/13$$

$$2x = 10/13$$

$$x = 5/13$$

$$x = 5/13, y = 14/13$$

∴ The real values of x and y are 5/13, 14/13

**(ii)  $(3x - 2iy)(2 + i)^2 = 10(1 + i)$**

Given:

$$(3x - 2iy)(2+i)^2 = 10(1+i)$$

$$(3x - 2iy)(2^2+i^2+2(2)(i)) = 10+10i$$

$$(3x - 2iy)(4 + (-1)+4i) = 10+10i \text{ [since, } i^2 = -1]$$

$$(3x - 2iy)(3+4i) = 10+10i$$

Let us divide with 3+4i on both sides we get,

$$(3x - 2iy) = (10+10i)/(3+4i)$$

$$= \text{Now multiply and divide with } (3-4i)$$

$$= [10(3-4i) + 10i(3-4i)] / (3^2 - (4i)^2)$$

$$\begin{aligned}
 &= [30-40i+30i-40i^2] / (9 - 16i^2) \\
 &= [30-10i-40(-1)] / (9-16(-1)) \\
 &= [70-10i]/25
 \end{aligned}$$

Now, equating Real and Imaginary parts on both sides we get

$$3x = 70/25 \text{ and } -2y = -10/25$$

$$x = 70/75 \text{ and } y = 1/5$$

$$x = 14/15 \text{ and } y = 1/5$$

∴ The real values of x and y are 14/15, 1/5

$$(iii) \frac{(1+i)x - 2i}{3+i} + \frac{(2-3i)y + i}{3-i} = i$$

Given:

$$\frac{(1+i)x - 2i}{3+i} + \frac{(2-3i)y + i}{3-i} = i$$

$$\frac{((1+i)x-2i)(3-i) + ((2-3i)y+i)(3+i)}{(3+i)(3-i)} = i$$

$$\frac{((1+i)(3-i)x) - (2i)(3-i) + ((2-3i)(3+i)y) + (i)(3+i)}{3^2 - i^2} = i$$

$$\frac{(3-i+3i-i^2)x - 6i + 2i^2 + (6+2i-9i-3i^2)y + 3i + i^2}{9 - (-1)} = i$$

$$\frac{(3+2i-(-1))x - 6i + 2(-1) + (6-7i-3(-1))y + 3i + (-1)}{10} = i \text{ [since, } i^2 = -1]$$

$$(4+2i)x - 3i - 3 + (9-7i)y = 10i$$

$$(4x+9y-3) + i(2x-7y-3) = 10i$$

Now, equating Real and Imaginary parts on both sides we get,

$$4x+9y-3 = 0 \dots (i)$$

$$\text{And } 2x-7y-3 = 10$$

$$2x-7y = 13 \dots (ii)$$

Multiply (i) by 7 and (ii) by 9 and add

On solving these equations we get

$$28x + 18x + 63y - 63y = 117 + 21$$

$$46x = 117 + 21$$

$$46x = 138$$

$$x = 138/46$$

$$= 3$$

Substitute the value of x in (i) we get,

$$4x+9y-3 = 0$$

$$9y = -9$$

$$y = -9/9$$

$$= -1$$

$$x = 3 \text{ and } y = -1$$

∴ The real values of x and y are 3 and -1

$$(iv) (1 + i)(x + iy) = 2 - 5i$$

Given:

$$(1 + i)(x + iy) = 2 - 5i$$

Divide with (1+i) on both the sides we get,

$$(x + iy) = (2 - 5i)/(1+i)$$

Multiply and divide by (1-i)

$$= (2 - 5i)/(1+i) \times (1-i)/(1-i)$$

$$= [2(1-i) - 5i(1-i)] / (1^2 - i^2)$$

$$= [2 - 7i + 5(-1)] / 2 \text{ [since, } i^2 = -1]$$

$$= (-3-7i)/2$$

Now, equating Real and Imaginary parts on both sides we get

$$x = -3/2 \text{ and } y = -7/2$$

∴ The real values of x and y are -3/2, -7/2

### 3. Find the conjugates of the following complex numbers:

(i)  $4 - 5i$

(ii)  $1 / (3 + 5i)$

(iii)  $1 / (1 + i)$

(iv)  $(3 - i)^2 / (2 + i)$

(v)  $[(1 + i)(2 + i)] / (3 + i)$

(vi)  $[(3 - 2i)(2 + 3i)] / [(1 + 2i)(2 - i)]$

**Solution:**

(i)  $4 - 5i$

Given:

$$4 - 5i$$

We know the conjugate of a complex number (a + ib) is (a - ib)

