NCERT Solutions for Class-XII Maths

Chapter-4.4

NCERT Math Class 12

1. Write minors and cofactors of the elements of following determinants:

(i) $\begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$

(ii) $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$

1. (i) $\begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$

The minor of element a_{ij} is M_{ij} and the cofactor is $A_{ij} = (-1)^{i+j} M_{ij}$, therefore,

The minor of element a_{11} is $M_{11} = 3$ and the cofactor is $A_{11} = (-1)^{1+1} M_{11} = 3$

The minor of element a_{12} is $M_{12} = 0$ and the cofactor is $A_{12} = (-1)^{1+2} M_{12} = 0$

The minor of element a_{21} is $M_{21} = -4$ and the cofactor is $A_{21} = (-1)^{2+1} M_{21} = 4$

The minor of element a_{22} is $M2_{22} = 2$ and the cofactor is $A_{22} = (-1)^{2+2} M_{22} = 2$

(ii) $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$

The minor of element a_{11} is $M_{11} = d$ and the cofactor is $A_{11} = (-1)^{1+1} M_{11} = d$

The minor of element a_{12} is $M_{12} = b$ and the cofactor is $A_{12} = (-1)^{1+2} M_{12} = -b$

The minor of element a_{21} is $M_{21} = c$ and the cofactor is $A_{21} = (-1)^{2+1} M_{21} = -c$

The minor of element a_{22} is $M_{22} = a$ and the cofactor is $A_{22} = (-1)^{2+2} M_{22} = a$

2. (i) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

(ii) $\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$

2. **(i)** $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

Minor of an element $a_{ij} = M_{ij}$

$a_{11} = 1$, Minor of element $a_{11} = M_{11} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = (1 \times 1) - (0 \times 0) = 1$

Here removing 1st row and 1st column from the determinant we are left out with the determinant $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$. Solving this we get $M_{11} = 1$

Similarly, finding other Minors of the determinant

$a_{12} = 0$, Minor of element $a_{12} = M_{12} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = (0 \times 1) - (0 \times 0) = 0$

$a_{13} = 0$, Minor of element $a_{13} = M_{13} = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = (0 \times 0) - (1 \times 0) = 0$

$a_{21} = 0$, Minor of element $a_{21} = M_{21} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = (0 \times 1) - (0 \times 0) = 0$

$a_{22} = 1$, Minor of element $a_{22} = M_{22} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = (1 \times 1) - (0 \times 0) = 1$

$a_{23} = 0$, Minor of element $a_{23} = M_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = (1 \times 0) - (0 \times 0) = 0$

$a_{31} = 0$, Minor of element $a_{31} = M_{31} = \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = (0 \times 0) - (0 \times 1) = 0$

$a_{32} = 0$, Minor of element $a_{32} = M_{32} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = (1 \times 0) - (0 \times 0) = 0$

$a_{33} = 1$, Minor of element $a_{33} = M_{33} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = (1 \times 1) - (0 \times 0) = 1$

Cofactor of an element a_{ij}, $A_{ij} = (-1)^{i+j} \times M_{ij}$

$A_{11} = (-1)^{1+1} \times M_{11} = 1 \times 1 = 1$

$A_{12} = (-1)^{1+2} \times M_{12} = (-1) \times 0 = 0$

$A_{13} = (-1)^{1+3} \times M_{13} = 1 \times 0 = 0$

$A_{21} = (-1)^{2+1} \times M_{21} = (-1) \times 0 = 0$

$A_{22} = (-1)^{2+2} \times M_{22} = 1 \times 1 = 1$

$A_{23} = (-1)^{2+3} \times M_{23} = (-1) \times 0 = 0$

$A_{31} = (-1)^{3+1} \times M_{31} = 1 \times 0 = 0$

$A_{32} = (-1)^{3+2} \times M_{32} = (-1) \times 0 = 0$

$A_{33} = (-1)^{3+3} \times M_{33} = 1 \times 1 = 1$

(ii) $\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$

Minor of an element $a_{ij} = M_{ij}$

$a_{11} = 1$, Minor of element $a_{11} = M_{11} = \begin{vmatrix} 5 & -1 \\ 1 & 2 \end{vmatrix} = (5 \times 2) - ((-1) \times 1) = 10 + 1 = 11$

Here removing 1st row and 1st column from the determinant we are left out with the determinant $\begin{vmatrix} 5 & -1 \\ 1 & 2 \end{vmatrix}$. Solving this we get $M_{11} = 11$

Similarly, finding other Minors of the determinant

$a_{12} = 0$, Minor of element $a_{12} = M_{12} = \begin{vmatrix} 3 & -1 \\ 0 & 2 \end{vmatrix} = (3 \times 2) - ((-1) \times 0) = (6 - 0) = 6$

$a_{13} = 4$, Minor of element $a_{13} = M_{13} = \begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = (3 \times 1) - (5 \times 0) = 3 - 0 = 3$

$a_{21} = 3$, Minor of element $a_{21} = M_{21} = \begin{vmatrix} 0 & 4 \\ 1 & 2 \end{vmatrix} = (0 \times 2) - (4 \times 1) = 0 - 4 = -4$

$a_{22} = 5$, Minor of element $a_{22} = M_{22} = \begin{vmatrix} 1 & 4 \\ 0 & 2 \end{vmatrix} = (1 \times 2) - (4 \times 0) = 2 - 0 = 2$