So,

∴ The conjugate of (4 - 5i) is (4 + 5i)

(ii)  $1 / (3 + 5i)$

Given:

$$1 / (3 + 5i)$$

Since the given complex number is not in the standard form of (a + ib)

Let us convert to standard form by multiplying and dividing with  $(3 - 5i)$

We get,

$$\begin{aligned}\frac{1}{3+5i} &= \frac{1}{3+5i} \times \frac{3-5i}{3-5i} \\ &= \frac{3-5i}{3^2-(5i)^2} \\ &= \frac{3-5i}{9-25i^2} \\ &= \frac{3-5i}{9-25(-1)} \quad [\text{Since, } i^2 = -1] \\ &= \frac{3-5i}{34}\end{aligned}$$

We know the conjugate of a complex number  $(a + ib)$  is  $(a - ib)$

So,

$\therefore$  The conjugate of  $(3 - 5i)/34$  is  $(3 + 5i)/34$

**(iii)**  $1 / (1 + i)$

Given:

$$1 / (1 + i)$$

Since the given complex number is not in the standard form of  $(a + ib)$

Let us convert to standard form by multiplying and dividing with  $(1 - i)$

We get,

$$\begin{aligned}\frac{1}{1+i} &= \frac{1}{1+i} \times \frac{1-i}{1-i} \\ &= \frac{1-i}{1^2-i^2} \\ &= \frac{1-i}{1-(-1)} \quad [\text{since, } i^2 = -1] \\ &= \frac{1-i}{2}\end{aligned}$$

We know the conjugate of a complex number  $(a + ib)$  is  $(a - ib)$

So,

$\therefore$  The conjugate of  $(1-i)/2$  is  $(1+i)/2$

**(iv)**  $(3 - i)^2 / (2 + i)$

Given:

$$(3 - i)^2 / (2 + i)$$

Since the given complex number is not in the standard form of  $(a + ib)$

Let us convert to standard form,

$$\begin{aligned}\frac{(3-i)^2}{2+i} &= \frac{3^2+i^2-2(3)(i)}{2+i} \\ &= \frac{9+(-1)-6i}{2+i} \quad [\text{Since, } i^2 = -1] \\ &= \frac{8-6i}{2+i}\end{aligned}$$

Now, let us multiply and divide with  $(2 - i)$  we get,

$$\begin{aligned}\frac{8-6i}{2+i} &= \frac{8-6i}{2+i} \times \frac{2-i}{2-i} \\ &= \frac{8(2-i)-6i(2-i)}{2^2-i^2} \\ &= \frac{16-8i-12i+6i^2}{4-(-1)} \quad [\text{Since, } i^2 = -1] \\ &= \frac{16-20i+6(-1)}{5} \\ &= \frac{10-20i}{5} \\ &= 10/5 - 20i/5 \\ &= 2 - 4i\end{aligned}$$

We know the conjugate of a complex number  $(a + ib)$  is  $(a - ib)$

So,

$\therefore$  The conjugate of  $(2 - 4i)$  is  $(2 + 4i)$

(v)  $[(1 + i)(2 + i)] / (3 + i)$

Given:

$$[(1 + i)(2 + i)] / (3 + i)$$

Since the given complex number is not in the standard form of  $(a + ib)$

Let us convert to standard form,

$$\begin{aligned}\frac{(1+i)(2+i)}{3+i} &= \frac{1(2+i)+i(2+i)}{3+i} \\ &= \frac{2+i+2i+i^2}{3+i} \\ &= \frac{2+3i+(-1)}{3+i} \quad [\text{Since, } i^2 = -1] \\ &= \frac{1+3i}{3+i}\end{aligned}$$

Now, let us multiply and divide with  $(3 - i)$  we get,

$$\frac{1+3i}{3+i} = \frac{1+3i}{3+i} \times \frac{3-i}{3-i}$$

$$\begin{aligned}
 &= \frac{1(3-i)+3i(3-i)}{3^2-i^2} \\
 &= \frac{3-i+9i-3i^2}{9-(-1)} \quad [\text{Since, } i^2 = -1] \\
 &= \frac{3+8i-3(-1)}{10} \\
 &= \frac{6+8i}{10} \\
 &= \frac{3}{5} + \frac{4i}{5}
 \end{aligned}$$

We know the conjugate of a complex number  $(a + ib)$  is  $(a - ib)$

So,

$\therefore$  The conjugate of  $(3 + 4i)/5$  is  $(3 - 4i)/5$

(vi)  $[(3 - 2i)(2 + 3i)] / [(1 + 2i)(2 - i)]$

Given:

$$[(3 - 2i)(2 + 3i)] / [(1 + 2i)(2 - i)]$$

Since the given complex number is not in the standard form of  $(a + ib)$

Let us convert to standard form,

$$\begin{aligned}
 \frac{(3-2i)(2+3i)}{(1+2i)(2-i)} &= \frac{3(2+3i)-2i(2+3i)}{1(2-i)+2i(2-i)} \\
 &= \frac{6+9i-4i-6i^2}{2-i+4i-2i^2} \\
 &= \frac{6+5i-6(-1)}{2+3i-2(-1)} \\
 &= \frac{12+5i}{4+3i}
 \end{aligned}$$

Now, let us multiply and divide with  $(4 - 3i)$  we get,

$$\begin{aligned}
 \frac{12+5i}{4+3i} &= \frac{12+5i}{4+3i} \times \frac{4-3i}{4-3i} \\
 &= \frac{12(4-3i)+5i(4-3i)}{4^2-(3i)^2} \\
 &= \frac{48-36i+20i-15i^2}{16-9i^2} \\
 &= \frac{48-16i-15(-1)}{16-9(-1)} \\
 &= \frac{63 - 16i}{25}
 \end{aligned}$$

We know the conjugate of a complex number  $(a + ib)$  is  $(a - ib)$

So,

∴ The conjugate of  $(63 - 16i)/25$  is  $(63 + 16i)/25$

**4. Find the multiplicative inverse of the following complex numbers:**

(i)  $1 - i$

(ii)  $(1 + i\sqrt{3})^2$

(iii)  $4 - 3i$

(iv)  $\sqrt{5} + 3i$

**Solution:**

(i)  $1 - i$

Given:

$1 - i$

We know the multiplicative inverse of a complex number (Z) is  $Z^{-1}$  or  $1/Z$

So,

$$Z = 1 - i$$

$$Z^{-1} = \frac{1}{1 - i}$$

Let us multiply and divide by  $(1 + i)$  we get,

$$= \frac{1}{1 - i} \times \frac{1 + i}{1 + i}$$

$$= \frac{1 + i}{1^2 - (i)^2}$$

$$= \frac{1 + i}{1 - (-1)} \text{ [Since, } i^2 = -1 \text{]}$$

$$= \frac{1 + i}{2}$$

∴ The multiplicative inverse of  $(1 - i)$  is  $(1 + i)/2$

(ii)  $(1 + i\sqrt{3})^2$

Given:

$(1 + i\sqrt{3})^2$

$Z = (1 + i\sqrt{3})^2$

$$= 1^2 + (i\sqrt{3})^2 + 2(1)(i\sqrt{3})$$

$$= 1 + 3i^2 + 2i\sqrt{3}$$

$$= 1 + 3(-1) + 2i\sqrt{3} \text{ [since, } i^2 = -1 \text{]}$$

$$= 1 - 3 + 2i\sqrt{3}$$

$$= -2 + 2i\sqrt{3}$$

We know the multiplicative inverse of a complex number (Z) is  $Z^{-1}$  or  $1/Z$