$a_{23} = -1$, Minor of element $a_{23} = M_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = (1 \times 1) - (0 \times 0) = 1$

$a_{31} = 0$, Minor of element $a_{31} = M_{31} = \begin{vmatrix} 0 & 4 \\ 5 & -1 \end{vmatrix} = (0 \times (-1)) - (4 \times 5) = 0 - 20 = -20$

$a_{32} = 1$, Minor of element $a_{32} = M_{32} = \begin{vmatrix} 1 & 4 \\ 3 & -1 \end{vmatrix} = (1 \times (-1)) - (4 \times 3) = -1 - 12 = -13$

$a_{33} = 2$, Minor of element $a_{33} = M_{33} = \begin{vmatrix} 1 & 0 \\ 3 & 5 \end{vmatrix} = (1 \times 5) - (0 \times 3) = (5 - 0) = 5$

Cofactor of an element a_{ij}, $A_{ij} = (-1)^{i+j} \times M_{ij}$

$A_{11} = (-1)^{1+1} \times M_{11} = 1 \times 11 = 11$

$A_{12} = (-1)^{1+2} \times M_{12} = (-1) \times 6 = -6$

$A_{13} = (-1)^{1+3} \times M_{13} = 1 \times 3 = 3$

$A_{21} = (-1)^{2+1} \times M_{21} = (-1) \times (-4) = 4$

$A_{22} = (-1)^{2+2} \times M_{22} = 1 \times 2 = 2$

$A_{23} = (-1)^{2+3} \times M_{23} = (-1) \times 1 = -1$

$A_{31} = (-1)^{3+1} \times M_{31} = 1 \times (-20) = -20$

$A_{32} = (-1)^{3+2} \times M_{32} = (-1) \times (-13) = 13$

$A_{33} = (-1)^{3+3} \times M_{33} = 1 \times 5 = 5$

3. using cofactors of elements of second row, evaluate $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$.

3. $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix} = a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23}$

Here, $a_{21} = 2$, $a_{22} = 0$, $a_{23} = 1$ and

$A_{21} = (-1)^{2+1} \begin{vmatrix} 3 & 8 \\ 2 & 3 \end{vmatrix} = -(9-16) = 7$

$A_{22} = (-1)^{2+2} \begin{vmatrix} 5 & 8 \\ 1 & 3 \end{vmatrix} = 15 - 8 = 7$

$A_{23} = (-1)^{2+3} \begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} = -(10-3) = -7$

Therefore, $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix} = 2(7) + 0(7) + 1(-7) = 7$

4. Using cofactors of elements of third column, evaluate $\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & x & xy \end{vmatrix}$

4. To evaluate a determinant using cofactors, Let

$B = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

Expanding along Row 1

$B = (-1)^{1+1} \times a_{11} \times \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} \times a_{12} \times \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} \times a_{13} \times \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

$B = a_{11} A_{11} + a_{12} A_{12} + a_{13} A_{13}$

[Where A_{ij} represents cofactors of a_{ij} of determinant B.]

B = Sum of product of elements of R_1 with their corresponding cofactors

Similarly, the determinant can be solved by expanding along column

So, B = sum of product of elements of any row or column with their corresponding cofactors

$\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$

Cofactors of third column

$A_{13} = (-1)^{1+3} \times M_{13} = 1 \times \begin{vmatrix} 1 & y \\ 1 & z \end{vmatrix} = 1 \times (1 \times z - 1 \times y) = (z - y)$

$A_{23} = (-1)^{2+3} \times M_{23} = (-1) \times \begin{vmatrix} 1 & x \\ 1 & z \end{vmatrix} = (-1) \times (1 \times z - 1 \times x) = -(z - x) = (x - z)$

$A_{33} = (-1)^{3+3} \times M_{33} = 1 \times \begin{vmatrix} 1 & x \\ 1 & y \end{vmatrix} = 1 \times (1 \times y - 1 \times x) = (y - x)$

[Where $A_{ij} = (-1)^{i+j} \times M_{ij}$, M_{ij}=Minor of i^{th} row & j^{th} column]

Therefore,

$\Delta = a_{13}A_{13} + a_{23}A_{23} + a_{33}A_{33}$

$\Delta = yz(z - y) + zx(x - z) + xy(y - x) = z[y(z - y) + x(x - z)] + xy(y - x)$

$\Delta = z(yz - y^2 + x^2 - xz) + xy(y - x) = z[(yz - xz) + (x^2 - y^2)] + xy(y - x)$

$\Delta = z[z \times (y - x) + (x + y) \times (x - y)] + xy(y - x)$

$\Delta = z \times (y - x) \times (z - x - y) + xy(y - x)$

$\Delta = (y - x) \times (z^2 - xz - yz + xy)$

$\Delta = (y - x) \times [z(z - x) - y(z - x)] = (y - x) \times (z - y) \times (z - x)$

$\Delta = (x - y)(y - z)(z - x)$

5. If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ A_{ij} is cofactors of a_{ij}, then value of Δ is given by

(A) $a_{11}A_{11} + a_{12}A_{32} + a_{13}A_{33}$

(B) $a_{11}A_{11} + a_{12}A_{21} + a_{13}A_{31}$

(C) $a_{21}A_{11} + a_{22}A_{12} + a_{23}A_{13}$

(D) $a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

5. The value of $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ is given by: $a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

Hence, the option (D) is correct.