So,

$$Z = -2 + 2i\sqrt{3}$$

$$Z^{-1} = \frac{1}{-2 + 2i\sqrt{3}}$$

Let us multiply and divide by  $-2 - 2i\sqrt{3}$ , we get

$$\begin{aligned} &= \frac{1}{-2+2\sqrt{3}i} \times \frac{-2-2\sqrt{3}i}{-2-2\sqrt{3}i} \\ &= \frac{-2-2\sqrt{3}i}{(-2)^2-(2\sqrt{3}i)^2} \\ &= \frac{-2-2\sqrt{3}i}{4-12i^2} \\ &= \frac{-2-2\sqrt{3}i}{4-12(-1)} \\ &= \frac{-2-2\sqrt{3}i}{16} \\ &= \frac{-1-i\sqrt{3}}{8} \end{aligned}$$

$\therefore$  The multiplicative inverse of  $(1 + i\sqrt{3})^2$  is  $(-1-i\sqrt{3})/8$

(iii)  $4 - 3i$

Given:

$$4 - 3i$$

We know the multiplicative inverse of a complex number (Z) is  $Z^{-1}$  or  $1/Z$ .

So,

$$Z = 4 - 3i$$

$$Z^{-1} = \frac{1}{4 - 3i}$$

Let us multiply and divide by  $(4 + 3i)$ , we get

$$\begin{aligned} &= \frac{1}{4-3i} \times \frac{4+3i}{4+3i} \\ &= \frac{4+3i}{4^2-(3i)^2} \\ &= \frac{4+3i}{16-9i^2} \\ &= \frac{4+3i}{16-9(-1)} \\ &= \frac{4+3i}{25} \end{aligned}$$

$\therefore$  The multiplicative inverse of  $(4 - 3i)$  is  $(4 + 3i)/25$

(iv)  $\sqrt{5} + 3i$

Given:

$$\sqrt{5} + 3i$$

We know the multiplicative inverse of a complex number ( $Z$ ) is  $Z^{-1}$  or  $1/Z$

So,

$$Z = \sqrt{5} + 3i$$

$$Z^{-1} = \frac{1}{\sqrt{5} + 3i}$$

Let us multiply and divide by  $(\sqrt{5} - 3i)$

$$= \frac{1}{\sqrt{5} + 3i} \times \frac{\sqrt{5} - 3i}{\sqrt{5} - 3i}$$

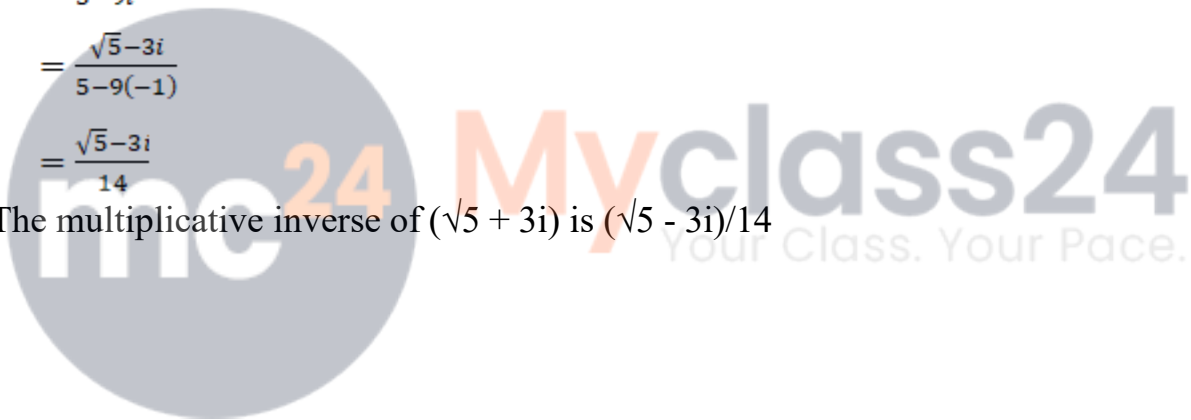
$$= \frac{\sqrt{5} - 3i}{(\sqrt{5})^2 - (3i)^2}$$

$$= \frac{\sqrt{5} - 3i}{5 - 9i^2}$$

$$= \frac{\sqrt{5} - 3i}{5 - 9(-1)}$$

$$= \frac{\sqrt{5} - 3i}{14}$$

$\therefore$  The multiplicative inverse of  $(\sqrt{5} + 3i)$  is  $(\sqrt{5} - 3i)/14$



$$\begin{aligned}
 &= \frac{\sqrt{4^2+0^2}}{\sqrt{1^2+(-1)^2}} \\
 &= \frac{4}{\sqrt{2}} \\
 &= 2\sqrt{2}
 \end{aligned}$$

∴ The value of  $\left| \frac{z_1+z_2+1}{z_1-z_2+i} \right|$  is  $2\sqrt{2}$

6. If  $z_1 = (2 - i)$ ,  $z_2 = (-2 + i)$ , find

- (i)  $\operatorname{Re} \left( \frac{z_1 z_2}{\bar{z}_1} \right)$   
 (ii)  $\operatorname{Im} \left( \frac{1}{z_1 \bar{z}_1} \right)$

**Solution:**

Given:

$$z_1 = (2 - i) \text{ and } z_2 = (-2 + i)$$

- (i)  $\operatorname{Re} \left( \frac{z_1 z_2}{\bar{z}_1} \right)$

We shall rationalise the denominator, we get

$$\begin{aligned}
 \frac{z_1 z_2}{\bar{z}_1} &= \frac{z_1 z_2}{z_1} \times \frac{z_1}{z_1} \\
 &= \frac{(z_1)^2 z_2}{z_1 \bar{z}_1} \\
 &= \frac{(2 - i)^2 (-2 + i)}{|z_1|^2} \quad [\text{since, } z\bar{z} = |z|^2] \\
 &= \frac{(2^2 + i^2 - 2 \times 2 \times i)(-2 + i)}{|2 - i|^2} \\
 &= \frac{(4 - 1 - 4i)(-2 + i)}{2^2 + (-1)^2} \\
 &= \frac{(3 - 4i)(-2 + i)}{4 + i} \\
 &= \frac{3(-2 + i) - 4i(-2 + i)}{4 + i} \\
 &= \frac{-6 + 3i + 8i + 4}{4 + i} \\
 &= \frac{-2 + 11i}{4 + i}
 \end{aligned}$$

∴ The real value of  $\left(\frac{z_1 z_2}{\bar{z}_1}\right)$  is  $\frac{-2}{5}$

$$\begin{aligned} \text{(ii) Im}\left(\frac{1}{z_1 \bar{z}_1}\right) \\ \frac{1}{z_1 \bar{z}_1} &= \frac{1}{|z_1|^2} \\ &= \frac{1}{|2 - i|^2} \\ &= \frac{1}{2^2 + (-1)^2} \\ &= \frac{1}{4 + 1} \\ &= \frac{1}{5} \end{aligned}$$

∴ The imaginary value of  $\left(\frac{1}{z_1 \bar{z}_1}\right)$  is 0

**7. Find the modulus of  $[(1 + i)/(1 - i)] - [(1 - i)/(1 + i)]$**

**Solution:**

Given:

$$[(1 + i)/(1 - i)] - [(1 - i)/(1 + i)]$$

So,

$$Z = [(1 + i)/(1 - i)] - [(1 - i)/(1 + i)]$$

Let us simplify, we get

$$\begin{aligned} &= [(1+i)(1+i) - (1-i)(1-i)] / (1^2 - i^2) \\ &= [1^2 + i^2 + 2(1)(i) - (1^2 + i^2 - 2(1)(i))] / (1 - (-1)) \text{ [Since, } i^2 = -1] \\ &= 4i/2 \\ &= 2i \end{aligned}$$

We know that for a complex number  $Z = (a+ib)$  it's magnitude is given by  $|z| = \sqrt{(a^2 + b^2)}$

So,

$$\begin{aligned} |Z| &= \sqrt{(0^2 + 2^2)} \\ &= 2 \end{aligned}$$

∴ The modulus of  $[(1 + i)/(1 - i)] - [(1 - i)/(1 + i)]$  is 2.

**8. If  $x + iy = (a+ib)/(a-ib)$ , prove that  $x^2 + y^2 = 1$**

**Solution:**

Given:

$$x + iy = (a+ib)/(a-ib)$$

We know that for a complex number  $Z = (a+ib)$  it's magnitude is given by  $|z| = \sqrt{a^2 + b^2}$

So,

$$|a/b| \text{ is } |a| / |b|$$

Applying Modulus on both sides we get,

$$|x + iy| = \left| \frac{a+ib}{a-ib} \right|$$

$$\sqrt{x^2 + y^2} = \frac{|a+ib|}{|a-ib|}$$

$$= \frac{\sqrt{a^2+b^2}}{\sqrt{a^2+(-b)^2}}$$

$$= \frac{\sqrt{a^2+b^2}}{\sqrt{a^2+b^2}}$$

$$= 1$$

Squaring on both sides we get,

$$(\sqrt{x^2 + y^2})^2 = 1^2$$

$$x^2 + y^2 = 1$$

∴ Hence Proved.

**9. Find the least positive integral value of n for which  $[(1+i)/(1-i)]^n$  is real.**

**Solution:**

Given:

$$[(1+i)/(1-i)]^n$$

$$Z = [(1+i)/(1-i)]^n$$

Now let us multiply and divide by  $(1+i)$ , we get

$$= \frac{1+i}{1-i} \times \frac{1+i}{1+i}$$

$$= \frac{(1+i)^2}{1^2 - i^2}$$

$$= \frac{1^2 + i^2 + 2(1)(i)}{1 - (-1)}$$

$$= \frac{1 - 1 + 2i}{2}$$

$$= \frac{2i}{2}$$

$$= i \text{ [which is not real]}$$

For  $n = 2$ , we have

$$[(1+i)/(1-i)]^2 = i^2$$

$$= -1 \text{ [which is real]}$$

So, the smallest positive integral 'n' that can make  $[(1+i)/(1-i)]^n$  real is 2.

∴ The smallest positive integral value of 'n' is 2.

**10. Find the real values of  $\theta$  for which the complex number  $(1 + i \cos \theta) / (1 - 2i \cos \theta)$  is purely real.**

**Solution:**

Given:

$$(1 + i \cos \theta) / (1 - 2i \cos \theta)$$

$$Z = (1 + i \cos \theta) / (1 - 2i \cos \theta)$$

Let us multiply and divide by  $(1 + 2i \cos \theta)$

$$= \frac{1+i\cos\theta}{1-2i\cos\theta} \times \frac{1+2i\cos\theta}{1+2i\cos\theta}$$

$$= \frac{1(1+2i\cos\theta)+i\cos\theta(1+2i\cos\theta)}{1^2-(2i\cos\theta)^2}$$

$$= \frac{1+2i\cos\theta+i\cos\theta+2i^2\cos^2\theta}{1-4i^2\cos^2\theta}$$

$$= \frac{1+3i\cos\theta+2(-1)\cos^2\theta}{1-4(-1)\cos^2\theta}$$

$$= \frac{1-2\cos^2\theta+3i\cos\theta}{1+4\cos^2\theta}$$

For a complex number to be purely real, the imaginary part should be equal to zero.

So,

$$\frac{3\cos\theta}{1+4\cos^2\theta} = 0$$

$$3\cos\theta = 0 \text{ (since, } 1 + 4\cos^2\theta \geq 1)$$

$$\cos\theta = 0$$

$$\cos\theta = \cos\pi/2$$

$$\theta = [(2n+1)\pi] / 2, \text{ for } n \in \mathbb{Z}$$

$$= 2n\pi \pm \pi/2, \text{ for } n \in \mathbb{Z}$$

∴ The values of  $\theta$  to get the complex number to be purely real is  $2n\pi \pm \pi/2$ , for  $n \in \mathbb{Z}$

**11. Find the smallest positive integer value of n for which  $(1+i)^n / (1-i)^{n-2}$  is a real number.**

**Solution:**

Given:

$$(1+i)^n / (1-i)^{n-2}$$

$$Z = (1+i)^n / (1-i)^{n-2}$$

Let us multiply and divide by  $(1 - i)^2$

$$\begin{aligned}
 &= \frac{(1+i)^n}{(1-i)^{n-2}} \times \frac{(1-i)^2}{(1-i)^2} \\
 &= \left(\frac{1+i}{1-i}\right)^n \times (1-i)^2 \\
 &= \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^n \times (1^2 + i^2 - 2(1)(i)) \\
 &= \left(\frac{(1+i)^2}{1^2-i^2}\right)^n \times (1 + i^2 - 2i) \\
 &= \left(\frac{1^2+i^2+2(1)(i)}{1-(-1)}\right)^n \times (1 + (-1) - 2i) \\
 &= \left(\frac{1-1+2i}{2}\right)^n \times (-2i) \\
 &= \left(\frac{2i}{2}\right)^n \times (-2i) \\
 &= i^n \times (-2i) \\
 &= -2i^{n+1}
 \end{aligned}$$

For  $n = 1$ ,

$$Z = -2i^{1+1}$$

$$= -2i^2$$

$$= 2, \text{ which is a real number.}$$

∴ The smallest positive integer value of  $n$  is 1.

**12. If  $[(1+i)/(1-i)]^3 - [(1-i)/(1+i)]^3 = x + iy$ , find  $(x, y)$**

**Solution:**

Given:

$$[(1+i)/(1-i)]^3 - [(1-i)/(1+i)]^3 = x + iy$$

Let us rationalize the denominator, we get

$$\left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^3 - \left(\frac{1-i}{1+i} \times \frac{1-i}{1-i}\right)^3 = x + iy$$

$$\left(\frac{(1+i)^2}{1^2-i^2}\right)^3 - \left(\frac{(1-i)^2}{1^2-i^2}\right)^3 = x + iy$$

$$\left(\frac{1^2+i^2+2(1)(i)}{1-(-1)}\right)^3 - \left(\frac{1^2+i^2-2(1)(i)}{1-(-1)}\right)^3 = x + iy$$

$$\left(\frac{1-1+2i}{2}\right)^3 - \left(\frac{1-1-2i}{2}\right)^3 = x + iy$$

$$\left(\frac{2i}{2}\right)^3 - \left(\frac{-2i}{2}\right)^3 = x + iy$$

$$i^3 - (-i)^3 = x + iy$$

$$2i^3 = x + iy$$

$$2i^2 \cdot i = x + iy$$

$$2(-1)i = x + iy$$

$$-2i = x + iy$$

Equating Real and Imaginary parts on both sides we get

$$x = 0 \text{ and } y = -2$$

$\therefore$  The values of  $x$  and  $y$  are  $0$  and  $-2$ .

**13. If  $(1+i)^2 / (2-i) = x + iy$ , find  $x + y$**

**Solution:**

Given:

$$(1+i)^2 / (2-i) = x + iy$$

Upon expansion we get,

$$\frac{1^2 + i^2 + 2(1)(i)}{2-i} = x + iy$$

$$\frac{1 + (-1) + 2i}{2-i} = x + iy$$

$$\frac{2i}{2-i} = x + iy$$

Now, let us multiply and divide by  $(2+i)$ , we get

$$\frac{2i}{2-i} \times \frac{2+i}{2+i} = x + iy$$

$$\frac{4i + 2i^2}{2^2 - i^2} = x + iy$$

$$\frac{2(-1) + 4i}{4 - (-1)} = x + iy$$

$$\frac{-2 + 4i}{5} = x + iy$$

Let us equate real and imaginary parts on both sides we get,

$$x = -2/5 \text{ and } y = 4/5$$

so,

$$x + y = -2/5 + 4/5$$

$$= (-2+4)/5$$

$$= 2/5$$

$\therefore$  The value of  $(x + y)$  is  $2/5$